BC Calculus – 6.10 Notes - Linear Partial Fractions

Let $F(x) = \frac{P(x)}{Q(x)}$ be a rational function where $P(x)$ and $Q(x)$ are polynomials. If the degree of the numerator is smaller than the degree of the denominator (**proper rational function**), then you might be able to use partial fraction decomposition to change the integrand to something easier to integrate. Let's "decompose" this fraction to make it easier to integrate.

Partial Fraction Decomposition

Every proper rational function can be written as a sum

$$\frac{P(x)}{Q(x)} = F_1(x) + F_2(x) + F_3(x) + \cdots + F_n(x).$$

Where $F_1(x), F_2(x), F_3(x), \ldots, F_n(x)$ are also rational functions, in which the denominators are factors of $Q(x)$. We are only concerned with nonrepeating linear factors.

With $Q(x)$ having *n* nonrepeating linear factors, we can write

$$Q(x) = (a_1x + b_1)(a_2x + b_2)(a_3x + b_3)\ldots(a_nx + b_n)$$

Therefore, using the process of partial fractions and with $Q(x)$ having *n* nonrepeating linear factors, we can write $\frac{P(x)}{Q(X)} =$

1. Evaluate $\int \frac{1}{(x-1)(x+2)}dx$

2. Evaluate $\int \frac{2x^2-7}{x^3-3x^2-4x}dx$

Practice Problems

Evaluate using partial fractions.

1. $\int \frac{x-12}{x^2-4x}\,dx$

2. $\int \frac{2x}{x^2-4}\,dx$

3. $\int \frac{1}{(x+2)(x-3)(x+1)}\,dx$

4. $\int \frac{x+2}{x^2+5x}\,dx$

5. $\int \frac{2}{x(x-2)}\,dx$

6. $\int \frac{x^3-11x-15}{x^2-2x-8}\,dx$

7. For $0 < P < 50$, what is the antiderivative of $\frac{1}{P(50-P)}$?

8. $\int_0^1 \frac{1}{(x+5)(x+1)}\,dx$

9. $\int_0^2 \frac{3}{(4x+1)(x+1)}\,dx$

10. $\int_2^3 \frac{3}{(x-1)(x+2)}\,dx$