6.1 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Verifying a Solution In Exercises 1–8, verify the solution of the differential equation.

Solution	Differential Equation
1. $y = Ce^{4x}$	$y' = 4y$
2. $y = e^{-2x}$	$3y' + 5y = -e^{-2x}$
3. $x^2 + y^2 = Cy$	$y' = \dfrac{2xy}{x^2 - y^2}$
4. $y^2 - 2 \ln y = x^2$	$\dfrac{dy}{dx} = \dfrac{xy}{y^2 - 1}$
5. $y = C_1 \sin x - C_2 \cos x$	$y'' + y = 0$
6. $y = C_1 e^{-x} \cos x + C_2 e^{-x} \sin x$	$y'' + 2y' + 2y = 0$
7. $y = -\cos x \ln\lvert \sec x + \tan x \rvert$	$y'' + y = \tan x$
8. $y = \frac{2}{5}(e^{-4x} + e^x)$	$y'' + 4y' = 2e^x$

Verifying a Particular Solution In Exercises 9–12, verify the particular solution of the differential equation.

Solution	Differential Equation and Initial Condition
9. $y = \sin x \cos x - \cos^2 x$	$2y + y' = 2 \sin(2x) - 1$ $y\left(\frac{\pi}{4}\right) = 0$
10. $y = 6x - 4 \sin x + 1$	$y' = 6 - 4 \cos x$ $y(0) = 1$
11. $y = 4e^{-6x^2}$	$y' = -12xy$ $y(0) = 4$
12. $y = e^{-\cos x}$	$y' = y \sin x$ $y\left(\frac{\pi}{2}\right) = 1$

Determining a Solution In Exercises 13–20, determine whether the function is a solution of the differential equation $y^{(4)} - 16y = 0$.

13. $y = 3 \cos x$
14. $y = 2 \sin x$
15. $y = 3 \cos 2x$
16. $y = 3 \sin 2x$
17. $y = e^{-2x}$
18. $y = 5 \ln x$
19. $y = C_1 e^{2x} + C_2 e^{-2x} + C_3 \sin 2x + C_4 \cos 2x$
20. $y = 3e^{2x} - 4 \sin 2x$

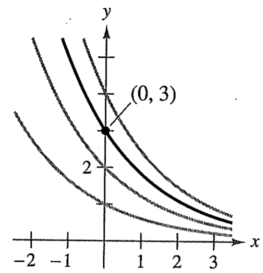
Determining a Solution In Exercises 21–28, determine whether the function is a solution of the differential equation $xy' - 2y = x^3 e^x$.

21. $y = x^2$
22. $y = x^3$
23. $y = x^2 e^x$
24. $y = x^2(2 + e^x)$
25. $y = \sin x$
26. $y = \cos x$
27. $y = \ln x$
28. $y = x^2 e^x - 5x^2$

Finding a Particular Solution In Exercises 29–32, some of the curves corresponding to different values of C in the general solution of the differential equation are shown in the graph. Find the particular solution that passes through the point shown on the graph.

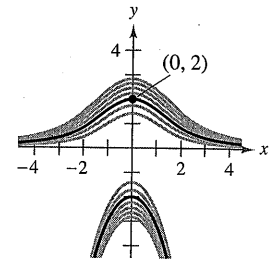
29. $y^2 = Ce^{-x/2}$

$2y' + y = 0$



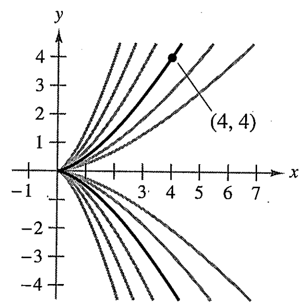
30. $y(x^2 + y) = C$

$2xy + (x^2 + 2y)y' = 0$



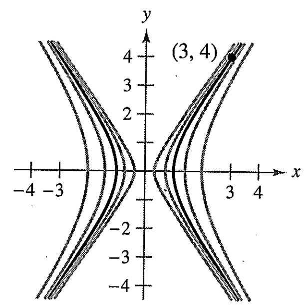
31. $y^2 = Cx^3$

$2xy' - 3y = 0$



32. $2x^2 - y^2 = C$

$yy' - 2x = 0$



Graphs of Particular Solutions In Exercises 33 and 34, the general solution of the differential equation is given. Use a graphing utility to graph the particular solutions for the given values of C.

33. $4yy' - x = 0$

$4y^2 - x^2 = C$

$C = 0, C = \pm 1, C = \pm 4$

34. $yy' + x = 0$

$x^2 + y^2 = C$

$C = 0, C = 1, C = 4$

Finding a Particular Solution In Exercises 35–40, verify that the general solution satisfies the differential equation. Then find the particular solution that satisfies the initial condition(s).

35. $y = Ce^{-2x}$

$y' + 2y = 0$

$y = 3$ when $x = 0$

36. $3x^2 + 2y^2 = C$

$3x + 2yy' = 0$

$y = 3$ when $x = 1$

37. $y = C_1 \sin 3x + C_2 \cos 3x$

$y'' + 9y = 0$

$y = 2$ when $x = \dfrac{\pi}{6}$

$y' = 1$ when $x = \dfrac{\pi}{6}$

38. $y = C_1 + C_2 \ln x$

$xy'' + y' = 0$

$y = 0$ when $x = 2$

$y' = \dfrac{1}{2}$ when $x = 2$

39. $y = C_1x + C_2x^3$
$x^2y'' - 3xy' + 3y = 0$
$y = 0$ when $x = 2$
$y' = 4$ when $x = 2$

40. $y = e^{2x/3}(C_1 + C_2x)$
$9y'' - 12y' + 4y = 0$
$y = 4$ when $x = 0$
$y = 0$ when $x = 3$

Finding a General Solution **In Exercises 41–52, use integration to find a general solution of the differential equation.**

41. $\dfrac{dy}{dx} = 6x^2$

42. $\dfrac{dy}{dx} = 10x^4 - 2x^3$

43. $\dfrac{dy}{dx} = \dfrac{x}{1 + x^2}$

44. $\dfrac{dy}{dx} = \dfrac{e^x}{4 + e^x}$

45. $\dfrac{dy}{dx} = \dfrac{x - 2}{x}$

46. $\dfrac{dy}{dx} = x \cos x^2$

47. $\dfrac{dy}{dx} = \sin 2x$

48. $\dfrac{dy}{dx} = \tan^2 x$

49. $\dfrac{dy}{dx} = x\sqrt{x - 6}$

50. $\dfrac{dy}{dx} = 2x\sqrt{4x^2 + 1}$

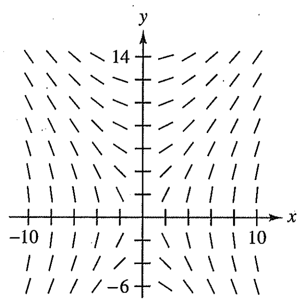
51. $\dfrac{dy}{dx} = xe^{x^2}$

52. $\dfrac{dy}{dx} = 5e^{-x/2}$

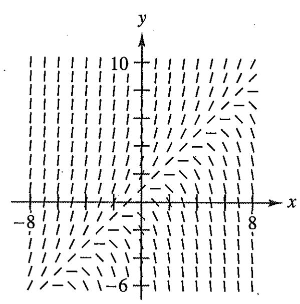
Slope Field **In Exercises 53–56, a differential equation and its slope field are given. Complete the table by determining the slopes (if possible) in the slope field at the given points.**

x	-4	-2	0	2	4	8
y	2	0	4	4	6	8
dy/dx						

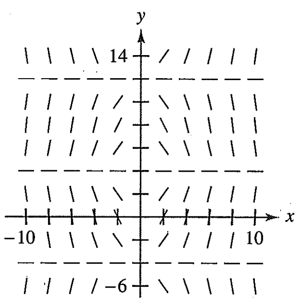
53. $\dfrac{dy}{dx} = \dfrac{2x}{y}$



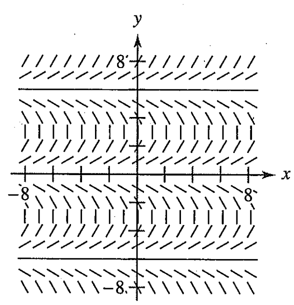
54. $\dfrac{dy}{dx} = y - x$



55. $\dfrac{dy}{dx} = x \cos \dfrac{\pi y}{8}$

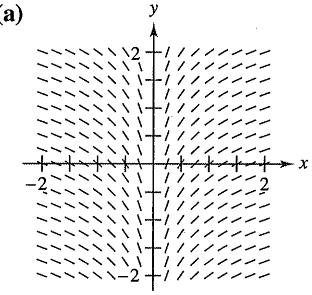


56. $\dfrac{dy}{dx} = \tan\left(\dfrac{\pi y}{6}\right)$

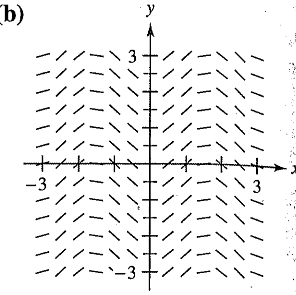


Matching **In Exercises 57–60, match the differential equation with its slope field. [The slope fields are labeled (a), (b), (c), and (d).]**

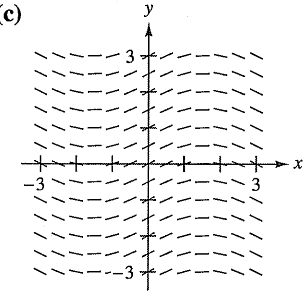
(a)



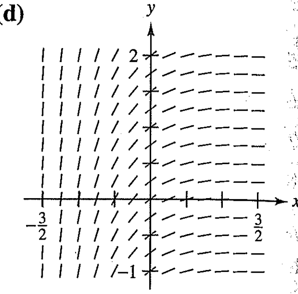
(b)



(c)



(d)



57. $\dfrac{dy}{dx} = \sin(2x)$

58. $\dfrac{dy}{dx} = \dfrac{1}{2} \cos x$

59. $\dfrac{dy}{dx} = e^{-2x}$

60. $\dfrac{dy}{dx} = \dfrac{1}{x}$

Slope Field **In Exercises 61–64, (a) sketch the slope field for the differential equation, (b) use the slope field to sketch the solution that passes through the given point, and (c) discuss the graph of the solution as $x \to \infty$ and $x \to -\infty$. Use a graphing utility to verify your results. To print a blank graph, go to *MathGraphs.com*.**

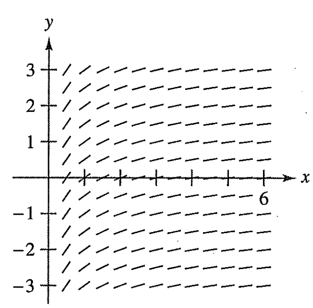
61. $y' = 3 - x$, $(4, 2)$

62. $y' = \frac{1}{3}x^2 - \frac{1}{2}x$, $(1, 1)$

63. $y' = y - 4x$, $(2, 2)$

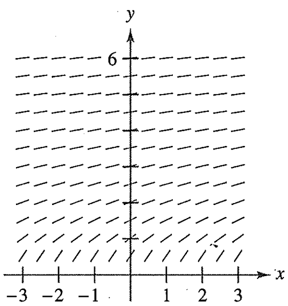
64. $y' = y + xy$, $(0, -4)$

65. Slope Field Use the slope field for the differential equation $y' = 1/x$, where $x > 0$, to sketch the graph of the solution that satisfies each given initial condition. Then make a conjecture about the behavior of a particular solution of $y' = 1/x$ as $x \to \infty$. To print an enlarged copy of the graph, go to *MathGraphs.com.*



(a) $(1, 0)$ (b) $(2, -1)$

66. **Slope Field** Use the slope field for the differential equation $y' = 1/y$, where $y > 0$, to sketch the graph of the solution that satisfies each given initial condition. Then make a conjecture about the behavior of a particular solution of $y' = 1/y$ as $x \to \infty$. To print an enlarged copy of the graph, go to *MathGraphs.com.*



(a) $(0, 1)$ (b) $(1, 1)$

Slope Field In Exercises 67–72, use a computer algebra system to (a) graph the slope field for the differential equation and (b) graph the solution satisfying the specified initial condition.

67. $\dfrac{dy}{dx} = 0.25y, \quad y(0) = 4$

68. $\dfrac{dy}{dx} = 4 - y, \quad y(0) = 6$

69. $\dfrac{dy}{dx} = 0.02y(10 - y), \quad y(0) = 2$

70. $\dfrac{dy}{dx} = 0.2x(2 - y), \quad y(0) = 9$

71. $\dfrac{dy}{dx} = 0.4y(3 - x), \quad y(0) = 1$

72. $\dfrac{dy}{dx} = \dfrac{1}{2}e^{-x/8}\sin\dfrac{\pi y}{4}, \quad y(0) = 2$

Euler's Method In Exercises 73–78, use Euler's Method to make a table of values for the approximate solution of the differential equation with the specified initial value. Use *n* steps of size *h*.

73. $y' = x + y, \quad y(0) = 2, \quad n = 10, \quad h = 0.1$

74. $y' = x + y, \quad y(0) = 2, \quad n = 20, \quad h = 0.05$

75. $y' = 3x - 2y, \quad y(0) = 3, \quad n = 10, \quad h = 0.05$

76. $y' = 0.5x(3 - y), \quad y(0) = 1, \quad n = 5, \quad h = 0.4$

77. $y' = e^{xy}, \quad y(0) = 1, \quad n = 10, \quad h = 0.1$

78. $y' = \cos x + \sin y, \quad y(0) = 5, \quad n = 10, \quad h = 0.1$

Euler's Method In Exercises 79–81, complete the table using the exact solution of the differential equation and two approximations obtained using Euler's Method to approximate the particular solution of the differential equation. Use $h = 0.2$ and $h = 0.1$, and compute each approximation to four decimal places.

x	0	0.2	0.4	0.6	0.8	1
$y(x)$ (exact)						
$y(x)$ $(h = 0.2)$						
$y(x)$ $(h = 0.1)$						

Table for 79–81

	Differential Equation	Initial Condition	Exact Solution
79.	$\dfrac{dy}{dx} = y$	$(0, 3)$	$y = 3e^x$
80.	$\dfrac{dy}{dx} = \dfrac{2x}{y}$	$(0, 2)$	$y = \sqrt{2x^2 + 4}$
81.	$\dfrac{dy}{dx} = y + \cos(x)$	$(0, 0)$	$y = \dfrac{1}{2}(\sin x - \cos x + e^x)$

82. **Euler's Method** Compare the values of the approximations in Exercises 79–81 with the values given by the exact solution. How does the error change as *h* increases?

83. **Temperature** At time $t = 0$ minutes, the temperature of an object is 140°F. The temperature of the object is changing at the rate given by the differential equation

$$\frac{dy}{dt} = -\frac{1}{2}(y - 72).$$

(a) Use a graphing utility and Euler's Method to approximate the particular solutions of this differential equation at $t = 1$, 2, and 3. Use a step size of $h = 0.1$. (A graphing utility program for Euler's Method is available at the website *college.hmco.com.*)

(b) Compare your results with the exact solution

$$y = 72 + 68e^{-t/2}.$$

(c) Repeat parts (a) and (b) using a step size of $h = 0.05$. Compare the results.

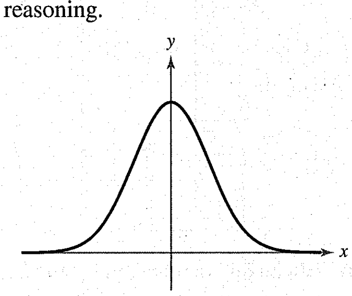
84. **HOW DO YOU SEE IT?** The graph shows a solution of one of the following differential equations. Determine the correct equation. Explain your reasoning.

(a) $y' = xy$

(b) $y' = \dfrac{4x}{y}$

(c) $y' = -4xy$

(d) $y' = 4 - xy$



WRITING ABOUT CONCEPTS

85. General and Particular Solutions In your own words, describe the difference between a general solution of a differential equation and a particular solution.

86. Slope Field Explain how to interpret a slope field.

87. Euler's Method Describe how to use Euler's Method to approximate a particular solution of a differential equation.

88. Finding Values It is known that $y = Ce^{kx}$ is a solution of the differential equation $y' = 0.07y$. Is it possible to determine C or k from the information given? If so, find its value.

True or False? **In Exercises 89–92, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

89. If $y = f(x)$ is a solution of a first-order differential equation, then $y = f(x) + C$ is also a solution.

90. The general solution of a differential equation is $y = -4.9x^2 + C_1x + C_2$. To find a particular solution, you must be given two initial conditions.

91. Slope fields represent the general solutions of differential equations.

92. A slope field shows that the slope at the point $(1, 1)$ is 6. This slope field represents the family of solutions for the differential equation $y' = 4x + 2y$.

93. Errors and Euler's Method The exact solution of the differential equation

$$\frac{dy}{dx} = -2y$$

where $y(0) = 4$, is $y = 4e^{-2x}$.

(a) Use a graphing utility to complete the table, where y is the exact value of the solution, y_1 is the approximate solution using Euler's Method with $h = 0.1$, y_2 is the approximate solution using Euler's Method with $h = 0.2$, e_1 is the absolute error $|y - y_1|$, e_2 is the absolute error $|y - y_2|$, and r is the ratio e_1/e_2.

x	0	0.2	0.4	0.6	0.8	1
y						
y_1						
y_2						
e_1						
e_2						
r						

(b) What can you conclude about the ratio r as h changes?

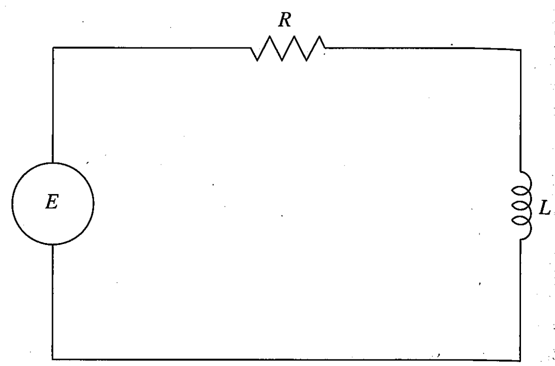
(c) Predict the absolute error when $h = 0.05$.

94. Errors and Euler's Method Repeat Exercise 93 for which the exact solution of the differential equation

$$\frac{dy}{dx} = x - y$$

where $y(0) = 1$, is $y = x - 1 + 2e^{-x}$.

95. Electric Circuit The diagram shows a simple electric circuit consisting of a power source, a resistor, and an inductor.



A model of the current I, in amperes (A), at time t is given by the first-order differential equation

$$L\frac{dI}{dt} + RI = E(t)$$

where $E(t)$ is the voltage (V) produced by the power source, R is the resistance, in ohms (Ω), and L is the inductance, in henrys (H). Suppose the electric circuit consists of a 24-V power source, a 12-Ω resistor, and a 4-H inductor.

(a) Sketch a slope field for the differential equation.

(b) What is the limiting value of the current? Explain.

96. Think About It It is known that $y = e^{kt}$ is a solution of the differential equation $y'' - 16y = 0$. Find the values of k.

97. Think About It It is known that $y = A \sin \omega t$ is a solution of the differential equation $y'' + 16y = 0$. Find the values of ω.

PUTNAM EXAM CHALLENGE

98. Let f be a twice-differentiable real-valued function satisfying

$$f(x) + f''(x) = -xg(x)f'(x)$$

where $g(x) \geq 0$ for all real x. Prove that $|f(x)|$ is bounded.

99. Prove that if the family of integral curves of the differential equation

$$\frac{dy}{dx} + p(x)y = q(x), \quad p(x) \cdot q(x) \neq 0$$

is cut by the line $x = k$, the tangents at the points of intersection are concurrent.