

# CHAPTER 6

## Differential Equations

### Section 6.1 Slope Fields and Euler's Method

1. Differential equation:  $y' = 4y$

Solution:  $y = Ce^{4x}$

Check:  $y' = 4Ce^{4x} = 4y$

2. Differential equation:  $3y' + 5y = -e^{-2x}$

Solution:  $y = e^{-2x}$

$y' = -2e^{-2x}$

Check:  $3(-2e^{-2x}) + 5(e^{-2x}) = -e^{-2x}$

3. Differential equation:  $y' = \frac{2xy}{x^2 - y^2}$

Solution:  $x^2 + y^2 = Cy$

Check:  $2x + 2yy' = Cy'$

$$y' = \frac{-2x}{(2y - C)}$$

$$y' = \frac{-2xy}{2y^2 - Cy}$$

$$= \frac{-2xy}{2y^2 - (x^2 + y^2)}$$

$$= \frac{-2xy}{y^2 - x^2}$$

$$= \frac{2xy}{x^2 - y^2}$$

5. Differential equation:  $y'' + y = 0$

Solution:  $y = C_1 \sin x - C_2 \cos x$

$y' = C_1 \cos x + C_2 \sin x$

$y'' = -C_1 \sin x + C_2 \cos x$

Check:  $y'' + y = (-C_1 \sin x + C_2 \cos x) + (C_1 \sin x - C_2 \cos x) = 0$

6. Differential equation:  $y'' + 2y' + 2y = 0$

Solution:  $y = C_1 e^{-x} \cos x + C_2 e^{-x} \sin x$

Check:  $y' = -(C_1 + C_2)e^{-x} \sin x + (-C_1 + C_2)e^{-x} \cos x$

$y'' = 2C_1 e^{-x} \sin x - 2C_2 e^{-x} \cos x$

$y'' + 2y' + 2y = 2C_1 e^{-x} \sin x - 2C_2 e^{-x} \cos x +$

$2(-C_1 + C_2)e^{-x} \sin x + (-C_1 + C_2)e^{-x} \cos x + 2(C_1 e^{-x} \cos x + C_2 e^{-x} \sin x)$

$= (2C_1 - 2C_1 - 2C_2 + 2C_2)e^{-x} \sin x + (-2C_2 - 2C_1 + 2C_2 + 2C_1)e^{-x} \cos x = 0$

4. Differential equation:  $\frac{dy}{dx} = \frac{xy}{y^2 - 1}$

Solution:  $y^2 - 2 \ln y = x^2$

Check:  $2yy' - \frac{2}{y}y' = 2x$

$$\left(y - \frac{1}{y}\right)y' = x$$

$$y' = \frac{x}{y - \frac{1}{y}}$$

$$y' = \frac{xy}{y^2 - 1}$$

7. Differential equation:  $y'' + y = \tan x$

Solution:  $y = -\cos x \ln|\sec x + \tan x|$

$$\begin{aligned} y' &= (-\cos x) \frac{1}{\sec x + \tan x} (\sec x \cdot \tan x + \sec^2 x) + \sin x \ln|\sec x + \tan x| \\ &= \frac{(-\cos x)}{\sec x + \tan x} (\sec x)(\tan x + \sec x) + \sin x \ln|\sec x + \tan x| \\ &= -1 + \sin x \ln|\sec x + \tan x| \end{aligned}$$

$$\begin{aligned} y'' &= (\sin x) \frac{1}{\sec x + \tan x} (\sec x \cdot \tan x + \sec^2 x) + \cos x \ln|\sec x + \tan x| \\ &= (\sin x)(\sec x) + \cos x \ln|\sec x + \tan x| \end{aligned}$$

Check:  $y'' + y = (\sin x)(\sec x) + \cos x \ln|\sec x + \tan x| - \cos x \ln|\sec x + \tan x| = \tan x$ .

8. Differential equation:  $y'' + 4y' = 2e^x$

Solution:  $y = \frac{2}{5}(e^{-4x} + e^x)$

$$y' = \frac{2}{5}(-4e^{-4x} + e^x) = -\frac{8}{5}e^{-4x} + \frac{2}{5}e^x$$

$$y'' = \frac{32}{5}e^{-4x} + \frac{2}{5}e^x$$

Check:  $y'' + 4y' = \left(\frac{32}{5}e^{-4x} + \frac{2}{5}e^x\right) + 4\left(-\frac{8}{5}e^{-4x} + \frac{2}{5}e^x\right) = \left(\frac{2}{5} + \frac{8}{5}\right)e^x = 2e^x$

9.  $y = \sin x \cos x - \cos^2 x$

$$\begin{aligned} y' &= -\sin^2 x + \cos^2 x + 2 \cos x \sin x \\ &= -1 + 2 \cos^2 x + \sin 2x \end{aligned}$$

Differential equation:

$$\begin{aligned} 2y + y' &= 2(\sin x \cos x - \cos^2 x) + (-1 + 2 \cos^2 x + \sin 2x) \\ &= 2 \sin x \cos x - 1 + \sin 2x \\ &= 2 \sin 2x - 1 \end{aligned}$$

Initial condition  $\left(\frac{\pi}{4}, 0\right)$ :

$$\sin \frac{\pi}{4} \cos \frac{\pi}{4} - \cos^2 \frac{\pi}{4} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} - \left(\frac{\sqrt{2}}{2}\right)^2 = 0$$

10.  $y = 6x - 4 \sin x + 1$

$$y' = 6 - 4 \cos x$$

Differential equation:  $y' = 6 - 4 \cos x$

Initial condition  $(0, 1)$ :  $0 - 0 + 1 = 1$

11.  $y = 4e^{-6x^2}$

$$y' = 4e^{-6x^2}(-12x) = -48xe^{-6x^2}$$

Differential equation:

$$y' = -12xy = -12x(4e^{-6x^2}) = -48xe^{-6x^2}$$

Initial condition  $(0, 4)$ :  $4e^0 = 4$

12.  $y = e^{-\cos x}$

$$y' = e^{-\cos x}(\sin x) = \sin x \cdot e^{-\cos x}$$

Differential equation:

$$y' = \sin x \cdot e^{-\cos x} = \sin x(y) = y \sin x$$

Initial condition  $\left(\frac{\pi}{2}, 1\right)$ :  $e^{-\cos(\pi/2)} = e^0 = 1$

In Exercises 13–20, the differential equation is  $y^{(4)} - 16y = 0$ .

13.  $y = 3 \cos x$   
 $y^{(4)} = 3 \cos x$   
 $y^{(4)} - 16y = -45 \cos x \neq 0$ ,  
 No

14.  $y = 2 \sin x$   
 $y^{(4)} = 2 \sin x$   
 $y^{(4)} - 16y = 2 \sin x - 16(2 \sin x) \neq 0$   
 No

15.  $y = 3 \cos 2x$   
 $y^{(4)} = 48 \cos 2x$   
 $y^{(4)} - 16y = 48 \cos 2x - 48 \cos 2x = 0$ ,  
 Yes

19.  $y = C_1 e^{2x} + C_2 e^{-2x} + C_3 \sin 2x + C_4 \cos 2x$   
 $y^{(4)} = 16C_1 e^{2x} + 16C_2 e^{-2x} + 16C_3 \sin 2x + 16C_4 \cos 2x$   
 $y^{(4)} - 16y = 0$ ,  
 Yes

20.  $y = 3e^{2x} - 4 \sin 2x$   
 $y^{(4)} = 48e^{2x} - 64 \sin 2x$   
 $y^{(4)} - 16y = (48e^{2x} - 64 \sin 2x) - 16(3e^{2x} - 4 \sin 2x) = 0$ ,  
 Yes

In Exercises 21–28, the differential equation is  $xy' - 2y = x^3 e^x$ .

21.  $y = x^2, y' = 2x$   
 $xy' - 2y = x(2x) - 2(x^2) = 0 \neq x^3 e^x$ ,  
 No

22.  $y = x^3, y' = 3x^2$   
 $xy' - 2y = x(3x^2) - 2x^3 = x^3 \neq x^3 e^x$   
 No

23.  $y = x^2 e^x, y' = x^2 e^x + 2x e^x = e^x(x^2 + 2x)$   
 $xy' - 2y = x(e^x(x^2 + 2x)) - 2(x^2 e^x) = x^3 e^x$ ,  
 Yes

24.  $y = x^2(2 + e^x), y' = x^2(e^x) + 2x(2 + e^x)$   
 $xy' - 2y = x[x^2 e^x + 2x e^x + 4x] - 2[x^2 e^x + 2x^2]$   
 $= x^3 e^x$ ,  
 Yes

16.  $y = 3 \sin 2x$   
 $y^{(4)} = 48 \sin 2x$   
 $y^{(4)} - 16y = 48 \sin 2x - 16(3 \sin 2x) = 0$   
 Yes

17.  $y = e^{-2x}$   
 $y^{(4)} = 16e^{-2x}$   
 $y^{(4)} - 16y = 16e^{-2x} - 16e^{-2x} = 0$ ,  
 Yes

18.  $y = 5 \ln x$   
 $y^{(4)} = -\frac{30}{x^4}$   
 $y^{(4)} - 16y = -\frac{30}{x^4} - 80 \ln x \neq 0$ ,  
 No

25.  $y = \sin x, y' = \cos x$   
 $xy' - 2y = x(\cos x) - 2(\sin x) \neq x^3 e^x$ ,  
 No

26.  $y = \cos x, y' = -\sin x$   
 $xy' - 2y = x(-\sin x) - 2 \cos x \neq x^3 e^x$   
 No

27.  $y = \ln x, y' = \frac{1}{x}$   
 $xy' - 2y = x\left(\frac{1}{x}\right) - 2 \ln x \neq x^3 e^x$ ,  
 No

28.  $y = x^2 e^x - 5x^2, y' = x^2 e^x + 2x e^x - 10x$   
 $xy' - 2y = x[x^2 e^x + 2x e^x - 10x] - 2[x^2 e^x - 5x^2]$   
 $= x^3 e^x$ ,  
 Yes

29.  $y = Ce^{-x/2}$  passes through  $(0, 3)$ .

$$3 = Ce^0 = C \Rightarrow C = 3$$

Particular solution:  $y = 3e^{-x/2}$

30.  $y(x^2 + y) = C$  passes through  $(0, 2)$ .

$$2(0 + 2) = C \Rightarrow C = 4$$

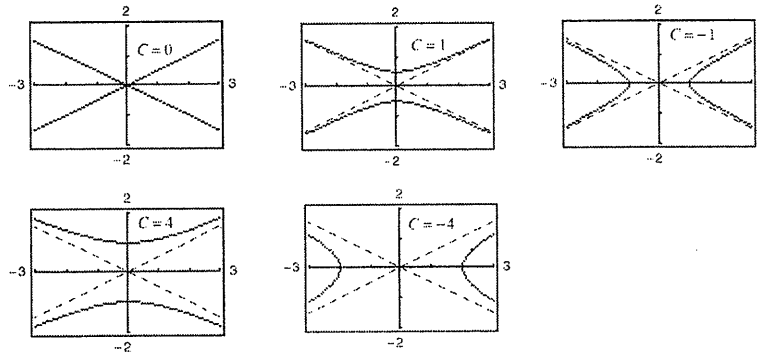
Particular solution:  $y(x^2 + y) = 4$

33. Differential equation:  $4yy' - x = 0$

General solution:  $4y^2 - x^2 = C$

Particular solutions:  $C = 0$ , Two intersecting lines

$C = \pm 1, C = \pm 4$ , Hyperbolas

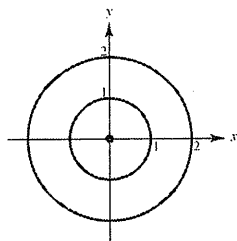


34. Differential equation:  $yy' + x = 0$

General solution:  $x^2 + y^2 = C$

Particular solutions:  $C = 0$ , Point

$C = 1, C = 4$ , Circles



31.  $y^2 = Cx^3$  passes through  $(4, 4)$ .

$$16 = C(64) \Rightarrow C = \frac{1}{4}$$

Particular solution:  $y^2 = \frac{1}{4}x^3$  or  $4y^2 = x^3$

32.  $2x^2 - y^2 = C$  passes through  $(3, 4)$ .

$$2(9) - 16 = C \Rightarrow C = 2$$

Particular solution:  $2x^2 - y^2 = 2$

35. Differential equation:  $y' + 2y = 0$

General solution:  $y = Ce^{-2x}$

$$y' + 2y = C(-2)e^{-2x} + 2(Ce^{-2x}) = 0$$

Initial condition  $(0, 3)$ :  $3 = Ce^0 = C$

Particular solution:  $y = 3e^{-2x}$

36. Differential equation:  $3x + 2yy' = 0$

General solution:  $3x^2 + 2y^2 = C$

$$6x + 4yy' = 0$$

$$2(3x + 2yy') = 0$$

$$3x + 2yy' = 0$$

Initial condition  $(1, 3)$ :

$$3(1)^2 + 2(3)^2 = 3 + 18 = 21 = C$$

Particular solution:  $3x^2 + 2y^2 = 21$

37. Differential equation:  $y'' + 9y = 0$ General solution:  $y = C_1 \sin 3x + C_2 \cos 3x$ 

$$y' = 3C_1 \cos 3x - 3C_2 \sin 3x,$$

$$y'' = -9C_1 \sin 3x - 9C_2 \cos 3x$$

$$y'' + 9y = (-9C_1 \sin 3x - 9C_2 \cos 3x) + 9(C_1 \sin 3x + C_2 \cos 3x) = 0$$

Initial conditions  $\left(\frac{\pi}{6}, 2\right)$  and  $y' = 1$  when  $x = \frac{\pi}{6}$ :

$$2 = C_1 \sin\left(\frac{\pi}{6}\right) + C_2 \cos\left(\frac{\pi}{6}\right) \Rightarrow C_1 = 2$$

$$y' = 3C_1 \cos 3x - 3C_2 \sin 3x$$

$$1 = 3C_1 \cos\left(\frac{\pi}{6}\right) - 3C_2 \sin\left(\frac{\pi}{6}\right) = -3C_2 \Rightarrow C_2 = -\frac{1}{3}$$

Particular solution:  $y = 2 \sin 3x - \frac{1}{3} \cos 3x$ 38. Differential equation:  $xy'' + y' = 0$ General solution:  $y = C_1 + C_2 \ln x$ 

$$y' = C_2 \left(\frac{1}{x}\right), y'' = -C_2 \left(\frac{1}{x^2}\right)$$

$$xy'' + y' = x \left(-C_2 \frac{1}{x^2}\right) + C_2 \frac{1}{x} = 0$$

Initial conditions  $(2, 0)$  and  $y' = \frac{1}{2}$  when  $x = 2$ :

$$0 = C_1 + C_2 \ln 2$$

$$y' = \frac{C_2}{x}$$

$$\frac{1}{2} = \frac{C_2}{2} \Rightarrow C_2 = 1, C_1 = -\ln 2$$

Particular solution:  $y = -\ln 2 + \ln x = \ln \frac{x}{2}$ 39. Differential equation:  $x^2 y'' - 3xy' + 3y = 0$ General solution:  $y = C_1 x + C_2 x^3$ 

$$y' = C_1 + 3C_2 x^2, y'' = 6C_2 x$$

$$x^2 y'' - 3xy' + 3y = x^2(6C_2 x) - 3x(C_1 + 3C_2 x^2) + 3(C_1 x + C_2 x^3) = 0$$

Initial conditions  $(2, 0)$  and  $y' = 4$  when  $x = 2$ :

$$0 = 2C_1 + 8C_2$$

$$y' = C_1 + 3C_2 x^2$$

$$4 = C_1 + 12C_2$$

$$\left. \begin{array}{l} C_1 + 4C_2 = 0 \\ C_1 + 12C_2 = 4 \end{array} \right\} C_2 = \frac{1}{2}, C_1 = -2$$

Particular solution:  $y = -2x + \frac{1}{2}x^3$

40. Differential equation:  $9y'' - 12y' + 4y = 0$

General solution:  $y = e^{2x/3}(C_1 + C_2x)$

$$y' = \frac{2}{3}e^{2x/3}(C_1 + C_2x) + C_2e^{2x/3} = e^{2x/3}\left(\frac{2}{3}C_1 + C_2 + \frac{2}{3}C_2x\right)$$

$$y'' = \frac{2}{3}e^{2x/3}\left(\frac{2}{3}C_1 + C_2 + \frac{2}{3}C_2x\right) + e^{2x/3}\frac{2}{3}C_2 = \frac{2}{3}e^{2x/3}\left(\frac{2}{3}C_1 + 2C_2 + \frac{2}{3}C_2x\right)$$

$$9y'' - 12y' + 4y = 9\left(\frac{2}{3}e^{2x/3}\right)\left(\frac{2}{3}C_1 + 2C_2 + \frac{2}{3}C_2x\right) - 12\left(e^{2x/3}\right)\left(\frac{2}{3}C_1 + C_2 + \frac{2}{3}C_2x\right) + 4\left(e^{2x/3}\right)(C_1 + C_2x) = 0$$

Initial conditions (0, 4) and (3, 0):

$$0 = e^2(C_1 + 3C_2)$$

$$4 = (1)(C_1 + 0) \Rightarrow C_1 = 4$$

$$0 = e^2(4 + 3C_2) \Rightarrow C_2 = -\frac{4}{3}$$

Particular solution:  $y = e^{2x/3}\left(4 - \frac{4}{3}x\right)$

41.  $\frac{dy}{dx} = 6x^2$

$$y = \int 6x^2 dx = 2x^3 + C$$

42.  $\frac{dy}{dx} = 10x^4 - 2x^3$

$$y = \int (10x^4 - 2x^3) dx = 2x^5 - \frac{x^4}{2} + C$$

43.  $\frac{dy}{dx} = \frac{x}{1+x^2}$

$$y = \int \frac{x}{1+x^2} dx = \frac{1}{2} \ln(1+x^2) + C$$

$$(u = 1+x^2, du = 2x dx)$$

44.  $\frac{dy}{dx} = \frac{e^x}{4+e^x}$

$$y = \int \frac{e^x}{4+e^x} dx = \ln(4+e^x) + C$$

45.  $\frac{dy}{dx} = \frac{x-2}{x} = 1 - \frac{2}{x}$

$$y = \int \left(1 - \frac{2}{x}\right) dx$$

$$= x - 2 \ln|x| + C = x - \ln x^2 + C$$

46.  $\frac{dy}{dx} = x \cos x^2$

$$y = \int x \cos(x^2) dx = \frac{1}{2} \sin(x^2) + C$$

$$(u = x^2, du = 2x dx)$$

47.  $\frac{dy}{dx} = \sin 2x$

$$y = \int \sin 2x dx = -\frac{1}{2} \cos 2x + C$$

$$(u = 2x, du = 2 dx)$$

48.  $\frac{dy}{dx} = \tan^2 x = \sec^2 x - 1$

$$y = \int (\sec^2 x - 1) dx = \tan x - x + C$$

49.  $\frac{dy}{dx} = x\sqrt{x-6}$

Let  $u = \sqrt{x-6}$ , then  $x = u^2 + 6$  and  $dx = 2u du$ .

$$\begin{aligned} y &= \int x\sqrt{x-6} dx = \int (u^2 + 6)(u)(2u) du \\ &= 2 \int (u^4 + 6u^2) du \\ &= 2 \left( \frac{u^5}{5} + 2u^3 \right) + C \\ &= \frac{2}{5}(x-6)^{5/2} + 4(x-6)^{3/2} + C \\ &= \frac{2}{5}(x-6)^{3/2}(x-6+10) + C \\ &= \frac{2}{5}(x-6)^{3/2}(x+4) + C \end{aligned}$$

50.  $\frac{dy}{dx} = 2x\sqrt{4x^2 + 1}$

$$y = \int 2x\sqrt{4x^2 + 1} dx$$

$$= \frac{1}{4} \int \sqrt{4x^2 + 1} (8x) dx$$

$$= \frac{1}{4} \frac{(4x^2 + 1)^{3/2}}{(3/2)} + C$$

$$= \frac{1}{6}(4x^2 + 1)^{3/2} + C$$

51.  $\frac{dy}{dx} = xe^{x^2}$

$$y = \int xe^{x^2} dx = \frac{1}{2}e^{x^2} + C$$

( $u = x^2, du = 2x dx$ )

52.  $\frac{dy}{dx} = 5e^{-x/2}$

$$y = \int 5e^{-x/2} dx = 5(-2) \int e^{-x/2} \left(-\frac{1}{2}\right) dx = -10e^{-x/2} + C$$

53.

$x$	-4	-2	0	2	4	8
$y$	2	0	4	4	6	8
$dy/dx$	-4	Undef.	0	1	$\frac{4}{3}$	2

54.

$x$	-4	-2	0	2	4	8
$y$	2	0	4	4	6	8
$dy/dx$	6	2	4	2	2	0

55.

$x$	-4	-2	0	2	4	8
$y$	2	0	4	4	6	8
$dy/dx$	$-2\sqrt{2}$	-2	0	0	$-2\sqrt{2}$	-8

56.

$x$	-4	-2	0	2	4	8
$y$	2	0	4	4	6	8
$dy/dx$	$\sqrt{3}$	0	$-\sqrt{3}$	$-\sqrt{3}$	0	$\sqrt{3}$

57.  $\frac{dy}{dx} = \sin 2x$

For  $x = 0, \frac{dy}{dx} = 0$ . Matches (b).

58.  $\frac{dy}{dx} = \frac{1}{2} \cos x$

For  $x = 0, \frac{dy}{dx} = \frac{1}{2}$ . Matches (c).

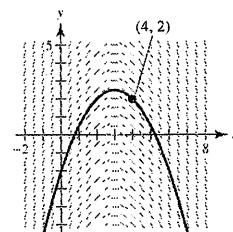
59.  $\frac{dy}{dx} = e^{-2x}$

As  $x \rightarrow \infty, \frac{dy}{dx} \rightarrow 0$ . Matches (d).

60.  $\frac{dy}{dx} = \frac{1}{x}$

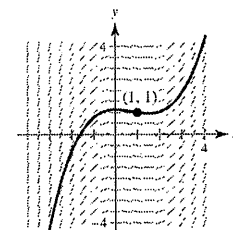
For  $x = 0, \frac{dy}{dx}$  is undefined (vertical tangent). Matches (a).

61. (a), (b)



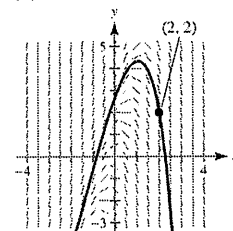
(c) As  $x \rightarrow \infty, y \rightarrow -\infty$   
As  $x \rightarrow -\infty, y \rightarrow -\infty$

62. (a), (b)



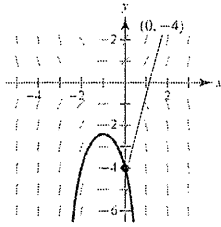
(c) As  $x \rightarrow \infty, y \rightarrow \infty$   
As  $x \rightarrow -\infty, y \rightarrow -\infty$

63. (a), (b)



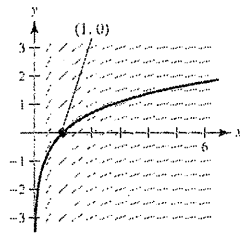
(c) As  $x \rightarrow \infty, y \rightarrow -\infty$   
As  $x \rightarrow -\infty, y \rightarrow -\infty$

64. (a), (b)



(c) As  $x \rightarrow \infty, y \rightarrow -\infty$   
As  $x \rightarrow -\infty, y \rightarrow -\infty$

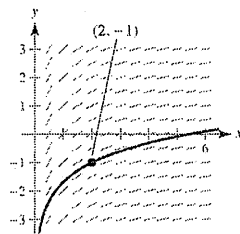
65. (a)  $y' = \frac{1}{x}, (1, 0)$



As  $x \rightarrow \infty, y \rightarrow \infty$

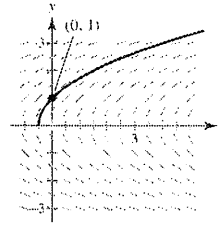
[Note: The solution is  $y = \ln x$ .]

(b)  $y' = \frac{1}{x}, (2, -1)$



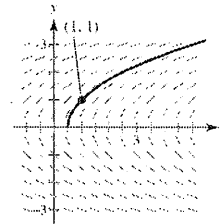
As  $x \rightarrow \infty, y \rightarrow \infty$

66. (a)  $y' = \frac{1}{y}, (0, 1)$



As  $x \rightarrow \infty, y \rightarrow \infty$

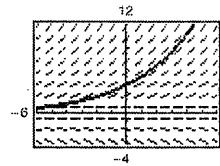
(b)  $y' = \frac{1}{y}, (1, 1)$



As  $x \rightarrow \infty, y \rightarrow \infty$

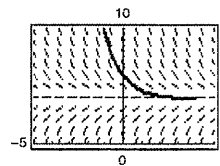
67.  $\frac{dy}{dx} = 0.25y, y(0) = 4$

(a), (b)



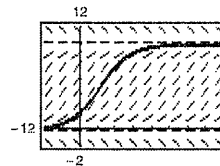
68.  $\frac{dy}{dx} = 4 - y, y(0) = 6$

(a), (b)



69.  $\frac{dy}{dx} = 0.02y(10 - y), y(0) = 2$

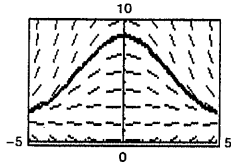
(a), (b)





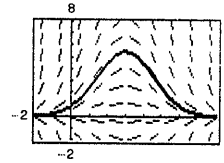
70.  $\frac{dy}{dx} = 0.2x(2 - y), y(0) = 9$

(a), (b)



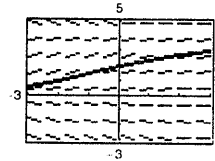
71.  $\frac{dy}{dx} = 0.4y(3 - x), y(0) = 1$

(a), (b)



72.  $\frac{dy}{dx} = \frac{1}{2}e^{-x/8} \sin \frac{\pi y}{4}, y(0) = 2$

(a), (b)



73.  $y' = x + y, y(0) = 2, n = 10, h = 0.1$

$y_1 = y_0 + hF(x_0, y_0) = 2 + (0.1)(0 + 2) = 2.2$

$y_2 = y_1 + hF(x_1, y_1) = 2.2 + (0.1)(0.1 + 2.2) = 2.43, \text{ etc.}$

$n$	0	1	2	3	4	5	6	7	8	9	10
$x_n$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$y_n$	2	2.2	2.43	2.693	2.992	3.332	3.715	4.146	4.631	5.174	5.781

74.  $y' = x + y, y(0) = 2, n = 20, h = 0.05$

$y_1 = y_0 + hF(x_0, y_0) = 2 + (0.05)(0 + 2) = 2.1$

$y_2 = y_1 + hF(x_1, y_1) = 2.1 + (0.05)(0.05 + 2.1) = 2.2075, \text{ etc.}$

The table shows the values for  $n = 0, 2, 4, \dots, 20$ .

$n$	0	2	4	6	8	10	12	14	16	18	20
$x_n$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$y_n$	2	2.208	2.447	2.720	3.032	3.387	3.788	4.240	4.749	5.320	5.960

75.  $y' = 3x - 2y, y(0) = 3, n = 10, h = 0.05$

$y_1 = y_0 + hF(x_0, y_0) = 3 + (0.05)(3(0) - 2(3)) = 2.7$

$y_2 = y_1 + hF(x_1, y_1) = 2.7 + (0.05)(3(0.05) + 2(2.7)) = 2.4375, \text{ etc.}$

$n$	0	1	2	3	4	5	6	7	8	9	10
$x_n$	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
$y_n$	3	2.7	2.438	2.209	2.010	1.839	1.693	1.569	1.464	1.378	1.308

76.  $y' = 0.5x(3 - y)$ ,  $y(0) = 1$ ,  $n = 5$ ,  $h = 0.4$

$$y_1 = y_0 + hF(x_0, y_0) = 1 + (0.4)(0.5(0)(3 - 1)) = 1$$

$$y_2 = y_1 + hF(x_1, y_1) = 1 + (0.4)(0.5(0.4)(3 - 1)) = 1.16, \text{ etc.}$$

$n$	0	1	2	3	4	5
$x_n$	0	0.4	0.8	1.2	1.6	2.0
$y_n$	1	1	1.16	1.454	1.825	2.201

77.  $y' = e^{xy}$ ,  $y(0) = 1$ ,  $n = 10$ ,  $h = 0.1$

$$y_1 = y_0 + hF(x_0, y_0) = 1 + (0.1)e^{0(1)} = 1.1$$

$$y_2 = y_1 + hF(x_1, y_1) = 1.1 + (0.1)e^{(0.1)(1.1)} \approx 1.2116, \text{ etc.}$$

$n$	0	1	2	3	4	5	6	7	8	9	10
$x_n$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$y_n$	1	1.1	1.212	1.339	1.488	1.670	1.900	2.213	2.684	3.540	5.958

78.  $y' = \cos x + \sin y$ ,  $y(0) = 5$ ,  $n = 10$ ,  $h = 0.1$

$$y_1 = y_0 + hF(x_0, y_0) = 5 + (0.1)(\cos 0 + \sin 5) \approx 5.0041$$

$$y_2 = y_1 + hF(x_1, y_1) = 5.0041 + (0.1)(\cos(0.1) + \sin(5.0041)) \approx 5.0078, \text{ etc.}$$

$n$	0	1	2	3	4	5	6	7	8	9	10
$x_n$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$y_n$	5	5.004	5.008	5.010	5.010	5.007	4.999	4.985	4.965	4.938	4.903

79.  $\frac{dy}{dx} = y$ ,  $y = 3e^x$ ,  $(0, 3)$

$x$	0	0.2	0.4	0.6	0.8	1
$y(x)$ (exact)	3	3.6642	4.4755	5.4664	6.6766	8.1548
$y(x)$ ( $h = 0.2$ )	3	3.6000	4.3200	5.1840	6.2208	7.4650
$y(x)$ ( $h = 0.1$ )	3	3.6300	4.3923	5.3147	6.4308	7.7812

80.  $\frac{dy}{dx} = \frac{2x}{y}$ ,  $y = \sqrt{2x^2 + 4}$ ,  $(0, 2)$

$x$	0	0.2	0.4	0.6	0.8	1
$y(x)$ (exact)	2	2.0199	2.0785	2.1726	2.2978	2.4495
$y(x)$ ( $h = 0.2$ )	2	2.000	2.0400	2.1184	2.2317	2.3751
$y(x)$ ( $h = 0.1$ )	2	2.0100	2.0595	2.1460	2.2655	2.4131

81.  $\frac{dy}{dx} = y + \cos x$ ,  $y = \frac{1}{2}(\sin x - \cos x + e^x)$ ,  $(0, 0)$

$x$	0	0.2	0.4	0.6	0.8	1
$y(x)$ (exact)	0	0.2200	0.4801	0.7807	1.1231	1.5097
$y(x)$ ( $h = 0.2$ )	0	0.2000	0.4360	0.7074	1.0140	1.3561
$y(x)$ ( $h = 0.1$ )	0	0.2095	0.4568	0.7418	1.0649	1.4273

82. As  $h$  increases (from 0.1 to 0.2), the error increases.

83.  $\frac{dy}{dt} = -\frac{1}{2}(y - 72)$ ,  $(0, 140)$ ,  $h = 0.1$

(a)

$t$	0	1	2	3
Euler	140	112.7	96.4	86.6

(b)  $y = 72 + 68e^{-t/2}$  exact

$t$	0	1	2	3
Exact	140	113.24	97.016	87.173

(c)  $\frac{dy}{dt} = -\frac{1}{2}(y - 72)$ ,  $(0, 140)$ ,  $h = 0.05$

$t$	0	1	2	3
Euler	140	112.98	96.7	86.9

The approximations are better using  $h = 0.05$ .

84. When  $x = 0$ ,  $y' = 0$ , therefore (d) is not possible.

When  $x, y > 0$ ,  $y' < 0$  (decreasing function) therefore (c) is the equation.

85. The general solution is a family of curves that satisfies the differential equation. A particular solution is one member of the family that satisfies given conditions.

86. A slope field for the differential equation  $y' = F(x, y)$  consists of small line segments at various points  $(x, y)$  in the plane. The line segment equals the slope  $y' = F(x, y)$  of the solution  $y$  at the point  $(x, y)$ .

87. Consider  $y' = F(x, y)$ ,  $y(x_0) = y_0$ . Begin with a point  $(x_0, y_0)$  that satisfies the initial condition,  $y(x_0) = y_0$ . Then, using a step size of  $h$ , find the point  $(x_1, y_1) = (x_0 + h, y_0 + hF(x_0, y_0))$ . Continue generating the sequence of points  $(x_{n+1}, y_{n+1}) = (x_n + h, y_n + hF(x_n, y_n))$ .

88.  $y = Ce^{kx}$

$$\frac{dy}{dx} = Cke^{kx}$$

Because  $dy/dx = 0.07y$ , you have  $Cke^{kx} = 0.07Ce^{kx}$ .

So,  $k = 0.07$ .

$C$  cannot be determined.

89. False. Consider Example 2.  $y = x^3$  is a solution to  $xy' - 3y = 0$ , but  $y = x^3 + 1$  is not a solution.

90. True

91. True

92. False. The slope field could represent many different differential equations, such as  $y' = 2x + 4y$ .

93.  $\frac{dy}{dx} = -2y, y(0) = 4, y = 4e^{-2x}$

(a)

$x$	0	0.2	0.4	0.6	0.8	1
$y$	4	2.6813	1.7973	1.2048	0.8076	0.5413
$y_1$	4	2.5600	1.6384	1.0486	0.6711	0.4295
$y_2$	4	2.4000	1.4400	0.8640	0.5184	0.3110
$e_1$	0	0.1213	0.1589	0.1562	0.1365	0.1118
$e_2$	0	0.2813	0.3573	0.3408	0.2892	0.2303
$r$		0.4312	0.4447	0.4583	0.4720	0.4855

(b) If  $h$  is halved, then the error is approximately halved ( $r \approx 0.5$ ).

(c) When  $h = 0.05$ , the errors will again be approximately halved.

94.  $\frac{dy}{dx} = x - y, y(0) = 1, y = x - 1 + 2e^{-x}$

(a)

$x$	0	0.2	0.4	0.6	0.8	1
$y$	1	0.8375	0.7406	0.6976	0.6987	0.7358
$y_1$	1	0.8200	0.7122	0.6629	0.6609	0.6974
$y_2$	1	0.8000	0.6800	0.6240	0.6192	0.6554
$e_1$	0	0.0175	0.0284	0.0347	0.0378	0.0384
$e_2$	0	0.0375	0.0606	0.0736	0.0795	0.0804
$r$		0.47	0.47	0.47	0.48	0.48

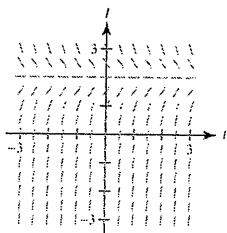
(b) If  $h$  is halved, then the error is halved ( $r \approx 0.5$ ).

(c) When  $h = 0.05$ , the error will again be approximately halved.

95. (a)  $L\frac{dI}{dt} + RI = E(t)$

$$4\frac{dI}{dt} + 12I = 24$$

$$\frac{dI}{dt} = \frac{1}{4}(24 - 12I) = 6 - 3I$$



(b) As  $t \rightarrow \infty, I \rightarrow 2$ . That is,  $\lim_{t \rightarrow \infty} I(t) = 2$ . In fact,

$I = 2$  is a solution to the differential equation.

96.  $y = e^{kt}$   
 $y' = ke^{kt}$   
 $y'' = k^2e^{kt}$

$$y'' - 16y = 0$$

$$k^2e^{kt} - 16e^{kt} = 0$$

$$k^2 - 16 = 0$$

(because  $e^{kt} \neq 0$ )

$$k = \pm 4$$

97.  $y = A \sin \omega t$

$$y' = A\omega \cos \omega t$$

$$y'' = -A\omega^2 \sin \omega t$$

$$y'' + 16y = 0$$

$$-A\omega^2 \sin \omega t + 16A \sin \omega t = 0$$

$$A \sin \omega t [16 - \omega^2] = 0$$

If  $A \neq 0$ , then  $\omega = \pm 4$