6.12 - Evaluating Improper Integrals

p.523–526 #7, 11, 15, 19, 23, 27, 31, 35, 45

7) $\int_0^\infty x^2 dx \quad \lim_{b\to\infty}\int_0^b x^2 dx \to \lim_{b\to\infty} \frac{x^3}{3}\Big]_0^b \to \lim_{b\to\infty} \frac{b^3}{3} - \frac{0}{3} = \infty$

$\boxed{\text{diverges to } \infty}$

Improper since limit of integration is ∞

11) $\int_0^1 \frac{1}{x}dx \quad \lim_{b\to 0^+}\int_b^1 \frac{1}{x}dx \to \ln|x|\Big]_b^1 = \lim_{b\to 0^+} \ln|1| - \ln|b| = 0 - \infty = \boxed{-\infty}$

diverges to $-\infty$

15) $\int_1^\infty \frac{dx}{x^3} \to \lim_{b\to\infty}\int_1^b x^{-3}dx \to \frac{x^{-2}}{-2}\Big]_1^b \to \lim_{b\to\infty} -\frac{1}{2b^2} - \left(-\frac{1}{2(1)^2}\right) = 0 + \frac{1}{2} = \boxed{\frac{1}{2}}$

$\boxed{\text{converges to } 1/2}$

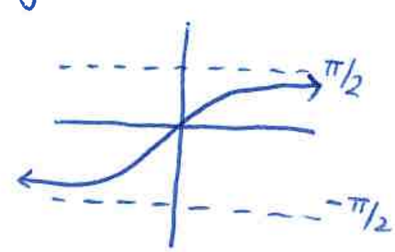
19) $\int_{-\infty}^{-1} \frac{4}{x}dx \quad \lim_{b\to-\infty}\int_b^{-1} \frac{4}{x}dx = 4\ln|x|\Big]_b^{-1} \quad \lim_{b\to-\infty} 4\ln|-1| - 4\ln|b|$

$= 0 - \infty = \boxed{-\infty}$

Diverges to $-\infty$

23) $\int_{-\infty}^{\infty} \frac{dx}{x^2+4} \quad \int_{-\infty}^{0} \frac{dx}{(x)^2+(2)^2} + \int_0^\infty \frac{dx}{(x)^2+(2)^2}$

$y = \arctan(x)$



$\lim_{b\to-\infty} \frac{1}{2}\arctan\left(\frac{x}{2}\right)\Big]_b^0 + \lim_{b\to\infty} \frac{1}{2}\arctan\left(\frac{x}{2}\right)\Big]_0^b$

$\frac{1}{2}\arctan(0) - \frac{1}{2}\arctan\left(\frac{b}{2}\right) + \frac{1}{2}\arctan\left(\frac{b}{2}\right) - \frac{1}{2}\arctan(0)$

$0 - \frac{1}{2}\left(\frac{-\pi}{2}\right) + \frac{1}{2}\left(\pi/2\right) - 0$

$\frac{\pi}{4} + \frac{\pi}{4} = \boxed{\frac{\pi}{2}}$

$\boxed{6.12}$ HW

27) $\int_0^1 \frac{dx}{x} = \ln|x|\Big]_0^1 \rightarrow \lim_{b \to 0^+} \ln|x|\Big]_b^1 = \ln|1| - \ln|b|$

$0 - (-\infty) = +\infty$

$\boxed{\text{diverges to } +\infty}$

31) $\int_{-1}^1 \frac{dx}{\sqrt[3]{x}}$ ← * vertical asymptote at $x=0$

$\lim_{a \to 0^-} \int_{-1}^a x^{-1/3} dx + \lim_{b \to 0^+} \int_b^1 x^{-1/3} dx \rightarrow \frac{x^{2/3}}{2/3}\Big]_b^1$

$\frac{x^{2/3}}{2/3}\Big]_{-1}^a = \frac{3}{2}a^{2/3} - \frac{3}{2}(-1)^{2/3} + \frac{3}{2}(1)^{2/3} - \frac{3}{2}(b)^{2/3}$

$= 0 - \frac{3}{2} + \frac{3}{2} - 0 = \boxed{0}$ Integral converges to 0.

35) $\int_{-\infty}^0 e^x dx = \lim_{b \to -\infty} \int_b^0 e^x dx \rightarrow \lim_{b \to -\infty} e^x\Big]_b^0 \rightarrow e^0 - e^b$

$\rightarrow 1 - \frac{1}{e^\infty} = \boxed{1}$

Integral converges to 1.

$\boxed{6.12}$ HW

45) $\displaystyle\int_0^{\infty} \frac{x}{(x+1)^{5/2}}\,dx$ $\quad u = x+1$ $\quad \frac{du}{dx} = 1$ $\quad \displaystyle\int \frac{x}{u^{5/2}}\,du$ $\quad x = u-1$ $\quad \displaystyle\int \frac{u-1}{u^{5/2}}\,du = \int \frac{u}{u^{5/2}} - \frac{1}{u^{5/2}}$

$$\int u^{-3/2} - u^{-5/2}\,du \qquad \frac{u^{-1/2}}{-1/2} - \frac{u^{-3/2}}{-3/2} \to \frac{-2}{u^{1/2}} + \frac{2}{3u^{3/2}}$$

$$\left.\frac{-2}{(x+1)^{1/2}} + \frac{2}{3(x+1)^{3/2}}\right]_0^b \to \lim_{b\to\infty} \frac{-2}{(b+1)^{1/2}} + \frac{2}{3(b+1)^{3/2}} - \left(\frac{-2}{1^{1/2}} + \frac{2}{3(1)^{3/2}}\right)$$

$$\to 0 + 0 + 2 - \frac{2}{3} = \frac{6}{3} - \frac{2}{3} = \boxed{\frac{4}{3}}$$

$\boxed{\text{Integral converges to } 4/3}$