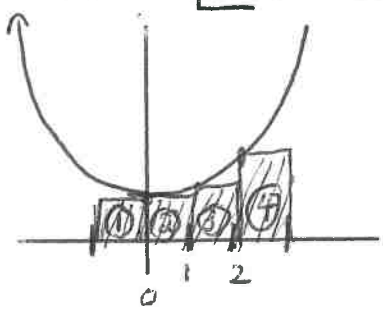
6.1 AP Practice Problems (p.398)

Key

1. The approximate area under the graph of $f(x) = x^2 + 1$ from -1 to 3 found by partitioning the interval $[-1, 3]$ into 4 subintervals of equal width and using lower sum s_4 (rectangles that lie under the graph) is

(A) 8 $\boxed{\text{(B)}}$ 9 (C) 10 (D) 18

$f(x) = x^2 + 1$

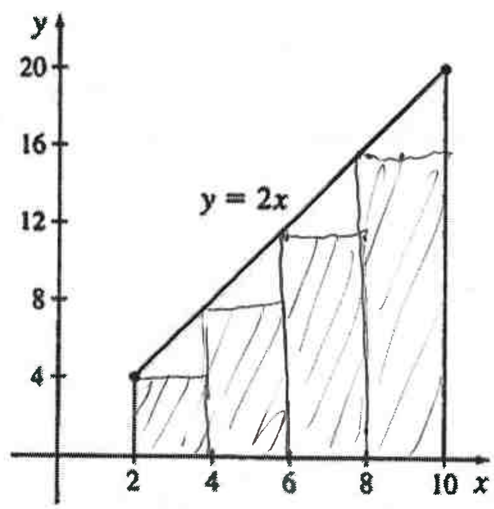


$$S_4 \approx \overbrace{1(f(0))}^{\text{Rectangle 1}} + \overbrace{1 \cdot f(0)}^{\text{Rectangle 2}} + \overbrace{1 \cdot f(1)}^{\text{Rectangle 3}} + \overbrace{1 \cdot f(2)}^{\text{Rectangle 4}}$$

$$1(1) + 1(1) + 1(2) + 1(5) = \boxed{9}$$

2. The graph of the function $f(x) = 2x$ from 2 to 10 is shown below.

a) 80
b) 112
c) 96



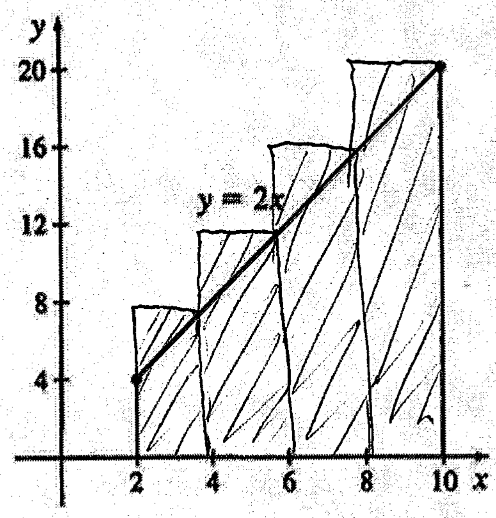
(a) Approximate the area under the graph of f by partitioning the interval [2, 10] into 4 subintervals of equal width and using lower sums s_4 (rectangles that lie under the graph of f).

$$S_4 = 2f(2) + 2f(4) + 2f(6) + 2f(8)$$

$$= 2(4) + 2(8) + 2(12) + 2(16)$$

$$= 8 + 16 + 24 + 32 = \boxed{80}$$

2. The graph of the function $f(x) = 2x$ from 2 to 10 is shown below.



(b) Approximate the area under the graph of f by partitioning the interval [2, 10] into 4 subintervals of equal width and using upper sums S_4 (rectangles that extend above the graph of f).

(c) Find the exact area under the graph using geometry.

b) $S_4 = 2(8) + 2(12) + 2(16) + 2(20) = \boxed{112}$

c) Exact Area → $A_{(\text{trapezoid})} = \frac{1}{2}w[h_1 + h_2]$

$= \frac{1}{2}(8)[4 + 20] = 4(24) = \boxed{96}$