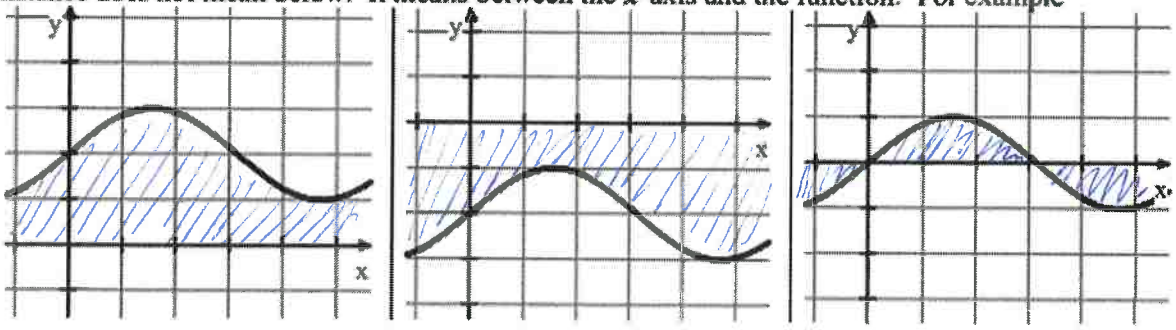
Key

Area Under the Curve:

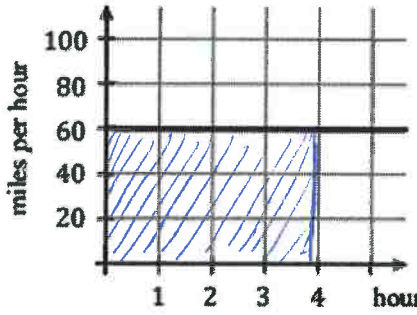
The region between a function and the x-axis is called the area under the curve. "Under" in this instance does not mean below. It means between the x-axis and the function. For example



Let's take a rate of change function and examine its graph. The area under the curve gives us the **accumulation of change.** (Adding up this "change" over time)

1. You are on a road trip and have your car on cruise control for 4 hours. You travel at 60 miles per hour. How far have you traveled?

$60 \times 4 = 240$ mi



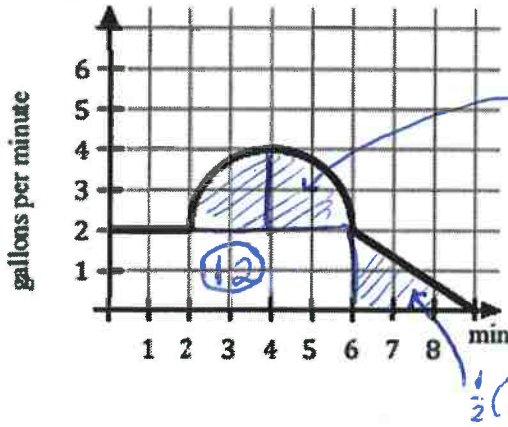
$\frac{60 \text{ miles}}{\cancel{\text{hr}}} \cdot 4 \cancel{\text{hrs}} = 240 \text{ miles}$

*How far we've traveled is the accumulation of the rate of change
Area represents how far we've gone on the trip

Units for Area Under the Curve: *Not the same as the starting units!

The dependent unit multiplied by the independent unit. In other words, the unit for y times the unit for x.

2. The graph below represents the rate at which water is leaking out of a tank. The units are given in the graph. How much water has leaked out of the tank after 9 minutes?

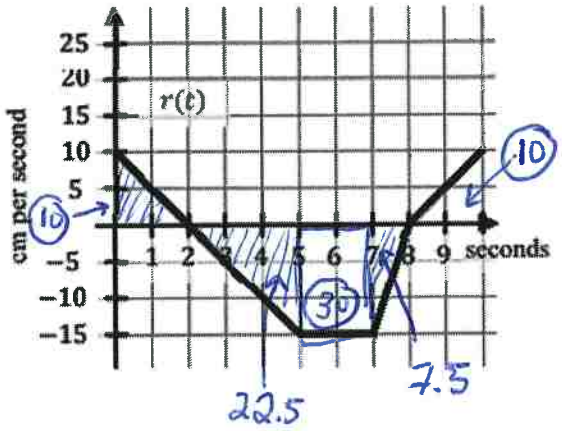


$12 + 3 + 2\pi$ gallons

$15 + 2\pi$ gallons

$\frac{\text{gallons}}{\text{min}} \times \text{mins} = \text{gallons}$

3. A particle is moving along the x-axis at a rate modeled by $r(t)$ and shown in the graph below.



$\frac{cm}{sec} \times sec = cm$

a. How far is the particle from its starting position after 10 seconds?

$10 - 22.5 - 30 - 7.5 + 10 = -40$ cm

(40 cm to the left)

b. How far is the particle from its position at $t = 2$ after $t = 8$ seconds?

$-22.5 - 30 - 7.5 = -60$ cm

(left 60 cm)

Practice: *Accumulation of change = Area

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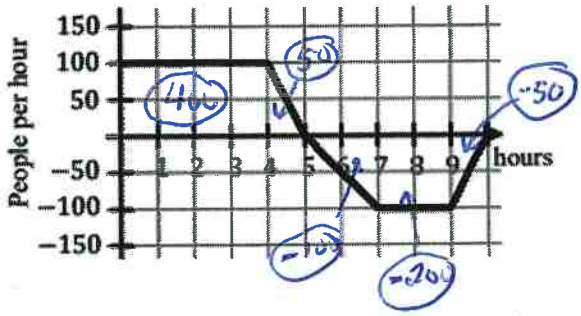
1. The graph below shows the rate of change for the number of people in a museum t hours after it opens.



a. How many people are in the museum after 5 hours?

$400 + 50 = 450$

b. How many people are in the museum after 10 hours?

$450 - 350 = 100$ people

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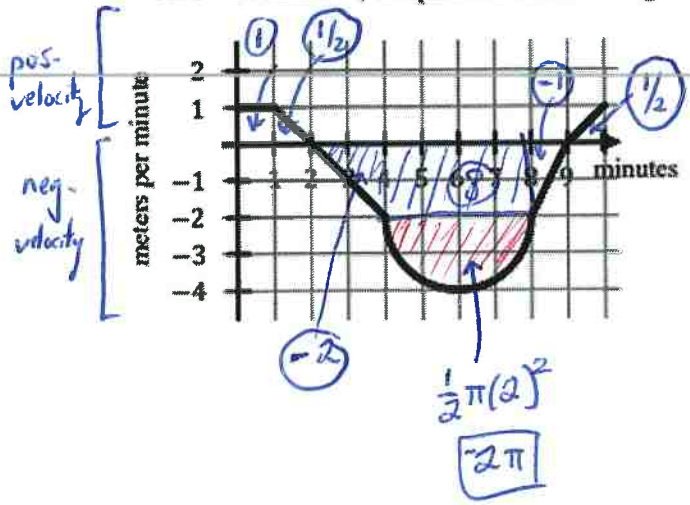
2. The graph below shows the velocity of a particle moving along the x-axis, measured in meters per minute. At $t = 0$ minutes, the particle is at the origin.



a. Where is the particle after two minutes?

1.5 meters to the right

b. Where is the particle after six minutes?

$1.5 - 2 - 4 - \pi = -4.5 - \pi$

$4.5 + \pi$ meters left

c. Where is the particle after ten minutes?

$1.5 - 2 - 8 - 2\pi - 1 + 1/2$

$= -9 - 2\pi$ or 9 + 2π meters to the left