BC Calculus – 6.12 Notes – Improper Integrals

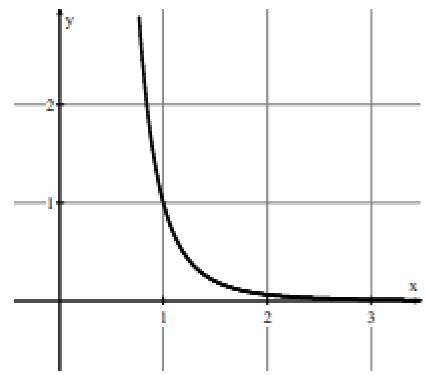
Improper integrals are integrals with infinite limits of integration or have an infinite discontinuity on the interval.

If $f(x)$ is continuous on $[a, \infty)$, then $\int_a^\infty f(x)\,dx =$

If $f(x)$ is continuous on $(-\infty, b]$, then $\int_{-\infty}^b f(x)\,dx =$

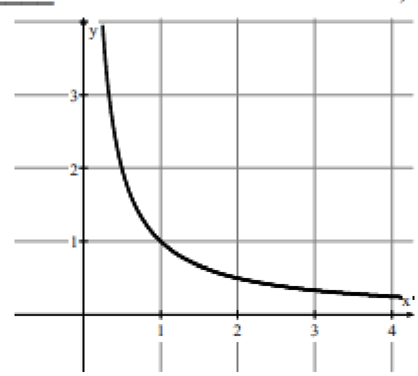
Provided the limits exist!

1. $\int_1^\infty \frac{1}{x^4} dx$



If the limit exists, the improper integral is said to __________. If the limit does not exist, the integral is said to _____________.

2. $\int_1^\infty \frac{1}{x} dx$



Improper *p-integral*: $\int_1^\infty \frac{1}{x^p} dx$ converges if and diverges if

Remember the definite integral $\int_a^b f(x)\,dx$, requires the interval to be finite and the FTC requires that $f(x)$ be continuous on $[a,b]$. If the integral does not meet these requirements, we may need to manipulate the problem.

Another form of the Improper Integral is $\int_{-\infty}^{\infty} f(x)\,dx$, with $f(x)$ continuous on $(-\infty,\infty)$. Let $x = c$ be any real number in the interval $(-\infty,\infty)$, then

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{c} f(x)\,dx + \int_{c}^{\infty} f(x)\,dx.$$

(It's easiest to use 0 here for c). If either of these integrals diverge, then the whole diverges.

3. $\int_{-\infty}^{\infty} e^x\,dx$

If $f(x)$ is continuous on $[a,b)$ and has an infinite discontinuity at b, then

$$\int_a^b f(x)\,dx =$$

If $f(x)$ is continuous on $(a,b]$ and has an infinite discontinuity at a, then

$$\int_a^b f(x)\,dx =$$

4. $\int_0^2 \frac{x+2}{\sqrt{x^2+4x}}\,dx$

If $f(x)$ is continuous on the interval $[a,b]$, except for some c in (a,b) at which f has an infinite discontinuity, then $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$.

5. $\int_{-1}^{1} \frac{1}{x}\,dx$

Evaluate each integral.

1. $\int_1^{\infty} \frac{1}{x^2}\,dx$

2. $\int_0^{\infty} \frac{2}{x^2+4x+3}\,dx$

3. $\int_0^{1} \frac{x+1}{\sqrt{x^2+2x}}\,dx$

4. $\int_1^{\infty} xe^{-x}\,dx$

5. $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$

6. $\int_{-1}^{0} \frac{1}{x^5}\,dx$

7. $\int_{0}^{\infty} e^{-x}\,dx$

8. Determine all the values of p for which $\int_{0}^{1} \frac{1}{x^p}\,dx$ converges.

9. If g is a twice-differentiable function, where $g(2) = 1$ and $\lim_{x\to\infty} g(x) = 8$, then $\int_2^\infty g'(x)\,dx$ is

A) -7 (B) 7 (C) 9 (D) nonexistent

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10. If R is the unbounded region between the graph of $y = \frac{x}{(1+x^2)^2}$ and the x-axis for $x \geq 0$, then the area of R is

A) -1 (B) 0 (C) $\frac{1}{2}$ (D) infinite

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11. For what values of p will $\int_1^\infty \frac{1}{x^{7p-3}}\,dx$ converge?

A) $p < 0$ (B) $p > 0$ (C) $p > \frac{4}{7}$ (D) $p < \frac{4}{7}$