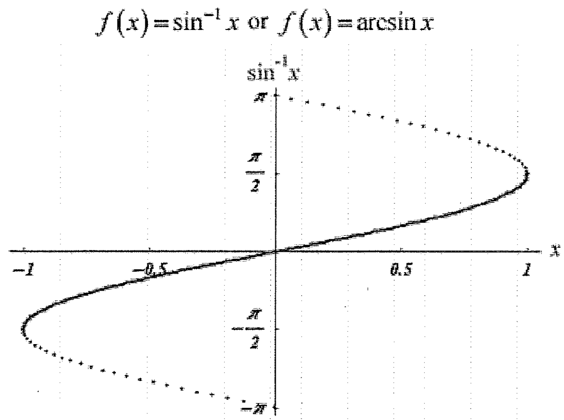
Calculus   Ch. 5.6 Notes   Derivatives of Inverse Trig Functions

None of the six trig functions is one-to-one, so to create inverse functions, their domains must be restricted to a convenient one-to-one interval covering all the ratio values.



Arcsinx and sinx are inverse functions. This means that arcsin(sinx) = x and sin(arcsinx) = x.

Since $f[f^{-1}(x)] = x$, then sin(arcsin x) = x   or $\sin[\sin^{-1} x] = x$

Inverse Trig Derivatives:

$$\frac{d}{dx}\arcsin u = \frac{u'}{\sqrt{1-u^2}} \qquad \frac{d}{dx}\arccos u = -\frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}\arctan u = \frac{u'}{1+u^2} \qquad \frac{d}{dx}arc\cot u = -\frac{u'}{1+u^2}$$

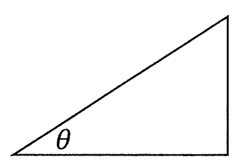
$$\frac{d}{dx}arc\sec u = \frac{u'}{|u|\sqrt{u^2-1}} \qquad \frac{d}{dx}arc\csc u = -\frac{u'}{|u|\sqrt{u^2-1}}$$

Ex. 1: Find $tan\left[arccos\left(\frac{\sqrt{2}}{2}\right)\right]$

This means "find the tangent of the angle whose cosine is $\frac{\sqrt{2}}{2}$"

Steps:

1. Draw the triangle and label angle θ
2. Label the sides according to arccos given
3. Use Pythagorean theorem to find the third side
4. Find tan θ



Ex. 2: Find $cos\left[arcsin\left(\frac{5}{13}\right)\right]$

Inverse Trig Derivatives:

$$\frac{d}{dx}\arcsin u = \frac{u'}{\sqrt{1-u^2}} \qquad \frac{d}{dx}\arccos u = -\frac{u'}{\sqrt{1-u^2}} \qquad \frac{d}{dx}arc\sec u = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx}\arctan u = \frac{u'}{1+u^2} \qquad \frac{d}{dx}arc\cot u = -\frac{u'}{1+u^2} \qquad \frac{d}{dx}arc\csc u = -\frac{u'}{|u|\sqrt{u^2-1}}$$

Ex. 3: Write the expression sec[arctan 3x] in algebraic form

Ex. 4: Find the derivative of y = arcsin x (using implicit method)

Steps:

1. Take the sine of both sides
2. Differentiate implicitly
3. Solve for dy/dx
4. Rewrite right side of the equation in terms of x

Ex. 5: Find y' for y = arctan(4x) (using implicit and derivative rule)

Ex. 6: Find y' for y = 2arcsec($3x^2$) (use derivative rules for ex.6 – ex.8)

Ex. 7: Find y' for y = xarccos(2x)

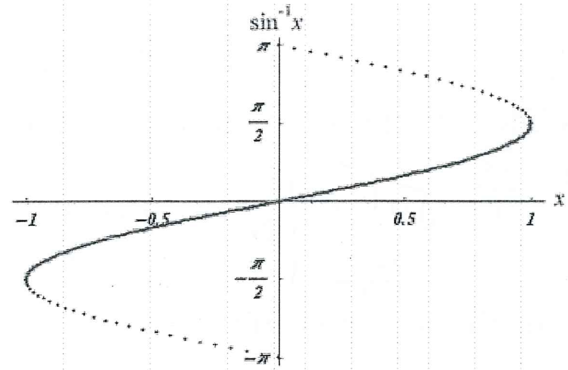
Ex. 8: Find y' for $cos(\arctan x)$

Calculus   Ch. 5.6 Notes   Derivatives of Inverse Trig Functions

Key

None of the six trig functions is one-to-one, so to create inverse functions, their domains must be restricted to a convenient one-to-one interval covering all the ratio values.

$f(x) = \sin^{-1} x$ or $f(x) = \arcsin x$



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$$\frac{d}{dx}\operatorname{arc}\sec u = \frac{u'}{|u|\sqrt{u^2-1}} \qquad \frac{d}{dx}\operatorname{arc}\csc u = -\frac{u'}{|u|\sqrt{u^2-1}}$$

Ex. 1: Find $tan\left[arccos\left(\frac{\sqrt{2}}{2}\right)\right]$ $\cos\theta = \frac{\sqrt{2}}{2}$

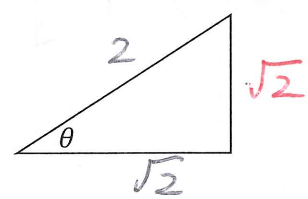
This means "find the tangent of the angle whose cosine is $\frac{\sqrt{2}}{2}$"

Steps:

1. Draw the triangle and label angle θ
2. Label the sides according to arccos given
3. Use Pythagorean theorem to find the third side
4. Find tan θ

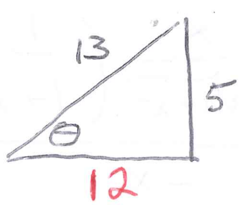
$\tan\left(\arccos\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{\sqrt{2}} = \boxed{1}$ $(\sqrt{2})^2 + y^2 = 2^2$

$y = \sqrt{2}$



Ex. 2: Find $cos\left[arcsin\left(\frac{5}{13}\right)\right]$ $\sin\theta = 5/13$

$\cos\left[\arcsin\left(\frac{5}{13}\right)\right] = \boxed{\frac{12}{13}}$



Inverse Trig Derivatives:

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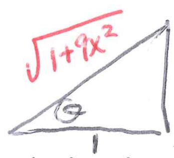
Ex. 3: Write the expression sec[arctan 3x] in algebraic form

$\tan\theta = \frac{3x}{1}$



$\sec[\arctan 3x] = \frac{\sqrt{1+9x^2}}{1} = \boxed{\sqrt{1+9x^2}}$

Ex. 4: Find the derivative of y = arcsin x

Steps:

1. Take the sine of both sides
2. Differentiate implicitly
3. Solve for dy/dx
4. Rewrite right side of the equation in terms of x

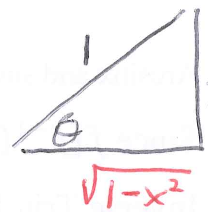
$\sin y = \sin[\arcsin x] \rightarrow \sin y = x$

$\cos y \cdot \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\cos y}$

$\frac{dy}{dx} = \sec y$

$\frac{dy}{dx} = \sec[\arcsin x]$



$\sec[\arcsin x] = \frac{1}{\sqrt{1-x^2}}$

$\boxed{\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}}$

Ex. 5: Find y' for y = arctan(4x)

$\tan y = \tan[\arctan(4x)]$

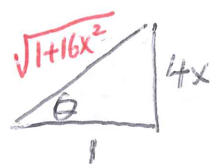
$\tan y = 4x$

$\sec^2 y\left(\frac{dy}{dx}\right) = 4$

$\frac{dy}{dx} = \frac{4}{\sec^2 y}$

$\frac{dy}{dx} = 4\cos^2 y$

$\frac{dy}{dx} = 4\cos^2[\arctan(4x)]$



$\frac{dy}{dx} = 4\left(\frac{1}{\sqrt{1+16x^2}}\right)^2$

$= \boxed{\frac{4}{1+16x^2}}$

using derivative rule

$\frac{dy}{dx} = \frac{u'}{1+u^2}$

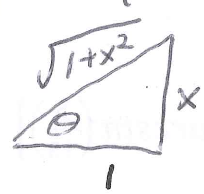
$= \frac{4}{1+(4x)^2} = \boxed{\frac{4}{1+16x^2}}$

Ex. 6: Find y' for y = 2arcsec($3x^2$)

$y' = 2\left[\frac{6x}{|3x^2|\sqrt{(3x^2)^2-1}}\right] = \boxed{\frac{4}{|x|\sqrt{9x^4-1}}}$

Ex. 8 $y = \cos(\arctan x)$ Find y'

$y' = -\sin(\arctan x) \cdot \frac{1}{1+x^2}$



$y' = -\frac{x}{\sqrt{1+x^2}}\left(\frac{1}{1+x^2}\right)$

$\boxed{y' = \frac{-x}{(1+x^2)^{3/2}}}$

Ex. 7: Find y' for y = xarccos(2x)

product Rule

$y' = 1(\arccos(2x)) + x\cdot\left(\frac{-2}{\sqrt{1-(2x)^2}}\right)$

$y' = \boxed{\arccos(2x) - \frac{2x}{\sqrt{1-4x^2}}}$