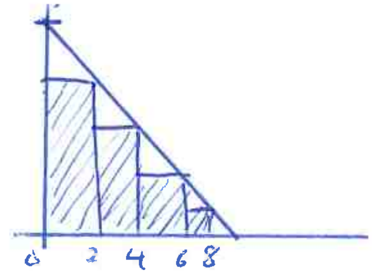
6.1 HW p.396-398 #17, 19, 29

Approximate Area using Lower Sums (A) and upper Sums (B)

17) $f(x) = -x + 10$ $[0, 8]$ $n = 4$ and $n = 8$ intervals

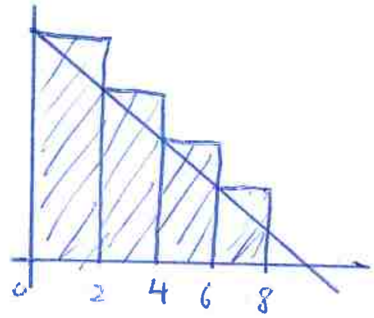
$\boxed{n = 4 \text{ intervals}}$ $w = \frac{8-0}{4} = 2$



a) Lower Sum $\rightarrow 2f(2) + 2f(4) + 2f(6) + 2f(8)$

$\rightarrow 2(8) + 2(6) + 2(4) + 2(2)$

$= \boxed{40}$



b) Upper Sum $\rightarrow 2f(0) + 2f(2) + 2f(4) + 2f(6)$

$\rightarrow 2(10) + 2(8) + 2(6) + 2(4)$

$\rightarrow \boxed{56}$

$\boxed{n = 8 \text{ intervals}}$ $w = \frac{8-0}{8} = 1$

a) Lower Sum $\rightarrow 1f(1) + 1f(2) + 1f(3) + \dots + 1f(7) + 1f(8)$

$\rightarrow 1(9) + 1(8) + 1(7) + \dots \quad 1(3) + 1(2)$

$\rightarrow \boxed{44}$

b) Upper Sum $\rightarrow 1f(0) + f(1) + f(2) + \dots \quad + f(6) + f(7)$

$\rightarrow \boxed{52}$

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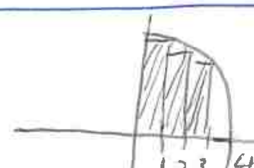
19) $f(x) = 16 - x^2$ $[0, 4]$ $w = \frac{b-a}{n} \rightarrow \frac{8-0}{4} = 2$



$\boxed{n = 4 \text{ intervals}}$

a) Lower Sum: $1f(1) + 1f(2) + 1f(3) + 1f(4)$

$= 15 + 12 + 7 + 0 = \boxed{34}$

b) Upper Sum: $1f(0) + 1f(1) + 1f(2) + 1f(3)$

$16 + 15 + 12 + 7 = \boxed{50}$

$\boxed{n = 8 \text{ intervals}}$ $w = \frac{4-0}{8} = \frac{1}{2}$

a) Lower Sum: $\frac{1}{2}f(\frac{1}{2}) + \frac{1}{2}f(1) + \frac{1}{2}f(\frac{3}{2}) + \frac{1}{2}f(2)$

$+ \frac{1}{2}f(\frac{5}{2}) + \frac{1}{2}f(3) + \frac{1}{2}f(\frac{7}{2}) + \frac{1}{2}f(4)$

$= \boxed{77/2}$

b) Upper Sum: $\frac{1}{2}f(0) + \frac{1}{2}f(\frac{1}{2}) + \dots + \frac{1}{2}f(3) + \frac{1}{2}f(\frac{7}{2})$

$= \boxed{93/2}$

$\boxed{6.1}$

29) $f(x) = 4x^2 \quad [0,2]$ $\qquad w = \frac{b-a}{n} \rightarrow \frac{2-0}{n} = \frac{2}{n}$

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{b-a}{n}\right] f\left[a + \frac{b-a}{n} i\right] \rightarrow \quad f\left[0 + \frac{2}{n} i\right] = f\left(\frac{2}{n} i\right)$$

$$f(\ ) = 4(\ )^2$$

$$f\left(\frac{2}{n} i\right) = 4\left(\frac{2}{n} i\right)^2$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n} \cdot 4\left(\frac{2}{n} i\right)^2$$

$$\sum_{i=1}^{n} \frac{8}{n} \cdot \frac{4}{n^2} i^2$$

$$\frac{32}{n^3} \sum i^2 \rightarrow \lim_{n\to\infty} \frac{32}{n^3} \sum_{i=1}^{n} \frac{n(n+1)(2n+1)}{6} \rightarrow \lim_{n\to\infty} \frac{32 \cdot 2n^3}{6n^3} \rightarrow \boxed{\frac{32}{3}}$$