123. $\log_e\left(1+\frac{1}{x}\right) = \ln\left(1+\frac{1}{x}\right) = \int_x^{1+x}\frac{dt}{t}$

$$> \int_x^{1+x}\frac{dt}{1+x} \quad \begin{pmatrix}\text{because } 1+x \ge t\\ \text{on } x \le t \le 1+x\end{pmatrix}$$

$$= \left[\frac{t}{1+x}\right]_x^{1+x} = \frac{1+x}{1+x} - \frac{x}{1+x} = \frac{1}{1+x}$$

Note: You can confirm this result by graphing $y_1 = \ln\left(1+\frac{1}{x}\right)$ and $y_2 = \frac{1}{1+x}$.

Section 5.6 Inverse Trigonometric Functions: Differentiation

1. $y = \arccos x$

$\left(-\frac{\sqrt{2}}{2}, \frac{3\pi}{4}\right)$ because $\cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$\left(\frac{1}{2}, \frac{\pi}{3}\right)$ because $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

$\left(\frac{\sqrt{3}}{2}, \frac{\pi}{6}\right)$ because $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

2. $y = \arctan x$

$\left(1, \frac{\pi}{4}\right)$ because $\tan\left(\frac{\pi}{4}\right) = 1$

$\left(-\frac{\sqrt{3}}{3}, -\frac{\pi}{6}\right)$ because $\tan\left(-\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3}$

$\left(-\sqrt{3}, -\frac{\pi}{3}\right)$ because $\tan\left(-\frac{\pi}{3}\right) = -\sqrt{3}$

3. $\arcsin\frac{1}{2} = \frac{\pi}{6}$

4. $\arcsin 0 = 0$

5. $\arccos\frac{1}{2} = \frac{\pi}{3}$

6. $\arccos 1 = 0$

7. $\arctan\frac{\sqrt{3}}{3} = \frac{\pi}{6}$

8. $\text{arccot}\left(-\sqrt{3}\right) = \frac{5\pi}{6}$

9. $\text{arccsc}\left(-\sqrt{2}\right) = -\frac{\pi}{4}$

10. $\text{arcsec}\left(-\sqrt{2}\right) = \frac{3\pi}{4}$

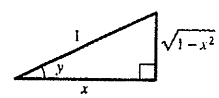
11. $\arccos(-0.8) \approx 2.50$

12. $\arcsin(-0.39) \approx -0.40$

13. $\text{arcsec}(1.269) = \arccos\left(\frac{1}{1.269}\right) \approx 0.66$

14. $\arctan(-5) \approx -1.37$

In Exercises 15–20, use the triangle.



15. $y = \arccos x$

$\cos y = x$

16. $\sin y = \sqrt{1-x^2}$

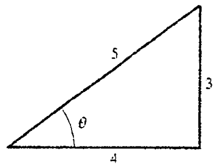
17. $\tan y = \frac{\sqrt{1-x^2}}{x}$

18. $\cot y = \frac{x}{\sqrt{1-x^2}}$

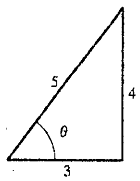
19. $\sec y = \frac{1}{x}$

20. $\csc y = \frac{1}{\sqrt{1-x^2}}$

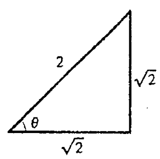
21. (a) $\sin\left(\arctan \frac{3}{4}\right) = \frac{3}{5}$



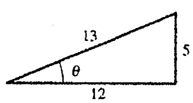
(b) $\sec\left(\arcsin \frac{4}{5}\right) = \frac{5}{3}$



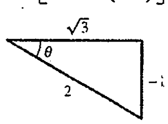
22. (a) $\tan\left(\arccos \frac{\sqrt{2}}{2}\right) = \tan\left(\frac{\pi}{4}\right) = 1$



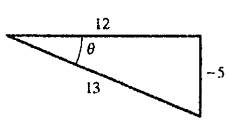
(b) $\cos\left(\arcsin \frac{5}{13}\right) = \frac{12}{13}$



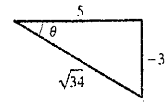
23. (a) $\cot\left[\arcsin\left(-\frac{1}{2}\right)\right] = \cot\left(-\frac{\pi}{6}\right) = -\sqrt{3}$



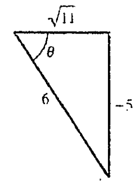
(b) $\csc\left[\arctan\left(-\frac{5}{12}\right)\right] = -\frac{13}{5}$



24. (a) $\sec\left[\arctan\left(-\frac{3}{5}\right)\right] = \frac{\sqrt{34}}{5}$



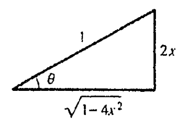
(b) $\tan\left[\arcsin\left(-\frac{5}{6}\right)\right] = -\frac{5\sqrt{11}}{11}$



25. $y = \cos(\arcsin 2x)$

$\theta = \arcsin 2x$

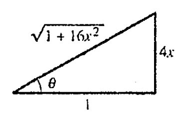
$y = \cos\theta = \sqrt{1 - 4x^2}$



26. $y = \sec(\arctan 4x)$

$\theta = \arctan 4x$

$y = \sec\theta = \sqrt{1 + 16x^2}$

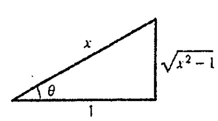


27. $y = \sin(\operatorname{arcsec} x)$

$\theta = \operatorname{arcsec} x,\ 0 \le \theta \le \pi, \theta \ne \frac{\pi}{2}$

$y = \sin\theta = \frac{\sqrt{x^2 - 1}}{|x|}$

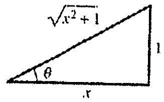
The absolute value bars on x are necessary because of the restriction $0 \le \theta \le \pi$, $\theta \ne \pi/2$, and $\sin\theta$ for this domain must always be nonnegative.



28. $y = \cos(\operatorname{arccot} x)$

$\theta = \operatorname{arccot} x$

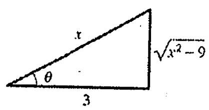
$y = \cos\theta = \dfrac{x}{\sqrt{x^2 + 1}}$



29. $y = \tan\left(\operatorname{arcsec}\dfrac{x}{3}\right)$

$\theta = \operatorname{arcsec}\dfrac{x}{3}$

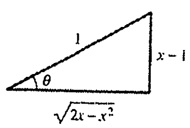
$y = \tan\theta = \dfrac{\sqrt{x^2 - 9}}{3}$



30. $y = \sec[\arcsin(x - 1)]$

$\theta = \arcsin(x - 1)$

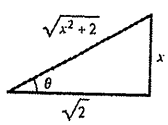
$y = \sec\theta = \dfrac{1}{\sqrt{2x - x^2}}$



31. $y = \csc\left(\arctan\dfrac{x}{\sqrt{2}}\right)$

$\theta = \arctan\dfrac{x}{\sqrt{2}}$

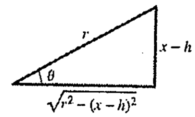
$y = \csc\theta = \dfrac{\sqrt{x^2 + 2}}{x}$



32. $y = \cos\left(\arcsin\dfrac{x - h}{r}\right)$

$\theta = \arcsin\dfrac{x - h}{r}$

$y = \cos\theta = \dfrac{\sqrt{r^2 - (x - h)^2}}{r}$



33. $\arcsin(3x - \pi) = \frac{1}{2}$

$3x - \pi = \sin\left(\frac{1}{2}\right)$

$x = \frac{1}{3}\left[\sin\left(\frac{1}{2}\right) + \pi\right] \approx 1.207$

34. $\arctan(2x - 5) = -1$

$2x - 5 = \tan(-1)$

$x = \frac{1}{2}(\tan(-1) + 5) \approx 1.721$

35. $\arcsin\sqrt{2x} = \arccos\sqrt{x}$

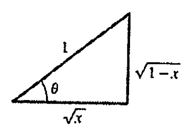
$\sqrt{2x} = \sin\left(\arccos\sqrt{x}\right)$

$\sqrt{2x} = \sqrt{1 - x}, \quad 0 \le x \le 1$

$2x = 1 - x$

$3x = 1$

$x = \frac{1}{3}$



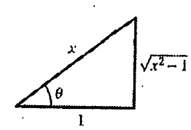
36. $\arccos x = \operatorname{arcsec} x$

$x = \cos(\operatorname{arcsec} x)$

$x = \dfrac{1}{x}$

$x^2 = 1$

$x = \pm 1$



37. (a) $\text{arccsc } x = \arcsin \frac{1}{x}, \quad |x| \ge 1$

Let $y = \text{arccsc } x$. Then for

$-\frac{\pi}{2} \le y < 0$ and $0 < y \le \frac{\pi}{2}$,

$\csc y = x \Rightarrow \sin y = 1/x$. So, $y = \arcsin(1/x)$.

Therefore, $\text{arccsc } x = \arcsin(1/x)$.

(b) $\arctan x + \arctan \frac{1}{x} = \frac{\pi}{2}, \quad x > 0$

Let $y = \arctan x + \arctan(1/x)$. Then,

$$\tan y = \frac{\tan(\arctan x) + \tan[\arctan(1/x)]}{1 - \tan(\arctan x)\tan[\arctan(1/x)]}$$

$$= \frac{x + (1/x)}{1 - x(1/x)}$$

$$= \frac{x + (1/x)}{0} \text{ (which is undefined).}$$

So, $y = \pi/2$. Therefore,

$\arctan x + \arctan(1/x) = \pi/2$.

38. (a) $\arcsin(-x) = -\arcsin x, \quad |x| \le 1$

Let $y = \arcsin(-x)$. Then,

$-x = \sin y \Rightarrow x = -\sin y \Rightarrow x = \sin(-y)$.

So, $-y = \arcsin x \Rightarrow y = -\arcsin x$. Therefore,

$\arcsin(-x) = -\arcsin x$.

(b) $\arccos(-x) = \pi - \arccos x, \quad |x| \le 1$

Let $y = \arccos(-x)$. Then,

$-x = \cos y \Rightarrow x = -\cos y \Rightarrow x = \cos(\pi - y)$.

So, $\pi - y = \arccos x \Rightarrow y = \pi - \arccos x$.

Therefore, $\arccos(-x) = \pi - \arccos x$.

39. $f(x) = 2\arcsin(x - 1)$

$$f'(x) = \frac{2}{\sqrt{1 - (x-1)^2}} = \frac{2}{\sqrt{2x - x^2}}$$

40. $f(t) = \arcsin t^2$

$$f'(t) = \frac{2t}{\sqrt{1 - t^4}}$$

41. $g(x) = 3\arccos \frac{x}{2}$

$$g'(x) = \frac{-3(1/2)}{\sqrt{1 - (x^2/4)}} = \frac{-3}{\sqrt{4 - x^2}}$$

42. $f(x) = \text{arcsec } 2x$

$$f'(x) = \frac{2}{|2x|\sqrt{4x^2 - 1}} = \frac{1}{|x|\sqrt{4x^2 - 1}}$$

43. $f(x) = \arctan(e^x)$

$$f'(x) = \frac{1}{1 + (e^x)^2}e^x = \frac{e^x}{1 + e^{2x}}$$

44. $f(x) = \arctan \sqrt{x}$

$$f'(x) = \left(\frac{1}{1 + x}\right)\left(\frac{1}{2\sqrt{x}}\right) = \frac{1}{2\sqrt{x}(1 + x)}$$

45. $g(x) = \frac{\arcsin 3x}{x}$

$$g'(x) = \frac{x\left(3/\sqrt{1 - 9x^2}\right) - \arcsin 3x}{x^2}$$

$$= \frac{3x - \sqrt{1 - 9x^2}\arcsin 3x}{x^2\sqrt{1 - 9x^2}}$$

46. $h(x) = x^2 \arctan(5x)$

$$h'(x) = 2x\arctan(5x) + x^2\frac{1}{1 + (5x)^2}(5)$$

$$= 2x\arctan(5x) + \frac{5x^2}{1 + 25x^2}$$

47. $h(t) = \sin(\arccos t) = \sqrt{1 - t^2}$

$$h'(t) = \frac{1}{2}(1 - t^2)^{-1/2}(-2t)$$

$$= \frac{-t}{\sqrt{1 - t^2}}$$

48. $f(x) = \arcsin x + \arccos x = \frac{\pi}{2}$

$f'(x) = 0$

49. $y = 2x\arccos x - 2\sqrt{1 - x^2}$

$$y' = 2\arccos x - 2x\frac{1}{\sqrt{1 - x^2}} - 2\left(\frac{1}{2}\right)(1 - x^2)^{-1/2}(-2x)$$

$$= 2\arccos x - \frac{2x}{\sqrt{1 - x^2}} + \frac{2x}{\sqrt{1 - x^2}} = 2\arccos x$$

50. $y = \ln(t^2 + 4) - \frac{1}{2}\arctan\frac{t}{2}$

$$y' = \frac{2t}{t^2 + 4} - \frac{1}{2}\cdot\frac{1}{1 + (t/2)^2}\left(\frac{1}{2}\right)$$

$$= \frac{2t}{t^2 + 4} - \frac{1}{t^2 + 4} = \frac{2t - 1}{t^2 + 4}$$

51. $y = \frac{1}{2}\left(\frac{1}{2}\ln\frac{x+1}{x-1} + \arctan x\right) = \frac{1}{4}\left[\ln(x+1) - \ln(x-1)\right] + \frac{1}{2}\arctan x$

$$\frac{dy}{dx} = \frac{1}{4}\left(\frac{1}{x+1} - \frac{1}{x-1}\right) + \frac{1/2}{1+x^2} = \frac{1}{1-x^4}$$

52. $y = \frac{1}{2}\left[x\sqrt{4-x^2} + 4\arcsin\left(\frac{x}{2}\right)\right]$

$$y' = \frac{1}{2}\left[x\frac{1}{2}(4-x^2)^{-1/2}(-2x) + \sqrt{4-x^2} + 2\frac{1}{\sqrt{1-(x/2)^2}}\right] = \frac{1}{2}\left[\frac{-x^2}{\sqrt{4-x^2}} + \sqrt{4-x^2} + \frac{4}{\sqrt{4-x^2}}\right] = \sqrt{4-x^2}$$

53. $y = x\arcsin x + \sqrt{1-x^2}$

$$\frac{dy}{dx} = x\left(\frac{1}{\sqrt{1-x^2}}\right) + \arcsin x - \frac{x}{\sqrt{1-x^2}} = \arcsin x$$

54. $y = x\arctan 2x - \frac{1}{4}\ln(1+4x^2)$

$$\frac{dy}{dx} = \frac{2x}{1+4x^2} + \arctan(2x) - \frac{1}{4}\left(\frac{8x}{1+4x^2}\right) = \arctan(2x)$$

55. $y = 8\arcsin\frac{x}{4} - \frac{x\sqrt{16-x^2}}{2}$

$$y' = 2\frac{1}{\sqrt{1-(x/4)^2}} - \frac{\sqrt{16-x^2}}{2} - \frac{x}{4}(16-x^2)^{-1/2}(-2x)$$

$$= \frac{8}{\sqrt{16-x^2}} - \frac{\sqrt{16-x^2}}{2} + \frac{x^2}{2\sqrt{16-x^2}} = \frac{16-(16-x^2)+x^2}{2\sqrt{16-x^2}} = \frac{x^2}{\sqrt{16-x^2}}$$

56. $y = 25\arcsin\frac{x}{5} - x\sqrt{25-x^2}$

$$y' = 5\frac{1}{\sqrt{1-(x/2)^2}} - \sqrt{25-x^2} - x\frac{1}{2}(25-x^2)^{-1/2}(-2x)$$

$$= \frac{25}{\sqrt{25-x^2}} - \frac{(25-x^2)}{\sqrt{25-x^2}} + \frac{x^2}{\sqrt{25-x^2}} = \frac{2x^2}{\sqrt{25-x^2}}$$

57. $y = \arctan x + \frac{x}{1+x^2}$

$$y' = \frac{1}{1+x^2} + \frac{(1+x^2)-x(2x)}{(1+x^2)^2}$$

$$= \frac{(1+x^2)+(1-x^2)}{(1+x^2)^2}$$

$$= \frac{2}{(1+x^2)^2}$$

58. $y = \arctan\frac{x}{2} - \frac{1}{2(x^2+4)}$

$$y' = \frac{1}{2}\frac{1}{1+(x/2)^2} + \frac{1}{2}(x^2+4)^{-2}(2x)$$

$$= \frac{2}{x^2+4} + \frac{x}{(x^2+4)^2}$$

$$= \frac{2x^2+8+x}{(x^2+4)^2}$$

59. $y = 2 \arcsin x, \quad \left(\frac{1}{2}, \frac{\pi}{3}\right)$

$$y' = \frac{2}{\sqrt{1 - x^2}}$$

At $\left(\frac{1}{2}, \frac{\pi}{3}\right)$, $y' = \frac{2}{\sqrt{1 - (1/4)}} = \frac{4}{\sqrt{3}}$.

Tangent line: $y - \frac{\pi}{3} = \frac{4}{\sqrt{3}}\left(x - \frac{1}{2}\right)$

$$y = \frac{4}{\sqrt{3}}x + \frac{\pi}{3} - \frac{2}{\sqrt{3}}$$

$$y = \frac{4\sqrt{3}}{3}x + \frac{\pi}{3} - \frac{2\sqrt{3}}{3}$$

60. $y = \frac{1}{2} \arccos x, \quad \left(-\frac{\sqrt{2}}{2}, \frac{3\pi}{8}\right)$

$$y' = \frac{-1}{2\sqrt{1 - x^2}}$$

At $\left(-\frac{\sqrt{2}}{2}, \frac{3\pi}{8}\right)$, $y' = \frac{-1}{2\sqrt{1/2}} = -\frac{\sqrt{2}}{2}$.

Tangent line: $y - \frac{3\pi}{8} = -\frac{\sqrt{2}}{2}\left(x + \frac{\sqrt{2}}{2}\right)$

$$y = -\frac{\sqrt{2}}{2}x + \frac{3\pi}{8} - \frac{1}{2}$$

61. $y = \arctan\left(\frac{x}{2}\right), \quad \left(2, \frac{\pi}{4}\right)$

$$y' = \frac{1}{1 + (x^2/4)}\left(\frac{1}{2}\right) = \frac{2}{4 + x^2}$$

At $\left(2, \frac{\pi}{4}\right)$, $y' = \frac{2}{4 + 4} = \frac{1}{4}$.

Tangent line: $y - \frac{\pi}{4} = \frac{1}{4}(x - 2)$

$$y = \frac{1}{4}x + \frac{\pi}{4} - \frac{1}{2}$$

62. $y = \operatorname{arcsec}(4x), \quad \left(\frac{\sqrt{2}}{4}, \frac{\pi}{4}\right)$

$$y' = \frac{4}{|4x|\sqrt{16x^2 - 1}} = \frac{1}{x\sqrt{16x^2 - 1}} \text{ for } x > 0$$

At $\left(\frac{\sqrt{2}}{4}, \frac{\pi}{4}\right)$, $y' = \frac{1}{(\sqrt{2}/4)\sqrt{2 - 1}} = 2\sqrt{2}$.

Tangent line: $y - \frac{\pi}{4} = 2\sqrt{2}\left(x - \frac{\sqrt{2}}{4}\right)$

$$y = 2\sqrt{2}x + \frac{\pi}{4} - 1$$

63. $y = 4x \arccos(x - 1), \quad (1, 2\pi)$

$$y' = 4x\frac{-1}{\sqrt{1 - (x - 1)^2}} + 4 \arccos(x - 1)$$

At $(1, 2\pi)$, $y' = -4 + 2\pi$.

Tangent line: $y - 2\pi = (2\pi - 4)(x - 1)$

$$y = (2\pi - 4)x + 4$$

64. $y = 3x \arcsin x, \quad \left(\frac{1}{2}, \frac{\pi}{4}\right)$

$$y' = 3x\frac{1}{\sqrt{1 - x^2}} + 3 \arcsin x$$

At $\left(\frac{1}{2}, \frac{\pi}{4}\right)$, $y' = \frac{3}{2}\frac{1}{\sqrt{3/4}} + 3\left(\frac{\pi}{6}\right) = \sqrt{3} + \frac{\pi}{2}$.

Tangent line: $y - \frac{\pi}{4} = \left(\sqrt{3} + \frac{\pi}{2}\right)\left(x - \frac{1}{2}\right)$

$$y = \left(\sqrt{3} + \frac{\pi}{2}\right)x - \frac{\sqrt{3}}{2}$$

65. $f(x) = \arctan x, \quad a = 0$

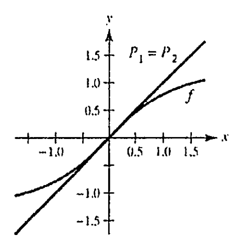
$f(0) = 0$

$f'(x) = \frac{1}{1 + x^2}, \quad f'(0) = 1$

$f''(x) = \frac{-2x}{(1 + x^2)^2}, f''(0) = 0$

$P_1(x) = f(0) + f'(0)x = x$

$P_2(x) = f(0) + f'(0)x + \frac{1}{2}f''(0)x^2 = x$



66. $f(x) = \arccos x, \quad a = 0$

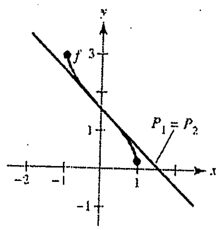
$$f(0) = \frac{\pi}{2}$$

$$f'(x) = \frac{-1}{\sqrt{1-x^2}}, \quad f'(0) = -1$$

$$f''(x) = \frac{-x}{\left(1-x^2\right)^{3/2}}, \quad f''(0) = 0$$

$$P_1(x) = f(0) + f'(0)x = \frac{\pi}{2} - x$$

$$P_2(x) = f(0) + f'(0)x + \frac{1}{2}f''(0)x^2 = \frac{\pi}{2} - x$$



67. $f(x) = \arcsin x, \quad a = \frac{1}{2}$

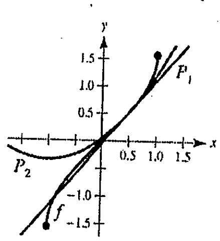
$$f'(x) = \frac{1}{\sqrt{1-x^2}}$$

$$f''(x) = \frac{x}{\left(1-x^2\right)^{3/2}}$$

$$P_1(x) = f\left(\frac{1}{2}\right) + f'\left(\frac{1}{2}\right)\left(x - \frac{1}{2}\right) = \frac{\pi}{6} + \frac{2\sqrt{3}}{3}\left(x - \frac{1}{2}\right)$$

$$P_2(x) = f\left(\frac{1}{2}\right) + f'\left(\frac{1}{2}\right)\left(x - \frac{1}{2}\right) + \frac{1}{2}f''\left(\frac{1}{2}\right)\left(x - \frac{1}{2}\right)^2$$

$$= \frac{\pi}{6} + \frac{2\sqrt{3}}{3}\left(x - \frac{1}{2}\right) + \frac{2\sqrt{3}}{9}\left(x - \frac{1}{2}\right)^2$$



68. $f(x) = \arctan x, \quad a = 1$

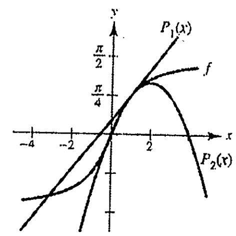
$$f'(x) = \frac{1}{1+x^2}$$

$$f''(x) = \frac{-2x}{\left(1+x^2\right)^2}$$

$$P_1(x) = f(1) + f'(1)(x-1) = \frac{\pi}{4} + \frac{1}{2}(x-1)$$

$$P_2(x) = f(1) + f'(1)(x-1) + \frac{1}{2}f''(1)(x-1)^2$$

$$= \frac{\pi}{4} + \frac{1}{2}(x-1) - \frac{1}{4}(x-1)^2$$



69. $f(x) = \operatorname{arcsec} x - x$

$$f'(x) = \frac{1}{|x|\sqrt{x^2-1}} - 1 = 0 \text{ when } |x|\sqrt{x^2-1} = 1$$

$$x^2\left(x^2-1\right) = 1$$

$$x^4 - x^2 - 1 = 0 \text{ when } x^2 = \frac{1+\sqrt{5}}{2} \text{ or}$$

$$x = \pm\sqrt{\frac{1+\sqrt{5}}{2}} = \pm 1.272$$

Relative maximum: $(1.272, -0.606)$

Relative minimum: $(-1.272, 3.747)$

70. $f(x) = \arcsin x - 2x$

$$f'(x) = \frac{1}{\sqrt{1-x^2}} - 2$$

$$= 0 \text{ when } \sqrt{1-x^2} = \frac{1}{2} \text{ or } x = \pm\frac{\sqrt{3}}{2}$$

$$f''(x) = \frac{x}{\left(1-x^2\right)^{3/2}}$$

$$f''\left(\frac{\sqrt{3}}{2}\right) > 0$$

Relative minimum: $\left(\frac{\sqrt{3}}{2}, -0.68\right)$

$$f''\left(-\frac{\sqrt{3}}{2}\right) < 0$$

Relative maximum: $\left(-\frac{\sqrt{3}}{2}, 0.68\right)$

71. $f(x) = \arctan x - \arctan(x - 4)$

$$f'(x) = \frac{1}{1 + x^2} - \frac{1}{1 + (x - 4)^2} = 0$$

$$1 + x^2 = 1 + (x - 4)^2$$

$$0 = -8x + 16$$

$$x = 2$$

By the First Derivative Test, $(2, 2.214)$ is a relative maximum.

72. $f(x) = \arcsin x - 2 \arctan x$

$$f'(x) = \frac{1}{\sqrt{1 - x^2}} - \frac{2}{1 + x^2} = 0$$

$$1 + x^2 = 2\sqrt{1 - x^2}$$

$$1 + 2x^2 + x^4 = 4(1 - x^2)$$

$$x^4 + 6x^2 - 3 = 0$$

$$x = \pm 0.681$$

By the First Derivative Test, $(-0.681, 0.447)$ is a relative maximum and $(0.681, -0.447)$ is a relative minimum.

73. $f(x) = \arcsin(x - 1)$

$$f'(x) = \frac{1}{\sqrt{1 - (x - 1)^2}} = \frac{1}{\sqrt{2x - x^2}}$$

$$f''(x) = \frac{x - 1}{(2x - x^2)^{3/2}}$$

Maximum: $\left(2, \frac{\pi}{2}\right)$

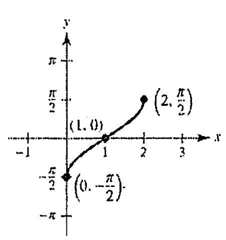
Minimum: $\left(0, -\frac{\pi}{2}\right)$

Point of inflection: $(1, 0)$

Domain: $[0, 2]$

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

The graph of f is $y = \arcsin x$ shifted 1 unit to the right.



74. $f(x) = \arctan x + \frac{\pi}{2}$

$$f'(x) = \frac{1}{1 + x^2}$$

$$f''(x) = \frac{-2x}{(1 + x^2)^2}$$

Increasing on $(-\infty, \infty)$

No relative extrema

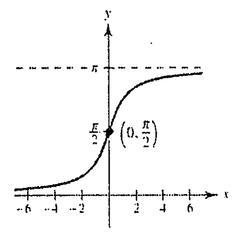
Point of inflection: $\left(0, \frac{\pi}{2}\right)$

Horizontal asymptotes: $y = 0$ and $y = \pi$

Domain: $(-\infty, \infty)$

Range: $(0, \pi)$

f is $\arctan x$ shifted $\frac{\pi}{2}$ units upward.



75. $f(x) = \operatorname{arcsec} 2x$

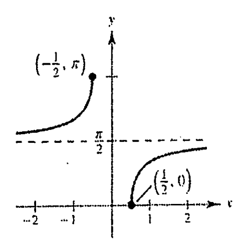
$$f'(x) = \frac{1}{|x|\sqrt{4x^2 - 1}}$$

Domain: $\left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$

Range: $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

Maximum: $\left(-\frac{1}{2}, \pi\right)$ Minimum: $\left(\frac{1}{2}, 0\right)$

Horizontal asymptote: $y = \frac{\pi}{2}$



76. $f(x) = \arccos \dfrac{x}{4}$

$$f'(x) = \frac{-1}{\sqrt{16 - x^2}} < 0$$

$$f''(x) = \frac{-x}{\left(16 - x^2\right)^{3/2}}$$

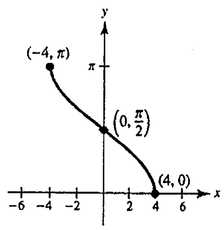
Domain: $[-4, 4]$

Range: $[0, \pi]$

Maximum: $(-4, \pi)$

Minimum: $(4, 0)$

Point of Inflection: $(0, \pi/2)$



77. $x^2 + x \arctan y = y - 1, \quad \left(-\dfrac{\pi}{4}, 1\right)$

$$2x + \arctan y + \frac{x}{1 + y^2} y' = y'$$

$$\left(1 - \frac{x}{1 + y^2}\right) y' = 2x + \arctan y$$

$$y' = \frac{2x + \arctan y}{1 - \dfrac{x}{1 + y^2}}$$

At $\left(-\dfrac{\pi}{4}, 1\right)$: $y' = \dfrac{-\dfrac{\pi}{2} + \dfrac{\pi}{4}}{1 - \dfrac{-\pi/4}{2}} = \dfrac{-\dfrac{\pi}{2}}{2 + \dfrac{\pi}{4}} = \dfrac{-2\pi}{8 + \pi}$

Tangent line: $y - 1 = \dfrac{-2\pi}{8 + \pi}\left(x + \dfrac{\pi}{4}\right)$

$$y = \frac{-2\pi}{8 + \pi} x + 1 - \frac{\pi^2}{16 + 2\pi}$$

78. $\arctan(xy) = \arcsin(x + y), \quad (0, 0)$

$$\frac{1}{1 + (xy)^2}[y + xy'] = \frac{1}{\sqrt{1 - (x + y)^2}}[1 + y']$$

At $(0, 0)$: $0 = 1 + y' \Rightarrow y' = -1$

Tangent line: $y = -x$

79. $\arcsin x + \arcsin y = \dfrac{\pi}{2}, \quad \left(\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$

$$\frac{1}{\sqrt{1 - x^2}} + \frac{1}{\sqrt{1 - y^2}} y' = 0$$

$$\frac{1}{\sqrt{1 - y^2}} y' = \frac{-1}{\sqrt{1 - x^2}}$$

At $\left(\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$: $y' = -1$

Tangent line: $y - \dfrac{\sqrt{2}}{2} = -1\left(x - \dfrac{\sqrt{2}}{2}\right)$

$$y = -x + \sqrt{2}$$

80. $\arctan(x + y) = y^2 + \dfrac{\pi}{4}, \quad (1, 0)$

$$\frac{1}{1 + (x + y)^2}[1 + y'] = 2yy'$$

At $(1, 0)$: $\dfrac{1}{2}[1 + y'] = 0 \Rightarrow y' = -1$

Tangent line: $y - 0 = -1(x - 1)$

$$y = -x + 1$$

81. The trigonometric functions are not one-to-one on $(-\infty, \infty)$, so their domains must be restricted to intervals on which they are one-to-one.

82. $\arctan 0 = 0$. π is not in the range of $y = \arctan x$.

83. (a) $\arcsin(\arcsin(0.5)) \approx 0.551$

$\arcsin(\arcsin(1.0))$ does not exist

(b) In order for $f(x) = \arcsin(\arcsin x)$ to be real, you must have $-1 \le \arcsin x \le 1$.

Because $\arcsin x = 1 \Rightarrow \sin(1) = x$ and $\arcsin x = -1 \Rightarrow \sin(-1) = -\sin 1 = x$, you have

$$-\sin(1) \le x \le \sin(1)$$
$$-0.84147 \le x \le 0.84147$$

84. (a) $\left(-\frac{\sqrt{2}}{2}, -\frac{\pi}{4}\right)$ and $(0, 0)$ lie on the graph of

$y = \arcsin x$ because $\sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$ and

$\sin(0) = 0$, and 0 and $-\frac{\pi}{4}$ lie in the internal $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$\left(\frac{\sqrt{3}}{2}, \frac{2\pi}{3}\right)$ does not lie on the graph of

$y = \arcsin x$ because $\frac{2\pi}{3}$ is not in the interval

$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

(b) $\left(-\frac{1}{2}, \frac{2\pi}{3}\right)$ and $\left(0, \frac{\pi}{2}\right)$ lie on the graph of

$y = \arccos x$ because both $\frac{2\pi}{3}$ and $\frac{\pi}{2}$ lie in the

interval $[0, \pi]$. $\left(\frac{1}{2}, -\frac{\pi}{3}\right)$ does not lie on the graph

of $y = \arccos x$ because $-\frac{\pi}{3}$ is not in the interval

$[0, \pi]$.

85. False

$\arccos \frac{1}{2} = \frac{\pi}{3}$

because the range is $[0, \pi]$.

86. False

$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$, so $\arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$.

87. True

$\frac{d}{dx}[\arctan x] = \frac{1}{1 + x^2} > 0$ for all x.

88. False

The range of $y = \arcsin x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

89. True

$\frac{d}{dx}[\arctan(\tan x)] = \frac{\sec^2 x}{1 + \tan^2 x} = \frac{\sec^2 x}{\sec^2 x} = 1$

90. False

$\arcsin^2 0 + \arccos^2 0 = 0 + \left(\frac{\pi}{2}\right)^2 \neq 1$

91. (a) $\cot \theta = \frac{x}{5}$

$\theta = \operatorname{arccot}\left(\frac{x}{5}\right)$

(b) $\frac{d\theta}{dt} = \frac{-1/5}{1 + (x/5)^2} \frac{dx}{dt} = \frac{-5}{x^2 + 25} \frac{dx}{dt}$

If $\frac{dx}{dt} = -400$ and $x = 10$, $\frac{d\theta}{dt} = 16$ rad/h.

If $\frac{dx}{dt} = -400$ and $x = 3$, $\frac{d\theta}{dt} \approx 58.824$ rad/h.

92. (a) $\cot \theta = \frac{x}{3}$

$\theta = \operatorname{arccot}\left(\frac{x}{3}\right)$

(b) $\frac{d\theta}{dt} = \frac{-3}{x^2 + 9} \frac{dx}{dt}$

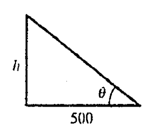
If $x = 10$, $\frac{d\theta}{dt} \approx 11.001$ rad/h.

If $x = 3$, $\frac{d\theta}{dt} \approx 66.667$ rad/h.

A lower altitude results in a greater rate of change of θ.

93. (a) $h(t) = -16t^2 + 256$

$-16t^2 + 256 = 0$ when $t = 4$ sec



(b) $\tan \theta = \frac{h}{500} = \frac{-16t^2 + 256}{500}$

$\theta = \arctan\left[\frac{16}{500}(-t^2 + 16)\right]$

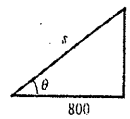
$\frac{d\theta}{dt} = \frac{-8t/125}{1 + \left[(4/125)(-t^2 + 16)\right]^2}$

$= \frac{-1000t}{15{,}625 + 16(16 - t^2)^2}$

When $t = 1$, $d\theta/dt \approx -0.0520$ rad/sec.

When $t = 2$, $d\theta/dt \approx -0.1116$ rad/sec.

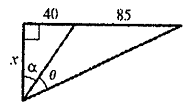
94. $\cos\theta = \dfrac{800}{s}$



$$\theta = \arccos\left(\frac{800}{s}\right)$$

$$\frac{d\theta}{dt} = \frac{d\theta}{ds}\cdot\frac{ds}{dt} = \frac{-1}{\sqrt{1-(800/s)^2}}\left(\frac{-800}{s^2}\right)\frac{ds}{dt} = \frac{800}{s\sqrt{s^2-800^2}}\frac{ds}{dt}, \qquad s > 800$$

95. $\tan\alpha = \dfrac{40}{x}$



$$\tan(\alpha+\theta) = \frac{40+85}{x} = \frac{125}{x}$$

$$\tan(\alpha+\theta) = \frac{\tan\alpha + \tan\theta}{1-\tan\alpha\tan\theta}$$

$$\frac{125}{x} = \frac{40/x + \tan\theta}{1-\frac{40}{x}\tan\theta} = \frac{40 + x\tan\theta}{x - 40\tan\theta}$$

$$125(x - 40\tan\theta) = x(40 + x\tan\theta)$$

$$85x = (x^2 + 5000)\tan\theta$$

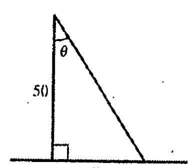
$$\theta = \arctan\left(\frac{85x}{x^2+5000}\right)$$

$$\frac{d\theta}{dx} = \frac{85(5000-x^2)}{(x^2+1600)(x^2+15625)} = 0 \Rightarrow x = \sqrt{5000} = 50\sqrt{2}$$

By the First Derivative Test, this is a maximum $x = 50\sqrt{2} \approx 70.71$ ft.

96. $\dfrac{d\theta}{dt} = 30(2\pi) = 60\pi$ rad/min

$$\tan\theta = \frac{x}{50}$$



$$\theta = \arctan\left(\frac{x}{50}\right)$$

$$\frac{d\theta}{dt} = \frac{d\theta}{dx}\frac{dx}{dt} = \frac{50}{x^2+2500}\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{x^2+2500}{50}\frac{d\theta}{dt}$$

When $\theta = 45° = \dfrac{\pi}{4}, x = 50$:

$$\frac{dx}{dt} = \frac{(50)^2+2500}{50}(60\pi) = 6000\pi \text{ ft/min}$$

97. (a) $\tan(\arctan x + \arctan y) = \dfrac{\tan(\arctan x) + \tan(\arctan y)}{1 - \tan(\arctan x)\tan(\arctan y)} = \dfrac{x+y}{1-xy}, \qquad xy \neq 1$

Therefore, $\arctan x + \arctan y = \arctan\left(\dfrac{x+y}{1-xy}\right), xy \neq 1.$

(b) Let $x = \dfrac{1}{2}$ and $y = \dfrac{1}{3}$.

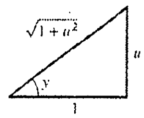
$$\arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right) = \arctan\frac{(1/2)+(1/3)}{1-[(1/2)\cdot(1/3)]} = \arctan\frac{5/6}{1-(1/6)} = \arctan\frac{5/6}{5/6} = \arctan 1 = \frac{\pi}{4}$$

98. (a) Let $y = \arctan u$. Then

$$\tan y = u$$

$$\sec^2 y \frac{dy}{dx} = u'$$

$$\frac{dy}{dx} = \frac{u'}{\sec^2 y} = \frac{u'}{1 + u^2}.$$

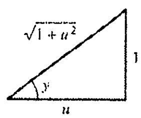


(b) Let $y = \operatorname{arccot} u$. Then

$$\cot y = u$$

$$-\csc^2 y \frac{dy}{dx} = u'$$

$$\frac{dy}{dx} = \frac{u'}{-\csc^2 y} = -\frac{u'}{1 + u^2}.$$

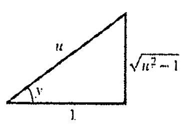


(c) Let $y = \operatorname{arcsec} u$. Then

$$\sec y = u$$

$$\sec y \tan y \frac{dy}{dx} = u'$$

$$\frac{dy}{dx} = \frac{u'}{\sec y \tan y} = \frac{u'}{|u|\sqrt{u^2 - 1}}.$$



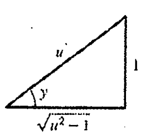
Note: The absolute value notation in the formula for the derivative of arcsec u is necessary because the inverse secant function has a positive slope at every value in its domain.

(d) Let $y = \operatorname{arccsc} u$. Then

$$\csc y = u$$

$$-\csc y \cot y \frac{dy}{dx} = u'$$

$$\frac{dy}{dx} = \frac{u'}{-\csc y \cot y} = -\frac{u'}{|u|\sqrt{u^2 - 1}}$$



Note: The absolute value notation in the formula for the derivative of arccsc u is necessary because the inverse cosecant function has a negative slope at every value in its domain.