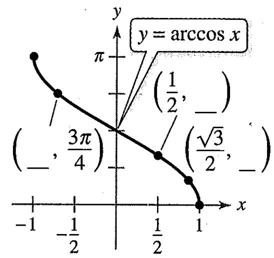
5.6 Exercises

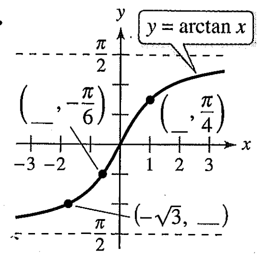
See **CalcChat.com** for tutorial help and worked-out solutions to odd-numbered exercises.

Finding Coordinates In Exercises 1 and 2, determine the missing coordinates of the points on the graph of the function.

1.



2.



Evaluating Inverse Trigonometric Functions In Exercises 3–10, evaluate the expression without using a calculator.

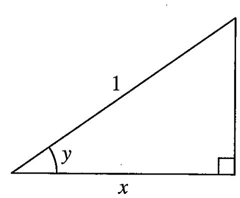
3. $\arcsin \frac{1}{2}$
4. $\arcsin 0$
5. $\arccos \frac{1}{2}$
6. $\arccos 1$
7. $\arctan \frac{\sqrt{3}}{3}$
8. $\operatorname{arccot}(-\sqrt{3})$
9. $\operatorname{arccsc}(-\sqrt{2})$
10. $\operatorname{arcsec}(-\sqrt{2})$

Approximating Inverse Trigonometric Functions In Exercises 11–14, use a calculator to approximate the value. Round your answer to two decimal places.

11. $\arccos(-0.8)$
12. $\arcsin(-0.39)$
13. $\operatorname{arcsec} 1.269$
14. $\arctan(-5)$

Using a Right Triangle In Exercises 15–20, use the figure to write the expression in algebraic form given $y = \arccos x$, where $0 < y < \pi/2$.

15. $\cos y$
16. $\sin y$
17. $\tan y$
18. $\cot y$
19. $\sec y$
20. $\csc y$



Evaluating an Expression In Exercises 21–24, evaluate each expression without using a calculator. (*Hint:* See Example 3.)

21. (a) $\sin\left(\arctan \frac{3}{4}\right)$ (b) $\sec\left(\arcsin \frac{4}{5}\right)$
22. (a) $\tan\left(\arccos \frac{\sqrt{2}}{2}\right)$ (b) $\cos\left(\arcsin \frac{5}{13}\right)$
23. (a) $\cot\left[\arcsin\left(-\frac{1}{2}\right)\right]$ (b) $\csc\left[\arctan\left(-\frac{5}{12}\right)\right]$
24. (a) $\sec\left[\arctan\left(-\frac{3}{5}\right)\right]$ (b) $\tan\left[\arcsin\left(-\frac{5}{6}\right)\right]$

Simplifying an Expression Using a Right Triangle In Exercises 25–32, write the expression in algebraic form. (*Hint:* Sketch a right triangle, as demonstrated in Example 3.)

25. $\cos(\arcsin 2x)$
26. $\sec(\arctan 4x)$
27. $\sin(\operatorname{arcsec} x)$
28. $\cos(\operatorname{arccot} x)$
29. $\tan\left(\operatorname{arcsec} \frac{x}{3}\right)$
30. $\sec[\arcsin(x - 1)]$
31. $\csc\left(\arctan \frac{x}{\sqrt{2}}\right)$
32. $\cos\left(\arcsin \frac{x - h}{r}\right)$

Solving an Equation In Exercises 33–36, solve the equation for x.

33. $\arcsin(3x - \pi) = \frac{1}{2}$
34. $\arctan(2x - 5) = -1$
35. $\arcsin\sqrt{2x} = \arccos\sqrt{x}$
36. $\arccos x = \operatorname{arcsec} x$

Verifying Identities In Exercises 37 and 38, verify each identity.

37. (a) $\operatorname{arccsc} x = \arcsin \frac{1}{x}, \quad x \geq 1$
(b) $\arctan x + \arctan \frac{1}{x} = \frac{\pi}{2}, \quad x > 0$
38. (a) $\arcsin(-x) = -\arcsin x, \quad |x| \leq 1$
(b) $\arccos(-x) = \pi - \arccos x, \quad |x| \leq 1$

Finding a Derivative In Exercises 39–58, find the derivative of the function.

39. $f(x) = 2\arcsin(x - 1)$
40. $f(t) = \arcsin t^2$
41. $g(x) = 3\arccos \frac{x}{2}$
42. $f(x) = \operatorname{arcsec} 2x$
43. $f(x) = \arctan e^x$
44. $f(x) = \arctan\sqrt{x}$
45. $g(x) = \frac{\arcsin 3x}{x}$
46. $h(x) = x^2 \arctan 5x$
47. $h(t) = \sin(\arccos t)$
48. $f(x) = \arcsin x + \arccos x$
49. $y = 2x \arccos x - 2\sqrt{1 - x^2}$
50. $y = \ln(t^2 + 4) - \frac{1}{2}\arctan \frac{t}{2}$
51. $y = \frac{1}{2}\left(\frac{1}{2}\ln\frac{x + 1}{x - 1} + \arctan x\right)$
52. $y = \frac{1}{2}\left[x\sqrt{4 - x^2} + 4\arcsin\left(\frac{x}{2}\right)\right]$
53. $y = x\arcsin x + \sqrt{1 - x^2}$
54. $y = x\arctan 2x - \frac{1}{4}\ln(1 + 4x^2)$
55. $y = 8\arcsin\frac{x}{4} - \frac{x\sqrt{16 - x^2}}{2}$
56. $y = 25\arcsin\frac{x}{5} - x\sqrt{25 - x^2}$
57. $y = \arctan x + \frac{x}{1 + x^2}$
58. $y = \arctan\frac{x}{2} - \frac{1}{2(x^2 + 4)}$

Finding an Equation of a Tangent Line **In Exercises 59–64, find an equation of the tangent line to the graph of the function at the given point.**

59. $y = 2\arcsin x, \quad \left(\frac{1}{2}, \frac{\pi}{3}\right)$

60. $y = \frac{1}{2}\arccos x, \quad \left(-\frac{\sqrt{2}}{2}, \frac{3\pi}{8}\right)$

61. $y = \arctan\frac{x}{2}, \quad \left(2, \frac{\pi}{4}\right)$

62. $y = \operatorname{arcsec} 4x, \quad \left(\frac{\sqrt{2}}{4}, \frac{\pi}{4}\right)$

63. $y = 4x\arccos(x - 1), \quad (1, 2\pi)$

64. $y = 3x\arcsin x, \quad \left(\frac{1}{2}, \frac{\pi}{4}\right)$

Linear and Quadratic Approximations **In Exercises 65–68, use a computer algebra system to find the linear approximation**

$$P_1(x) = f(a) + f'(a)(x - a)$$

and the quadratic approximation

$$P_2(x) = f(a) + f'(a)(x - a) + \tfrac{1}{2}f''(a)(x - a)^2$$

of the function f at $x = a$. Sketch the graph of the function and its linear and quadratic approximations.

65. $f(x) = \arctan x, \quad a = 0$

66. $f(x) = \arccos x, \quad a = 0$

67. $f(x) = \arcsin x, \quad a = \frac{1}{2}$

68. $f(x) = \arctan x, \quad a = 1$

Finding Relative Extrema **In Exercises 69–72, find any relative extrema of the function.**

69. $f(x) = \operatorname{arcsec} x - x$

70. $f(x) = \arcsin x - 2x$

71. $f(x) = \arctan x - \arctan(x - 4)$

72. $h(x) = \arcsin x - 2\arctan x$

Analyzing an Inverse Trigonometric Graph **In Exercises 73–76, analyze and sketch a graph of the function. Identify any relative extrema, points of inflection, and asymptotes. Use a graphing utility to verify your results.**

73. $f(x) = \arcsin(x - 1)$

74. $f(x) = \arctan x + \frac{\pi}{2}$

75. $f(x) = \operatorname{arcsec} 2x$

76. $f(x) = \arccos\frac{x}{4}$

Implicit Differentiation **In Exercises 77–80, use implicit differentiation to find an equation of the tangent line to the graph of the equation at the given point.**

77. $x^2 + x\arctan y = y - 1, \quad \left(-\frac{\pi}{4}, 1\right)$

78. $\arctan(xy) = \arcsin(x + y), \quad (0, 0)$

79. $\arcsin x + \arcsin y = \frac{\pi}{2}, \quad \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

80. $\arctan(x + y) = y^2 + \frac{\pi}{4}, \quad (1, 0)$

WRITING ABOUT CONCEPTS

81. **Restricted Domains** Explain why the domains of the trigonometric functions are restricted when finding the inverse trigonometric functions.

82. **Inverse Trigonometric Functions** Explain why $\tan\pi = 0$ does not imply that $\arctan 0 = \pi$.

83. **Finding Values**

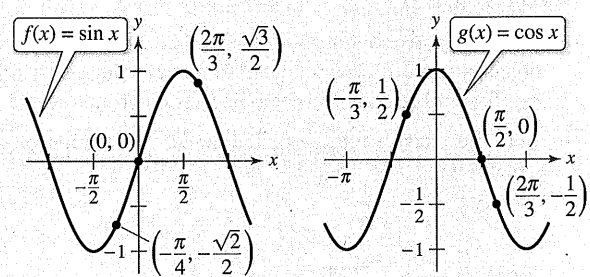
(a) Use a graphing utility to evaluate $\arcsin(\arcsin 0.5)$ and $\arcsin(\arcsin 1)$.

(b) Let

$$f(x) = \arcsin(\arcsin x).$$

Find the values of x in the interval $-1 \le x \le 1$ such that $f(x)$ is a real number.

84. **HOW DO YOU SEE IT?** The graphs of $f(x) = \sin x$ and $g(x) = \cos x$ are shown below.



(a) Explain whether the points

$$\left(-\frac{\sqrt{2}}{2}, -\frac{\pi}{4}\right), \quad (0, 0), \quad \text{and} \quad \left(\frac{\sqrt{3}}{2}, \frac{2\pi}{3}\right)$$

lie on the graph of $y = \arcsin x$.

(b) Explain whether the points

$$\left(-\frac{1}{2}, \frac{2\pi}{3}\right), \quad \left(0, \frac{\pi}{2}\right), \quad \text{and} \quad \left(\frac{1}{2}, -\frac{\pi}{3}\right)$$

lie on the graph of $y = \arccos x$.

True or False? **In Exercises 85–90, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

85. Because $\cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$, it follows that $\arccos\frac{1}{2} = -\frac{\pi}{3}$.

86. $\arcsin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

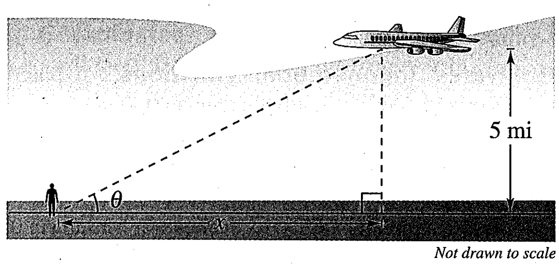
87. The slope of the graph of the inverse tangent function is positive for all x.

88. The range of $y = \arcsin x$ is $[0, \pi]$.

89. $\frac{d}{dx}[\arctan(\tan x)] = 1$ for all x in the domain.

90. $\arcsin^2 x + \arccos^2 x = 1$

91. Angular Rate of Change An airplane flies at an altitude of 5 miles toward a point directly over an observer. Consider θ and x as shown in the figure.



(a) Write θ as a function of x.

(b) The speed of the plane is 400 miles per hour. Find $d\theta/dt$ when $x = 10$ miles and $x = 3$ miles.

92. Writing Repeat Exercise 91 for an altitude of 3 miles and describe how the altitude affects the rate of change of θ.

93. Angular Rate of Change In a free-fall experiment, an object is dropped from a height of 256 feet. A camera on the ground 500 feet from the point of impact records the fall of the object (see figure).

(a) Find the position function that yields the height of the object at time t, assuming the object is released at time $t = 0$. At what time will the object reach ground level?

(b) Find the rates of change of the angle of elevation of the camera when $t = 1$ and $t = 2$.

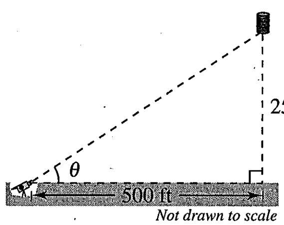


Figure for 93

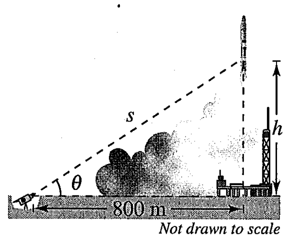


Figure for 94

94. Angular Rate of Change A television camera at ground level is filming the lift-off of a rocket at a point 800 meters from the launch pad. Let θ be the angle of elevation of the rocket and let s be the distance between the camera and the rocket (see figure). Write θ as a function of s for the period of time when the rocket is moving vertically. Differentiate the result to find $d\theta/dt$ in terms of s and ds/dt.

95. Maximizing an Angle A billboard 85 feet wide is perpendicular to a straight road and is 40 feet from the road (see figure). Find the point on the road at which the angle θ subtended by the billboard is a maximum.

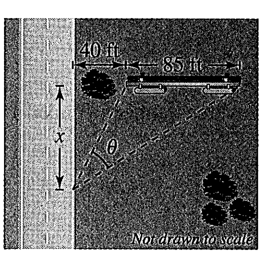


Figure for 95

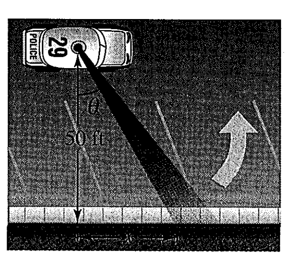


Figure for 96

96. Angular Speed A patrol car is parked 50 feet from a long warehouse (see figure). The revolving light on top of the car turns at a rate of 30 revolutions per minute. Write θ as a function of x. How fast is the light beam moving along the wall when the beam makes an angle of $\theta = 45°$ with the line perpendicular from the light to the wall?

97. Proof

(a) Prove that $\arctan x + \arctan y = \arctan \dfrac{x + y}{1 - xy}$, $xy \neq 1$.

(b) Use the formula in part (a) to show that

$$\arctan \frac{1}{2} + \arctan \frac{1}{3} = \frac{\pi}{4}.$$

98. Proof Prove each differentiation formula.

(a) $\dfrac{d}{dx}[\arctan u] = \dfrac{u'}{1 + u^2}$

(b) $\dfrac{d}{dx}[\operatorname{arccot} u] = \dfrac{-u'}{1 + u^2}$

(c) $\dfrac{d}{dx}[\operatorname{arcsec} u] = \dfrac{u'}{|u|\sqrt{u^2 - 1}}$

(d) $\dfrac{d}{dx}[\operatorname{arccsc} u] = \dfrac{-u'}{|u|\sqrt{u^2 - 1}}$

99. Describing a Graph

(a) Graph the function $f(x) = \arccos x + \arcsin x$ on the interval $[-1, 1]$.

(b) Describe the graph of f.

(c) Verify the result of part (b) analytically.

100. Think About It Use a graphing utility to graph $f(x) = \sin x$ and $g(x) = \arcsin(\sin x)$.

(a) Why isn't the graph of g the line $y = x$?

(b) Determine the extrema of g.

101. Maximizing an Angle In the figure, find the value of c in the interval $[0, 4]$ on the x-axis that maximizes angle θ.

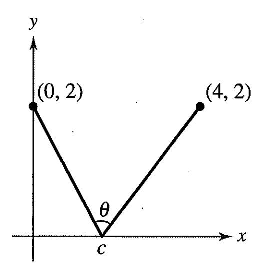


Figure for 101

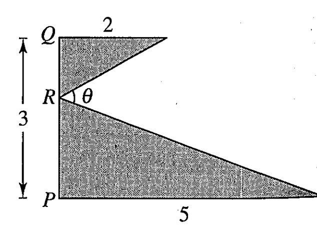


Figure for 102

102. Finding a Distance In the figure, find PR such that $0 \le PR \le 3$ and $m\angle\theta$ is a maximum.

103. Proof Prove that $\arcsin x = \arctan\left(\dfrac{x}{\sqrt{1 - x^2}}\right)$, $|x| < 1$.

104. Inverse Secant Function Some calculus textbooks define the inverse secant function using the range $[0, \pi/2) \cup [\pi, 3\pi/2)$.

(a) Sketch the graph $y = \operatorname{arcsec} x$ using this range.

(b) Show that $y' = \dfrac{1}{x\sqrt{x^2 - 1}}$.