Key

6.2 AP Practice Problems (p.411-412)

1. For the function $f(x) = 2 - 3x$, $0 \le x \le 4$, the interval [0, 4] is partitioned into four subintervals [0, 1], [1, 2], [2, 3], [3, 4]. The Right Riemann sum equals

   (A) −88 (B) −24 **[(C)]** −22 (D) −10

   $1f(1) + 1f(2) + 1f(3) + 1f(4)$

   $= (-1) + (-4) + (-7) + (-10) = \boxed{-22}$

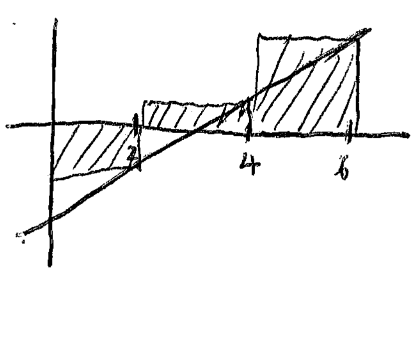
2. The Riemann sums for a function f on the interval [1, 5] are given as $\sum_{i=1}^{n} (2 - u_i^2)\Delta x_i$ where [1, 5] is partitioned into n subintervals $[x_{i-1}, x_i]$ of width Δx_i and u_i is some number in $[x_{i-1}, x_i]$. If $\lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} (2 - u_i^2)\Delta x_i$ exists, it equals

   (A) $\int_1^5 (2 - u_i^2)\Delta x_i$ (B) $\sum_{i=1}^{n} (2 - u_i^2)\Delta x_i$

   [(C)] $\int_1^5 (2 - x^2)dx$ (D) $\int_5^1 (2 - x^2)dx$

3. Approximate $\int_0^6 (2x - 5)dx$ by partitioning the interval [0, 6] into three subintervals each of width 2 and using a Right Riemann sum.

   (A) −10 (B) 9 **[(C)]** 18 (D) 22



$\int_0^6 (2x-5)dx \approx 2\cdot f(2) + 2f(4) + 2f(6)$

$= 2(-1) + 2(3) + 2(7)$

$= -2 + 6 + 14 = \boxed{18}$

4. $\int_{-3}^{5}(3-x^2)dx =$

$w = \frac{b-a}{n} \rightarrow \frac{5-(-3)}{n} \rightarrow \frac{8}{n}$

(A) $\lim_{\Delta x_i \to 0} \sum_{i=1}^{n} (3-u_i^2)\Delta x_i$

(B) $\lim_{n \to \infty} \sum_{i=1}^{n} (3-x^2)\frac{2}{n}$

(C) $\lim_{n \to \infty} \sum_{i=1}^{n} (3-u_i^2)\frac{8}{n}$

(D) $\lim_{n \to \infty} \sum_{i=1}^{n} (3-u_i^2)\frac{2}{n}$

5. A function f is continuous on the closed interval [0, 10] and has values

(widths: 1, 3, 4, 2)

x	0	1	4	8	10
$f(x)$	4	5	10	12	8

Find an approximation to $\int_0^{10} f(x)\,dx$ using a Right Riemann sum with the four subintervals [0, 1], [1, 4], [4, 8], [8, 10].

(A) 70 **(B) 99** (C) 83 (D) 62

$\int_0^{10} f(x)dx \approx 1(5)+3(10)+4(12)+2(8)$
$= 5+30+48+16$
$= \boxed{99}$

6. $\lim_{n\to\infty} \frac{1}{n}\left[2\left(\frac{1}{n}\right)^{2/3} + 2\left(\frac{2}{n}\right)^{2/3} + 2\left(\frac{3}{n}\right)^{2/3} + \cdots + 2\left(\frac{n}{n}\right)^{2/3}\right] =$

(A) $\int_0^1 2x^{2/3}dx$ (B) $\int_0^2 x^{2/3}dx$

(C) $2\int_0^1 \left(\frac{1}{x}\right)^{2/3} dx$ (D) $\frac{2}{n^{2/3}}\int_0^1 dx$

$w = \frac{1}{n} \rightarrow \frac{b-a}{n} \rightarrow \frac{1-0}{n}$

$f\left[a + \frac{1}{n}i\right]$

$f\left(0 + \frac{1}{n}i\right)$

$f(x) = 2x^{2/3}$

7. The expression

$$\frac{2}{25}\left[\sqrt{\frac{2}{25}} + \sqrt{\frac{4}{25}} + \sqrt{\frac{6}{25}} + \cdots + \sqrt{\frac{48}{25}} + \sqrt{\frac{50}{25}}\right]$$

is a Riemann sum approximation for

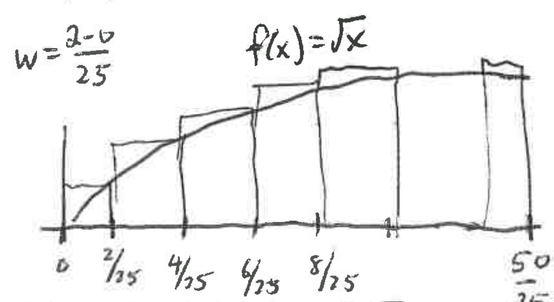
(A) $\frac{1}{25}\int_0^2 \sqrt{x}dx$ **(B)** $\int_0^2 \sqrt{x}dx$

(C) $\int_0^{50} \sqrt{x}dx$ (D) $\int_0^1 \sqrt{2x}dx$

$w = \frac{b-a}{n} \rightarrow \frac{2-0}{25}$

*25 means an arbitrary 25 rectangles

$f(x) = \sqrt{x}$



$\frac{2}{25}\cdot\sqrt{\frac{2}{25}} + \frac{2}{25}\sqrt{\frac{4}{25}} + \frac{2}{25}\sqrt{\frac{6}{25}}$

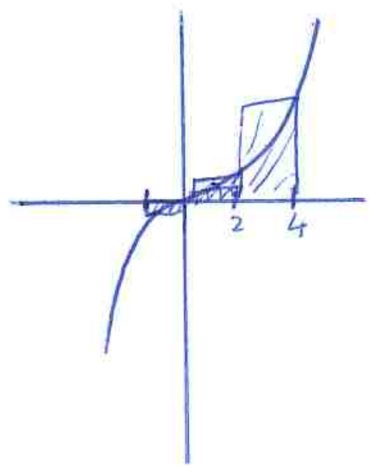
8. The integral $\int_{-2}^{4}(x^3-4)dx$ is approximated by partitioning the closed interval $[-2, 4]$ into three subintervals of equal width and using a Right Riemann sum. Which of the following is true?

(A) The Right Riemann sum $= -24$; it underestimates $\int_{-2}^{4}(x^3-4)dx$.

(B) The Right Riemann sum $= 60$; it overestimates $\int_{-2}^{4}(x^3-4)dx$.

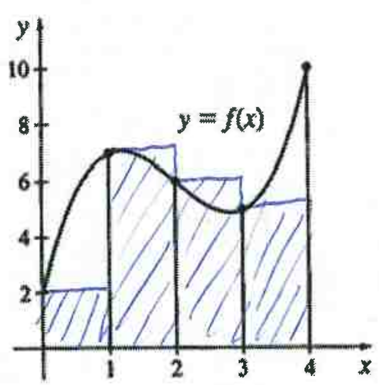
(C) The Right Riemann sum $= 120$; it overestimates $\int_{-2}^{4}(x^3-4)dx$.

(D) The Right Riemann sum $= 120$; there is not enough information to determine whether it overestimates or underestimates $\int_{-2}^{4}(x^3-4)dx$.



$$\int_{-2}^{4}(x^3-4)dx \approx 2f(0)+2f(2)+2f(4)$$
$$= 2(-4)+2(4)+2(60) = \boxed{120}$$

9. The graph of $f(x) = (x-2)^3 - 2x + 10$ is shown below.



a) 20

b) $\int_0^6 (x-2)^3 - 2x + 10$

c) 24

(a) Approximate the area under the graph of f using a Left Riemann sum with $n = 4$ subintervals of width 1.

(b) Express the area under the graph of f as a definite integral.

(c) Use technology to find the area under the graph of f.

a) $1 \cdot f(0) + 1f(1) + 1f(2) + 1f(3)$

$1(2) + 1(7) + 1(6) + 1(5) = \boxed{20}$

b) $\int_0^6 (x-2)^3 - 2x + 10\,dx$

c) $= \boxed{24}$

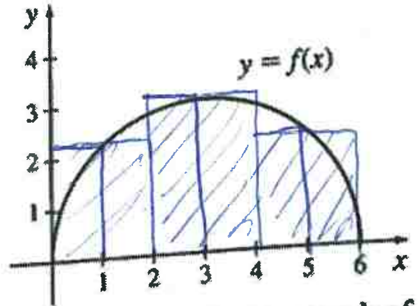
a) $(\sqrt{5}+\sqrt{9}+\sqrt{5})(2)$

b) $\int_0^6 \sqrt{6x-x^2}\,dx$

c) ≈ 14.137

d) $\frac{1}{2}\pi r^2 \to \frac{1}{2}\pi(3)^2 \approx 14.137$

10. The graph of $f(x)=\sqrt{6x-x^2}$ is shown below.



(a) Approximate the area under the graph of $f(x)=\sqrt{6x-x^2}$ using a Midpoint Riemann sum with three subintervals of equal width.

(b) Express the area under the graph of f as a definite integral.

(c) Evaluate the integral.

(d) Confirm the answer to (c) using geometry.

a) $2f(1)+2f(3)+2f(5)$

$2\sqrt{5}+2\sqrt{9}+2\sqrt{5}$

b) $\int_0^6 \sqrt{6x-x^2}\,dx$

c) $= 14.137$

d) semicircle $\to \frac{1}{2}\pi r^2 \to \frac{1}{2}\pi(3)^2 = \frac{9}{2}\pi \approx 14.137$

11. Express $\int_0^5 e^x\,dx$ as the limit of Riemann sums.

$$\lim_{n\to\infty}\sum_{i=1}^{n} e^{x_i}\left(\frac{5}{n}\right)$$