

6.10 Partial Fractions p. 502-504 #3, 5, 7, 21, 31, 49

$$3) \int \frac{dx}{(x-2)(x+1)} \rightarrow \frac{A}{x-2} + \frac{B}{x+1} \quad \left| \begin{array}{l} 1 = A(x+1) + B(x-2) \\ x=-1 \rightarrow B = -1/3 \\ x=2 \rightarrow A = 1/3 \end{array} \right.$$

$$\frac{1}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$$

$$\int \frac{1/3}{x-2} + \frac{-1/3}{x+1} dx = \frac{1}{3} \ln|x-2| - \frac{1}{3} \ln|x+1| + C \quad \text{or} \quad \frac{1}{3} \ln \left| \frac{x-2}{x+1} \right| + C$$

$$5) \int \frac{x}{(x-1)(x-2)} dx = \frac{A}{x-1} + \frac{B}{x-2} \quad \begin{array}{l} x=1 \quad A=-1 \\ x=2 \quad B=2 \end{array}$$

$$\int \frac{-1}{x-1} + \frac{2}{x-2} dx \rightarrow -\ln|x-1| + 2\ln|x-2| + C$$

$$7) \int \frac{x dx}{(3x-2)(2x+1)} = \frac{A}{3x-2} + \frac{B}{2x+1}$$

$$\begin{array}{l} x=2/3 \\ x=-1/2 \end{array}$$

$$A = \frac{2/3}{7/3} = \frac{2}{7}$$

$$B = \frac{-1/2}{-3/2 - 1/2} = \frac{-1}{-2} = 1/2$$

$$\int \frac{2}{7} \cdot \frac{1}{3x-2} dx + \frac{1}{7} \int \frac{1}{2x+1} dx$$

$$\begin{array}{l} u=3x+1 \\ \frac{du}{dx} = 2 \\ dx = \frac{du}{2} \end{array}$$

$$\begin{array}{l} u=3x-2 \\ \frac{du}{dx} = 3 \\ dx = \frac{du}{3} \end{array}$$

$$\int \frac{2}{7} \cdot \frac{1}{u} \cdot \frac{du}{3}$$

$$+ \frac{1}{7} \int \frac{1}{u} \cdot \frac{du}{2}$$

$$\int \frac{2}{21} \cdot \frac{1}{u} du$$

$$+ \frac{1}{14} \int \frac{1}{u} du$$

$$\frac{2}{21} \ln|3x-2| + \frac{1}{14} \ln|2x+1| + C$$

6.10 HW

$$21) \int \frac{x}{x^2+2x-3} dx \rightarrow \int \frac{x}{(x+3)(x-1)} dx \rightarrow \int \frac{A}{x+3} + \frac{B}{x-1} dx$$

$A = -\frac{3}{4}$ $B = \frac{1}{4}$
 $x = -3$ $x = 1$

$$\int \frac{3}{4} \left(\frac{1}{x+3} \right) dx + \frac{1}{4} \left(\frac{1}{x-1} \right) dx$$

$$\frac{3}{4} \ln|x+3| + \frac{1}{4} \ln|x-1| + C$$

$$31) \int_0^1 \frac{dx}{x^2-9} \rightarrow \int \frac{dx}{(x+3)(x-3)} \rightarrow \frac{A}{x+3} + \frac{B}{x-3}$$

$A = -\frac{1}{6}$ $B = \frac{1}{6}$
 $x = -3$ $x = 3$

$$\int \left[-\frac{1}{6} \left(\frac{1}{x+3} \right) + \frac{1}{6} \left(\frac{1}{x-3} \right) \right] dx \rightarrow \left[-\frac{1}{6} \ln|x+3| + \frac{1}{6} \ln|x-3| \right]_0^1$$

$$-\frac{1}{6} \ln(4) + \frac{1}{6} \ln(2) - \left(-\frac{1}{6} \ln(3) + \frac{1}{6} \ln(3) \right)$$

$$\frac{1}{6} \ln 2 - \frac{1}{6} \ln 4 = \frac{1}{6} \ln\left(\frac{1}{2}\right) \text{ or } \frac{1}{6} \ln(2)^{-1} = -\frac{1}{6} \ln(2)$$

$$49) \int_3^5 \frac{4}{x^2-4} dx \rightarrow \frac{A}{x+2} + \frac{B}{x-2} \rightarrow \int \frac{-1}{x+2} + \frac{1}{x-2} dx$$

$A = -1$ $B = 1$
 $x = -2$ $x = 2$

$$\int \frac{4}{(x+2)(x-2)} dx \rightarrow$$

$$\left[-\ln|x+2| + \ln|x-2| \right]_3^5$$

$$-\ln|7| + \ln|3| - \left(-\ln(5) + \ln(1) \right)$$

$$\ln 3 + \ln 5 - \ln 7 = \ln\left(\frac{3 \cdot 5}{7}\right) = \ln\left(\frac{15}{7}\right)$$