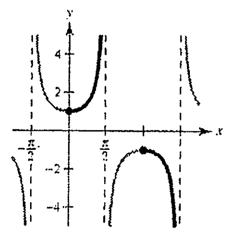
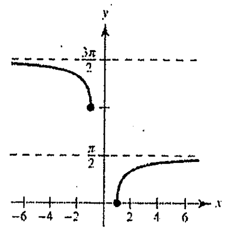
104. $f(x) = \sec x, \quad 0 \le x < \frac{\pi}{2}, \pi \le x < \frac{3\pi}{2}$



(a) $y = \text{arcsec } x, \quad x \le -1 \quad \text{or} \quad x \ge 1$

$$0 \le y < \frac{\pi}{2} \quad \text{or} \quad \pi \le y < \frac{3\pi}{2}$$



(b)
$$\begin{aligned} y &= \text{arcsec } x \\ x &= \sec y \\ 1 &= \sec y \tan y \cdot y' \\ y' &= \frac{1}{\sec y \tan y} \\ &= \frac{1}{x\sqrt{x^2 - 1}} \\ \tan^2 y + 1 &= \sec^2 y \\ \tan y &= \pm\sqrt{\sec^2 y - 1} \end{aligned}$$

On $0 \le y < \pi/2$ and $\pi \le y < 3\pi/2$, $\tan y \ge 0$.

Section 5.7 Inverse Trigonometric Functions: Integration

1. $\int \frac{dx}{\sqrt{9 - x^2}} = \arcsin\left(\frac{x}{3}\right) + C$

2. $\int \frac{dx}{\sqrt{1 - 4x^2}} = \frac{1}{2}\int \frac{2}{\sqrt{1 - 4x^2}}\,dx = \frac{1}{2}\arcsin(2x) + C$

3. $\int \frac{1}{x\sqrt{4x^2 - 1}}\,dx = \int \frac{2}{2x\sqrt{(2x)^2 - 1}}\,dx = \text{arcsec}|2x| + C$

4. $\int \frac{12}{1 + 9x^2}\,dx = 4\int \frac{3}{1 + 9x^2}\,dx = 4\arctan(3x) + C$

5. $\int \frac{1}{\sqrt{1 - (x + 1)^2}}\,dx = \arcsin(x + 1) + C$

6. $\int \frac{1}{4 + (x - 3)^2}\,dx = \frac{1}{2}\arctan\left(\frac{x - 3}{2}\right) + C$

7. Let $u = t^2$, $du = 2t\,dt$.

$$\int \frac{t}{\sqrt{1 - t^4}}\,dt = \frac{1}{2}\int \frac{1}{\sqrt{1 - (t^2)^2}}(2t)\,dt = \frac{1}{2}\arcsin t^2 + C$$

8. Let $u = x^2$, $du = 2x\,dx$.

$$\begin{aligned} \int \frac{1}{x\sqrt{x^4 - 4}}\,dx &= \frac{1}{2}\int \frac{1}{x^2\sqrt{(x^2)^2 - 2^2}}(2x)\,dx \\ &= \frac{1}{4}\,\text{arcsec}\,\frac{x^2}{2} + C \end{aligned}$$

9.
$$\begin{aligned} \int \frac{t}{t^4 + 25}\,dt &= \frac{1}{2}\int \frac{1}{(t^2)^2 + 5^2}(2)\,dt \\ &= \frac{1}{2}\frac{1}{5}\arctan\left(\frac{t^2}{5}\right) + C \\ &= \frac{1}{10}\arctan\left(\frac{t^2}{5}\right) + C \end{aligned}$$

10.
$$\begin{aligned} \int \frac{1}{x\sqrt{1 - (\ln x)^2}}\,dx &= \int \frac{1}{\sqrt{1 - (\ln x)^2}} \cdot \frac{1}{x}\,dx \\ &= \arcsin(\ln x) + C \end{aligned}$$

11. Let $u = e^{2x}$, $du = 2e^{2x}\,dx$.

$$\int \frac{e^{2x}}{4 + e^{4x}}\,dx = \frac{1}{2}\int \frac{2e^{2x}}{4 + (e^{2x})^2}\,dx = \frac{1}{4}\arctan\frac{e^{2x}}{2} + C$$

12. $u = 3x, du = 3\,dx, a = 5$

$$\int \frac{2}{x\sqrt{9x^2 - 25}}\,dx = 2\int \frac{1}{(3x)\sqrt{(3x)^2 - 5^2}}\,3\,dx$$
$$= \frac{2}{5}\operatorname{arcsec}\frac{|3x|}{5} + C$$

13. $$\int \frac{\sec^2 x}{\sqrt{25 - \tan^2 x}}\,dx = \int \frac{\sec^2 x}{\sqrt{5^2 - (\tan x)^2}}\,dx$$
$$= \arcsin\left(\frac{\tan x}{5}\right) + C$$

14. $$\int \frac{\sin x}{7 + \cos^2 x}\,dx = \int \frac{-1}{\left(\sqrt{7}\right)^2 + \cos^2 x}(-\sin x)\,dx$$
$$= -\frac{1}{\sqrt{7}}\arctan\left(\frac{\cos x}{\sqrt{7}}\right) + C$$
$$= -\frac{\sqrt{7}}{7}\arctan\left(\frac{\sqrt{7}\cos x}{7}\right) + C$$

15. $$\int \frac{1}{\sqrt{x}\sqrt{1 - x}}\,dx, u = \sqrt{x}, x = u^2, dx = 2u\,du$$
$$\int \frac{1}{u\sqrt{1 - u^2}}(2u\,du) = 2\int \frac{du}{\sqrt{1 - u^2}} = 2\arcsin u + C = 2\arcsin\sqrt{x} + C$$

16. $$\int \frac{3}{2\sqrt{x}(1 + x)}\,dx, u = \sqrt{x}, du = \frac{1}{2\sqrt{x}}\,dx, dx = 2u\,du$$
$$\frac{3}{2}\int \frac{2u\,du}{u(1 + u^2)} = 3\int \frac{du}{1 + u^2} = 3\arctan u + C = 3\arctan\sqrt{x} + C$$

17. $$\int \frac{x - 3}{x^2 + 1}\,dx = \frac{1}{2}\int \frac{2x}{x^2 + 1}\,dx - 3\int \frac{1}{x^2 + 1}\,dx = \frac{1}{2}\ln(x^2 + 1) - 3\arctan x + C$$

18. $$\int \frac{x^2 + 3}{x\sqrt{x^2 - 4}}\,dx = \int \frac{x^2}{x\sqrt{x^2 - 4}}\,dx + \int \frac{3}{x\sqrt{x^2 - 4}}\,dx$$
$$= \frac{1}{2}\int (x^2 - 4)^{-1/2}2x\,dx + 3\int \frac{1}{x\sqrt{x^2 - 4}}\,dx$$
$$= \sqrt{x^2 - 4} + \frac{3}{2}\operatorname{arcsec}\frac{|x|}{2} + C$$

19. $$\int \frac{x + 5}{\sqrt{9 - (x - 3)^2}}\,dx = \int \frac{(x - 3)}{\sqrt{9 - (x - 3)^2}}\,dx + \int \frac{8}{\sqrt{9 - (x - 3)^2}}\,dx$$
$$= -\sqrt{9 - (x - 3)^2} + 8\arcsin\left(\frac{x - 3}{3}\right) + C = -\sqrt{6x - x^2} + 8\arcsin\left(\frac{x}{3} - 1\right) + C$$

20. $$\int \frac{x - 2}{(x + 1)^2 + 4}\,dx = \frac{1}{2}\int \frac{2x + 2}{(x + 1)^2 + 4}\,dx - \int \frac{3}{(x + 1)^2 + 4}\,dx$$
$$= \frac{1}{2}\ln(x^2 + 2x + 5) - \frac{3}{2}\arctan\left(\frac{x + 1}{2}\right) + C$$

21. Let $u = 3x, du = 3\,dx$.

$$\int_0^{1/6} \frac{3}{\sqrt{1 - 9x^2}}\,dx = \int_0^{1/6} \frac{1}{\sqrt{1 - (3x)^2}}(3)\,dx$$
$$= \left[\arcsin(3x)\right]_0^{1/6} = \frac{\pi}{6}$$

22. $$\int_0^{\sqrt{2}} \frac{1}{\sqrt{4 - x^2}}\,dx = \left[\arcsin\frac{x}{2}\right]_0^{\sqrt{2}}$$
$$= \arcsin\frac{\sqrt{2}}{2} - \arcsin 0$$
$$= \frac{\pi}{4}$$

23. Let $u = 2x$, $du = 2\,dx$.

$$\int_0^{\sqrt{3}/2} \frac{1}{1+4x^2}\,dx = \frac{1}{2}\int_0^{\sqrt{3}/2} \frac{2}{1+(2x)^2}\,dx$$
$$= \left[\frac{1}{2}\arctan(2x)\right]_0^{\sqrt{3}/2} = \frac{\pi}{6}$$

24. $$\int_{\sqrt{3}}^{3} \frac{1}{x\sqrt{4x^2-9}}\,dx = \left[\frac{1}{3}\operatorname{arcsec}\frac{|2x|}{3}\right]_{\sqrt{3}}^{3}$$
$$= \frac{1}{3}\operatorname{arcsec}(2) - \frac{1}{3}\operatorname{arcsec}\frac{2\sqrt{3}}{3}$$
$$= \frac{1}{3}\left(\frac{\pi}{3}\right) - \frac{1}{3}\left(\frac{\pi}{6}\right) = \frac{\pi}{18}$$

25. $$\int_3^6 \frac{1}{25+(x-3)^2}\,dx = \left[\frac{1}{5}\arctan\left(\frac{x-3}{5}\right)\right]_3^6$$
$$= \frac{1}{5}\arctan(3/5)$$
$$\approx 0.108$$

26. $$\int_1^4 \frac{1}{x\sqrt{16x^2-5}}\,dx = \int_1^4 \frac{4\,dx}{(4x)\sqrt{(4x)^2-\left(\sqrt{5}\right)^2}}$$
$$= \left[\left(\frac{1}{\sqrt{5}}\right)\operatorname{arcsec}\frac{|4x|}{\sqrt{5}}\right]_1^4 = \frac{1}{\sqrt{5}}\operatorname{arcsec}\frac{16}{\sqrt{5}} - \frac{1}{\sqrt{5}}\operatorname{arcsec}\left(\frac{4}{\sqrt{5}}\right) \approx 0.091$$

27. Let $u = e^x$, $du = e^x\,dx$

$$\int_0^{\ln 5} \frac{e^x}{1+e^{2x}}\,dx = \left[\arctan\left(e^x\right)\right]_0^{\ln 5} = \arctan 5 - \frac{\pi}{4} \approx 0.588$$

28. Let $u = e^{-x}$, $du = -e^{-x}\,dx$

$$\int_{\ln 2}^{\ln 4} \frac{e^{-x}}{\sqrt{1-e^{-2x}}}\,dx = \left[-\arcsin\left(e^{-x}\right)\right]_{\ln 2}^{\ln 4} = -\arcsin\left(\frac{1}{4}\right) + \arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6} - \arcsin\left(\frac{1}{4}\right) \approx 0.271$$

29. Let $u = \cos x$, $du = -\sin x\,dx$.

$$\int_{\pi/2}^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx = -\int_{\pi/2}^{\pi} \frac{-\sin x}{1+\cos^2 x}\,dx = \left[-\arctan(\cos x)\right]_{\pi/2}^{\pi} = \frac{\pi}{4}$$

30. $$\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x}\,dx = \left[\arctan(\sin x)\right]_0^{\pi/2} = \frac{\pi}{4}$$

31. Let $u = \arcsin x$, $du = \dfrac{1}{\sqrt{1-x^2}}\,dx$.

$$\int_0^{1/\sqrt{2}} \frac{\arcsin x}{\sqrt{1-x^2}}\,dx = \left[\frac{1}{2}\arcsin^2 x\right]_0^{1/\sqrt{2}} = \frac{\pi^2}{32} \approx 0.308$$

32. Let $u = \arccos x$, $du = -\dfrac{1}{\sqrt{1-x^2}}\,dx$.

$$\int_0^{1/\sqrt{2}} \frac{\arccos x}{\sqrt{1-x^2}}\,dx = -\int_0^{1/\sqrt{2}} \frac{-\arccos x}{\sqrt{1-x^2}}\,dx = \left[-\frac{1}{2}\arccos^2 x\right]_0^{1/\sqrt{2}} = \frac{3\pi^2}{32} \approx 0.925$$

33. $$\int_0^2 \frac{dx}{x^2-2x+2} = \int_0^2 \frac{1}{1+(x-1)^2}\,dx = \left[\arctan(x-1)\right]_0^2 = \frac{\pi}{2}$$

34. $\int_{-2}^{2} \frac{dx}{x^2+4x+13} = \int_{-2}^{2} \frac{dx}{(x+2)^2+9} = \left[\frac{1}{3}\arctan\left(\frac{x+2}{3}\right)\right]_{-2}^{2} = \frac{1}{3}\arctan\left(\frac{4}{3}\right)$

35. $\int \frac{2x}{x^2+6x+13}\,dx = \int \frac{2x+6}{x^2+6x+13}\,dx - 6\int \frac{1}{x^2+6x+13}\,dx$

$= \int \frac{2x+6}{x^2+6x+13}\,dx - 6\int \frac{1}{4+(x+3)^2}\,dx = \ln\left|x^2+6x+13\right| - 3\arctan\left(\frac{x+3}{2}\right) + C$

36. $\int \frac{2x-5}{x^2+2x+2}\,dx = \int \frac{2x+2}{x^2+2x+2}\,dx - 7\int \frac{1}{1+(x+1)^2}\,dx = \ln\left|x^2+2x+2\right| - 7\arctan(x+1) + C$

37. $\int \frac{1}{\sqrt{-x^2-4x}}\,dx = \int \frac{1}{\sqrt{4-(x+2)^2}}\,dx = \arcsin\left(\frac{x+2}{2}\right) + C$

38. $\int \frac{2}{\sqrt{-x^2+4x}}\,dx = \int \frac{2}{\sqrt{4-(x^2-4x+4)}}\,dx = \int \frac{2}{\sqrt{4-(x-2)^2}}\,dx = 2\arcsin\left(\frac{x-2}{2}\right) + C$

39. $\int_2^3 \frac{2x-3}{\sqrt{4x-x^2}}\,dx = \int_2^3 \frac{2x-4}{\sqrt{4x-x^2}}\,dx + \int_2^3 \frac{1}{\sqrt{4x-x^2}}\,dx$

$= -\int_2^3 \left(4x-x^2\right)^{-1/2}(4-2x)\,dx + \int_2^3 \frac{1}{\sqrt{4-(x-2)^2}}\,dx$

$= \left[-2\sqrt{4x-x^2} + \arcsin\left(\frac{x-2}{2}\right)\right]_2^3 = 4 - 2\sqrt{3} + \frac{\pi}{6} \approx 1.059$

40. $\int \frac{1}{(x-1)\sqrt{x^2-2x}}\,dx = \int \frac{1}{(x-1)\sqrt{(x-1)^2-1}}\,dx = \operatorname{arcsec}\left|x-1\right| + C$

41. Let $u = x^2 + 1$, $du = 2x\,dx$.

$\int \frac{x}{x^4+2x^2+2}\,dx = \frac{1}{2}\int \frac{2x}{\left(x^2+1\right)^2+1}\,dx = \frac{1}{2}\arctan\left(x^2+1\right) + C$

42. Let $u = x^2 - 4$, $du = 2x\,dx$.

$\int \frac{x}{\sqrt{9+8x^2-x^4}}\,dx = \frac{1}{2}\int \frac{2x}{\sqrt{25-\left(x^2-4\right)^2}}\,dx = \frac{1}{2}\arcsin\left(\frac{x^2-4}{5}\right) + C$

43. Let $u = \sqrt{e^t - 3}$. Then $u^2 + 3 = e^t$, $2u\,du = e^t\,dt$, and $\frac{2u\,du}{u^2+3} = dt$.

$\int \sqrt{e^t-3}\,dt = \int \frac{2u^2}{u^2+3}\,du = \int 2\,du - \int 6\frac{1}{u^2+3}\,du$

$= 2u - 2\sqrt{3}\arctan\frac{u}{\sqrt{3}} + C = 2\sqrt{e^t-3} - 2\sqrt{3}\arctan\sqrt{\frac{e^t-3}{3}} + C$

44. Let $u = \sqrt{x-2}$, $u^2 + 2 = x$, $2u\,du = dx$.

$\int \frac{\sqrt{x-2}}{x+1}\,dx = \int \frac{2u^2}{u^2+3}\,du = \int \frac{2u^2+6-6}{u^2+3}\,du = 2\int du - 6\int \frac{1}{u^2+3}\,du$

$= 2u - \frac{6}{\sqrt{3}}\arctan\frac{u}{\sqrt{3}} + C = 2\sqrt{x-2} - 2\sqrt{3}\arctan\sqrt{\frac{x-2}{3}} + C$

45. $\int_1^3 \frac{dx}{\sqrt{x}(1+x)}$

Let $u = \sqrt{x}, u^2 = x, 2u\,du = dx, 1 + x = 1 + u^2$.

$$\int_1^{\sqrt{3}} \frac{2u\,du}{u(1+u^2)} = \int_1^{\sqrt{3}} \frac{2}{1+u^2}\,du$$
$$= \left[2\arctan(u)\right]_1^{\sqrt{3}}$$
$$= 2\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \frac{\pi}{6}$$

46. $\int_0^1 \frac{dx}{2\sqrt{3-x}\sqrt{x+1}}$

Let $u = \sqrt{x+1}, u^2 = x + 1, 2u\,du = dx$, $\sqrt{3-x} = \sqrt{4-u^2}$.

$$\int_1^{\sqrt{2}} \frac{2u\,du}{2\sqrt{4-u^2}u} = \int_1^{\sqrt{2}} \frac{du}{\sqrt{4-u^2}}$$
$$= \arcsin\left(\frac{u}{2}\right)\Big]_1^{\sqrt{2}}$$
$$= \arcsin\left(\frac{\sqrt{2}}{2}\right) - \arcsin\left(\frac{1}{2}\right)$$
$$= \frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{12}$$

47. (a) $\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C, \quad u = x$

(b) $\int \frac{x}{\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2} + C, \quad u = 1 - x^2$

(c) $\int \frac{1}{x\sqrt{1-x^2}}\,dx$ cannot be evaluated using the basic integration rules.

48. (a) $\int e^{x^2}\,dx$ cannot be evaluated using the basic integration rules.

(b) $\int xe^{x^2}\,dx = \frac{1}{2}e^{x^2} + C, \quad u = x^2$

(c) $\int \frac{1}{x^2}e^{1/x}\,dx = -e^{1/x} + C, \quad u = \frac{1}{x}$

49. (a) $\int \sqrt{x-1}\,dx = \frac{2}{3}(x-1)^{3/2} + C, \quad u = x - 1$

(b) Let $u = \sqrt{x-1}$. Then $x = u^2 + 1$ and $dx = 2u\,du$.

$$\int x\sqrt{x-1}\,dx = \int (u^2+1)(u)(2u)\,du$$
$$= 2\int (u^4 + u^2)\,du$$
$$= 2\left(\frac{u^5}{5} + \frac{u^3}{3}\right) + C$$
$$= \frac{2}{15}u^3(3u^2 + 5) + C$$
$$= \frac{2}{15}(x-1)^{3/2}\left[3(x-1) + 5\right] + C$$
$$= \frac{2}{15}(x-1)^{3/2}(3x+2) + C$$

(c) Let $u = \sqrt{x-1}$. Then $x = u^2 + 1$ and $dx = 2u\,du$.

$$\int \frac{x}{\sqrt{x-1}}\,dx = \int \frac{u^2+1}{u}(2u)\,du$$
$$= 2\int (u^2+1)\,du$$
$$= 2\left(\frac{u^3}{3} + u\right) + C$$
$$= \frac{2}{3}u(u^2+3) + C$$
$$= \frac{2}{3}\sqrt{x-1}(x+2) + C$$

Note: In (b) and (c), substitution was necessary *before* the basic integration rules could be used.

50. (a) $\int \frac{1}{1+x^4}\,dx$ cannot be evaluated using the basic integration rules.

(b) $\int \frac{x}{1+x^4}\,dx = \frac{1}{2}\int \frac{2x}{1+(x^2)^2}\,dx$

$$= \frac{1}{2}\arctan(x^2) + C, \quad u = x^2$$

(c) $\int \frac{x^3}{1+x^4}\,dx = \frac{1}{4}\int \frac{4x^3}{1+x^4}\,dx$

$$= \frac{1}{4}\ln(1+x^4) + C, \quad u = 1 + x^4$$

51. No. This integral does not correspond to any of the basic differentiation rules.

52. The area is approximately the area of a square of side 1. So, (c) best approximates the area.

53. $y' = \dfrac{1}{\sqrt{4 - x^2}}, \quad (0, \pi)$

$$y = \int \frac{1}{\sqrt{4 - x^2}}\,dx = \arcsin\left(\frac{x}{2}\right) + C$$

When $x = 0, y = \pi \Rightarrow C = \pi$

$$y = \arcsin\left(\frac{x}{2}\right) + \pi$$

54. $y' = \dfrac{1}{4 + x^2}, \quad (2, \pi)$

$$y = \int \frac{1}{4 + x^2}\,dx = \frac{1}{2}\arctan\frac{x}{2} + C$$

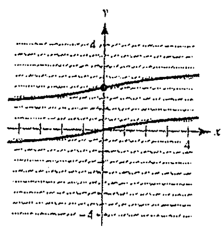
When $x = 2, y = \pi$:

$$\pi = \frac{1}{2}\arctan\left(\frac{2}{2}\right) + C$$

$$\pi = \frac{\pi}{8} + C \Rightarrow C = \frac{7\pi}{8}$$

$$y = \frac{1}{2}\arctan\left(\frac{x}{2}\right) + \frac{7\pi}{8}$$

55. (a)

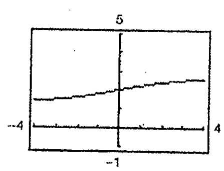


(b) $y' = \dfrac{2}{9 + x^2}, \quad (0, 2)$

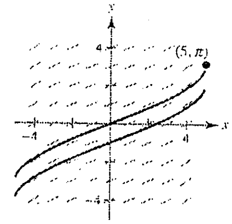
$$y = \int \frac{2}{9 + x^2}\,dx = \frac{2}{3}\arctan\left(\frac{x}{3}\right) + C$$

$$2 = C$$

$$y = \frac{2}{3}\arctan\left(\frac{x}{3}\right) + 2$$



56. (a)

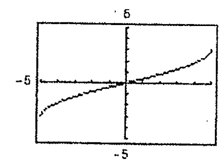


(b) $y' = \dfrac{2}{\sqrt{25 - x^2}}, \quad (5, \pi)$

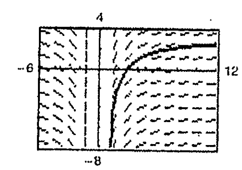
$$y = \int \frac{2}{\sqrt{25 - x^2}}\,dx = 2\arcsin\left(\frac{x}{5}\right) + C$$

$$\pi = 2\arcsin(1) + C \Rightarrow C = 0$$

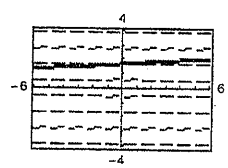
$$y = 2\arcsin\left(\frac{x}{5}\right)$$



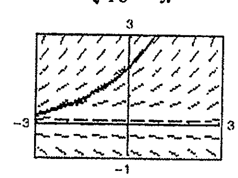
57. $\dfrac{dy}{dx} = \dfrac{10}{x\sqrt{x^2 - 1}}, \quad (3, 0)$



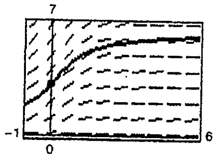
58. $\dfrac{dy}{dx} = \dfrac{1}{12 + x^2}, \quad (4, 2)$



59. $\dfrac{dy}{dx} = \dfrac{2y}{\sqrt{16 - x^2}}, \quad (0, 2)$



60. $\dfrac{dy}{dx} = \dfrac{\sqrt{y}}{1+x^2}$, $(0, 4)$



61. $\text{Area} = \displaystyle\int_0^1 \frac{2}{\sqrt{4-x^2}}\,dx$

$= \left[2\arcsin\left(\dfrac{x}{2}\right)\right]_0^1$

$= 2\arcsin\left(\dfrac{1}{2}\right) - 2\arcsin(0)$

$= 2\left(\dfrac{\pi}{6}\right) = \dfrac{\pi}{3}$

62. $\text{Area} = \displaystyle\int_{2/\sqrt{2}}^{2} \frac{1}{x\sqrt{x^2-1}}\,dx$

$= \left[\operatorname{arcsec} x\right]_{2/\sqrt{2}}^{2}$

$= \operatorname{arcsec}(2) - \operatorname{arcsec}\left(\dfrac{2}{\sqrt{2}}\right)$

$= \dfrac{\pi}{3} - \dfrac{\pi}{4} = \dfrac{\pi}{12}$

63. $\text{Area} = \displaystyle\int_1^3 \frac{1}{x^2-2x+5}\,dx = \int_1^3 \frac{1}{(x-1)^2+4}\,dx$

$= \left[\dfrac{1}{2}\arctan\left(\dfrac{x-1}{2}\right)\right]_1^3$

$= \dfrac{1}{2}\arctan(1) - \dfrac{1}{2}\arctan(0)$

$= \dfrac{\pi}{8}$

64. $\text{Area} = \displaystyle\int_{-2}^{0} \frac{2}{x^2+4x+8}\,dx = \int_{-2}^{0} \frac{2}{(x+2)^2+4}\,dx$

$= \left[\arctan\left(\dfrac{x+2}{2}\right)\right]_{-2}^{0}$

$= \arctan(1) - \arctan(0)$

$= \dfrac{\pi}{4}$

65. $\text{Area} = \displaystyle\int_{-\pi/2}^{\pi/2} \frac{3\cos x}{1+\sin^2 x}\,dx = 3\int_{-\pi/2}^{\pi/2} \frac{1}{1+\sin^2 x}(\cos x\,dx)$

$= \left[3\arctan(\sin x)\right]_{-\pi/2}^{\pi/2}$

$= 3\arctan(1) - 3\arctan(-1)$

$= \dfrac{3\pi}{4} + \dfrac{3\pi}{4} = \dfrac{3\pi}{2}$

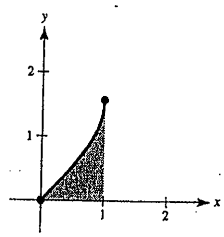
66. $\text{Area} = \displaystyle\int_0^{\ln\sqrt{3}} \frac{4e^x}{1+e^{2x}}\,dx$, $(u = e^x)$

$= 4\left[\arctan(e^x)\right]_0^{\ln\sqrt{3}}$

$= 4\left[\arctan(\sqrt{3}) - \arctan(1)\right]$

$= 4\left(\dfrac{\pi}{3} - \dfrac{\pi}{4}\right) = \dfrac{\pi}{3}$

67. (a)



Shaded area is given by $\int_0^1 \arcsin x\,dx$.

(b) $\int_0^1 \arcsin x\,dx \approx 0.5708$

(c) Divide the rectangle into two regions.

Area rectangle $= (\text{base})(\text{height}) = 1\left(\dfrac{\pi}{2}\right) = \dfrac{\pi}{2}$

Area rectangle $= \displaystyle\int_0^1 \arcsin x\,dx + \int_0^{\pi/2} \sin y\,dy$

$\dfrac{\pi}{2} = \displaystyle\int_0^1 \arcsin x\,dx + (-\cos y)\Big]_0^{\pi/2} = \int_0^1 \arcsin x\,dx + 1$

So, $\displaystyle\int_0^1 \arcsin x\,dx = \frac{\pi}{2} - 1$, (≈ 0.5708).

68. (a) $\int_0^1 \frac{4}{1+x^2}\,dx = \left[4 \arctan x\right]_0^1 = 4 \arctan 1 - 4 \arctan 0 = 4\left(\frac{\pi}{4}\right) - 4(0) = \pi$

(b) Let $n = 6$.

$$4\int_0^1 \frac{1}{1+x^2}\,dx \approx 4\left(\frac{1}{18}\right)\left[1 + \frac{4}{1+(1/36)} + \frac{2}{1+(1/9)} + \frac{4}{1+(1/4)} + \frac{2}{1+(4/9)} + \frac{4}{1+(25/36)} + \frac{1}{2}\right] \approx 3.1415918$$

(c) 3.1415927

69. $F(x) = \frac{1}{2}\int_x^{x+2} \frac{2}{t^2+1}\,dt$

(a) $F(x)$ represents the average value of $f(x)$ over the interval $[x, x+2]$. Maximum at $x = -1$, because the graph is greatest on $[-1, 1]$.

(b) $F(x) = \left[\arctan t\right]_x^{x+2} = \arctan(x+2) - \arctan x$

$$F'(x) = \frac{1}{1+(x+2)^2} - \frac{1}{1+x^2} = \frac{(1+x^2) - (x^2+4x+5)}{(x^2+1)(x^2+4x+5)} = \frac{-4(x+1)}{(x^2+1)(x^2+4x+5)} = 0 \text{ when } x = -1.$$

70. $\int \frac{1}{\sqrt{6x - x^2}}\,dx$

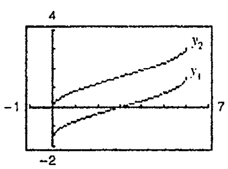
(a) $6x - x^2 = 9 - (x^2 - 6x + 9) = 9 - (x-3)^2$

$$\int \frac{1}{\sqrt{6x-x^2}}\,dx = \int \frac{dx}{\sqrt{9-(x-3)^2}} = \arcsin\left(\frac{x-3}{3}\right) + C$$

(b) $u = \sqrt{x},\ u^2 = x,\ 2u\,du = dx$

$$\int \frac{1}{\sqrt{6u^2 - u^4}}(2u\,du) = \int \frac{2}{\sqrt{6-u^2}}\,du = 2\arcsin\left(\frac{u}{\sqrt{6}}\right) + C = 2\arcsin\left(\frac{\sqrt{x}}{\sqrt{6}}\right) + C$$

(c)



The antiderivatives differ by a constant, $\pi/2$.

Domain: $[0, 6]$

71. False, $\int \frac{dx}{3x\sqrt{9x^2-16}} = \frac{1}{12}\operatorname{arcsec}\frac{|3x|}{4} + C$

72. False, $\int \frac{dx}{25+x^2}\,dx = \frac{1}{5}\arctan\frac{x}{5} + C$

73. True

$$\frac{d}{dx}\left[-\arccos\frac{x}{2} + C\right] = \frac{1/2}{\sqrt{1-(x/2)^2}} = \frac{1}{\sqrt{4-x^2}}$$

74. False. Use substitution: $u = 9 - e^{2x},\ du = -2e^{2x}dx$

75.

$$\frac{d}{dx}\left[\arcsin\left(\frac{u}{a}\right) + C\right] = \frac{1}{\sqrt{1-(u^2/a^2)}}\left(\frac{u'}{a}\right) = \frac{u'}{\sqrt{a^2-u^2}}$$

So, $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right) + C.$

76. $$\frac{d}{dx}\left[\frac{1}{a}\arctan\frac{u}{a} + C\right] = \frac{1}{a}\left[\frac{u'/a}{1 + (u/a)^2}\right]$$

$$= \frac{1}{a^2}\left[\frac{u'}{(a^2 + u^2)/a^2}\right] = \frac{u'}{a^2 + u^2}$$

So, $\int\frac{du}{a^2 + u^2} = \int\frac{u'}{a^2 + u^2}\,dx = \frac{1}{a}\arctan\frac{u}{a} + C.$

77. Assume $u > 0$.

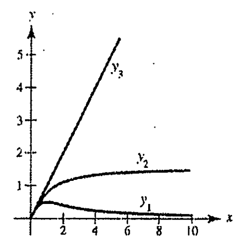
$$\frac{d}{dx}\left[\frac{1}{a}\operatorname{arcsec}\frac{u}{a} + C\right] = \frac{1}{a}\left[\frac{u'/a}{(u/a)\sqrt{(u/a)^2 - 1}}\right] = \frac{1}{a}\left[\frac{u'}{u\sqrt{(u^2 - a^2)/a^2}}\right] = \frac{u'}{u\sqrt{u^2 - a^2}}.$$

The case $u < 0$ is handled in a similar manner.

So, $\int\frac{du}{u\sqrt{u^2 - a^2}} = \int\frac{u'}{u\sqrt{u^2 - a^2}}\,dx = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C.$

78. Let $f(x) = \arctan x - \dfrac{x}{1 + x^2}$.

$$f'(x) = \frac{1}{1 + x^2} - \frac{1 - x^2}{(1 + x^2)^2} = \frac{2x^2}{(1 + x^2)} > 0 \text{ for } x > 0.$$



Because $f(0) = 0$ and f is increasing for $x > 0$,

$\arctan x - \dfrac{x}{1 + x^2} > 0$ for $x > 0$. So, $\arctan x > \dfrac{x}{1 + x^2}$.

Let $g(x) = x - \arctan x$

$$g'(x) = 1 - \frac{1}{1 + x^2} = \frac{x^2}{1 + x^2} > 0 \text{ for } x > 0.$$

Because $g(0) = 0$ and g is increasing for $x > 0$, $x - \arctan x > 0$ for $x > 0$. So, $x > \arctan x$. Therefore,

$$\frac{x}{1 + x^2} < \arctan x < x.$$

79. (a) Area $= \int_0^1 \frac{1}{1 + x^2}\,dx$

(b) Trapezoidal Rule: $n = 8, b - a = 1 - 0 = 1$

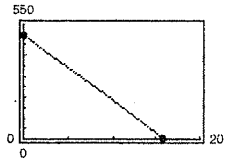
Area ≈ 0.7847

(c) Because

$$\int_0^1 \frac{1}{1 + x^2}\,dx = \left[\arctan x\right]_0^1 = \frac{\pi}{4},$$

you can use the Trapezoidal Rule to approximate $\pi/4$, and therefore, π. For example, using $n = 200$, you obtain $\pi \approx 4(0.785397) = 3.141588$.

80. (a) $v(t) = -32t + 500$



(b) $s(t) = \int v(t)\,dt = \int(-32t + 500)\,dt$

$= -16t^2 + 500t + C$

$s(0) = -16(0) + 500(0) + C = 0 \Rightarrow C = 0$

$s(t) = -16t^2 + 500t$

When the object reaches its maximum height, $v(t) = 0$.

$v(t) = -32t + 500 = 0$

$-32t = -500$

$t = 15.625$

$s(15.625) = -16(15.625)^2 + 500(15.625)$

$= 3906.25$ ft (Maximum height)

(c) $$\int \frac{1}{32 + kv^2}\,dv = -\int dt$$

$$\frac{1}{\sqrt{32k}}\arctan\left(\sqrt{\frac{k}{32}}v\right) = -t + C_1$$

$$\arctan\left(\sqrt{\frac{k}{32}}v\right) = -\sqrt{32k}t + C$$

$$\sqrt{\frac{k}{32}}v = \tan\left(C - \sqrt{32kt}\right)$$

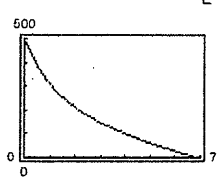
$$v = \sqrt{\frac{32}{k}}\tan\left(C - \sqrt{32kt}\right)$$

When $t = 0$, $v = 500$, $C = \arctan\left(500\sqrt{k/32}\right)$, and you have

$$v(t) = \sqrt{\frac{32}{k}}\tan\left[\arctan\left(500\sqrt{\frac{k}{32}}\right) - \sqrt{32kt}\right].$$

(d) When $k = 0.001$:

$$v(t) = \sqrt{32{,}000}\,\tan\left[\arctan\left(500\sqrt{0.00003125}\right) - \sqrt{0.032t}\right]$$



$v(t) = 0$ when $t_0 \approx 6.86$ sec.

(e) $h = \int_0^{6.86} \sqrt{32{,}000}\,\tan\left[\arctan\left(500\sqrt{0.00003125}\right) - \sqrt{0.032}\,t\right]dt$

Simpson's Rule: $n = 10$; $h \approx 1088$ feet

(f) Air resistance lowers the maximum height.