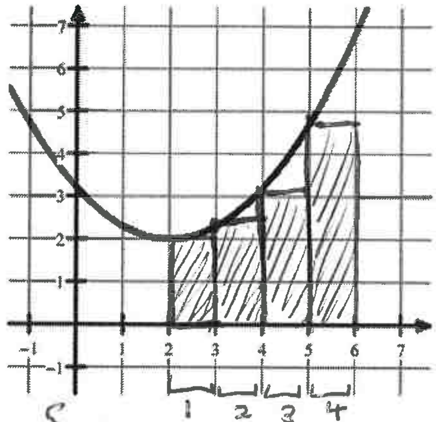
AP Calculus – 6.2 Notes – Approximating Area with Riemann Sums

Key

The graph of the function $g(x)$ is shown to the right. Approximate the area under the curve on the interval $[2, 6]$ with n subintervals by using a left-rectangular approximation method.

$n = 4$ subintervals $\overset{a\ \ b}{[2, 6]}$

$\text{width} = \frac{b-a}{n} = \frac{6-2}{4} = \frac{4}{4} = 1$



*Area of rectangle is width x height

This approximation method is called a Riemann Sum. It was named after a German mathematician named Bernhard Riemann.

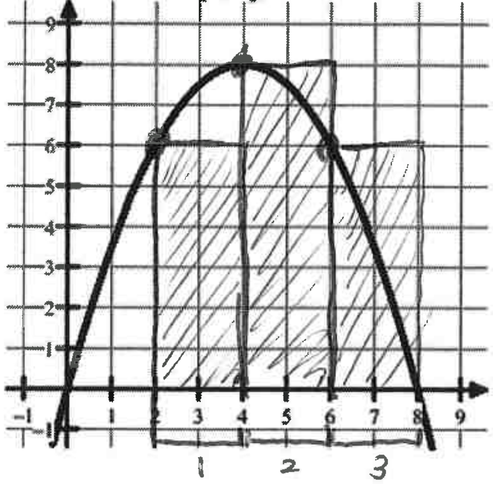
Below is the graph of $f(x) = 4x - \frac{1}{2}x^2$. Use Riemann Sums to find the approximation of the area under the curve.

Left-Riemann Sum

On the interval $[2, 8]$, use 3 subintervals

$W = \frac{b-a}{n} \rightarrow \frac{8-2}{3}$

$W = \frac{6}{3} = 2$



$\text{Area} \approx 2 \cdot f(2) + 2f(4) + 2f(6)$

$2(6) + 2(8) + 2(6) = \boxed{40}$

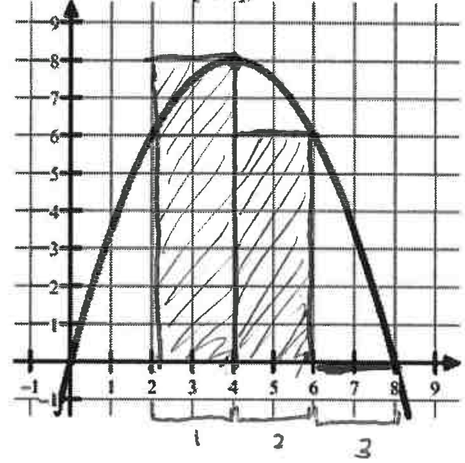
Right-Riemann Sum

On the interval $[2, 8]$, use 3 subintervals

$W = \frac{b-a}{n}$

$W = \frac{8-2}{3}$

$W = 3$



$\text{Area} \approx 2 \cdot f(4) + 2f(6) + 2f(8)$

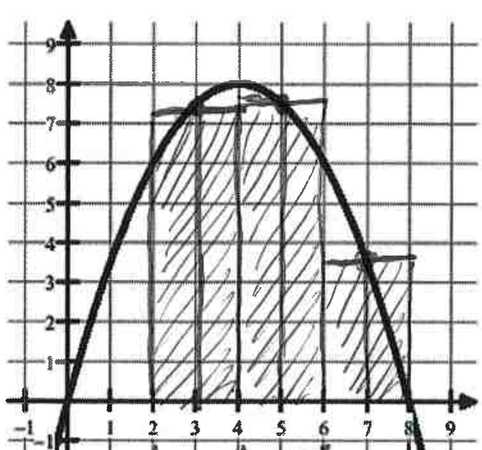
$2(8) + 2(6) + 2(0)$

$= \boxed{28}$

Midpoint-Riemann Sum

On the interval $[2, 8]$, use 3 subintervals

$W = \frac{8-2}{3} = \frac{6}{3} = 2$



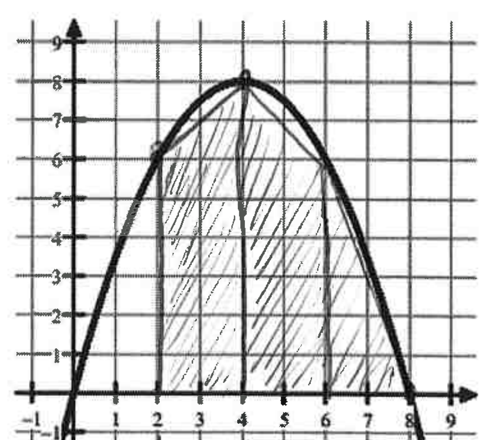
$2f(3) + 2f(5) + 2f(7)$

$2(7.5) + 2(7.5) + 2(3.5) = \boxed{37}$

Trapezoidal Sum

On the interval $[2, 8]$, use 3 subintervals

$A = \frac{1}{2}w[h_1 + h_2]$



$\boxed{\text{Area} = 34}$

$\frac{1}{2}(2)[f(2)+f(4)] + \frac{1}{2}(2)[f(4)+f(6)] + \frac{1}{2}(2)[f(6)+f(8)]$

Using Riemann Sums with a Table of Values

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The rate at which water is being pumped into a tank is given by the continuous and increasing function $R(t)$. A table of selected values of $R(t)$, for the time interval $0 < t < 12$ minutes, is given below.

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Time (minutes)	0	3	6	9	12
$R(t)$ (gallons/min)	7	13	18	23	27

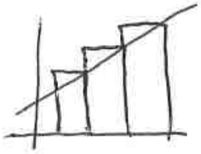
Use the following Riemann sums (with the given intervals), to estimate the number of gallons of water pumped into the tank during the 12 minutes.

Right-Riemann sum with 4 subintervals

$3(13)+3(18)+3(23)+3(27)$

$\boxed{\approx 243 \text{ gallons}}$

Is the approximation greater or less than the true value? Why?



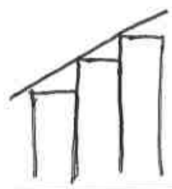
Overapproximation because Right Riemann Sum on an increasing function

Left-Riemann sum with 4 subintervals

$3(7)+3(13)+3(18)+3(23)$

$\boxed{183 \text{ gallons}}$

Is the approximation greater or less than the true value? Why?



Underapproximation because left Riemann Sum on an increasing function

Midpoint-Riemann sum with 2 subintervals

$6(13)+6(23)$

$\boxed{216 \text{ gallons}}$

Trapezoidal sum with 4 subintervals $\left(A=\frac{1}{2}w[h_1+h_2]\right)$

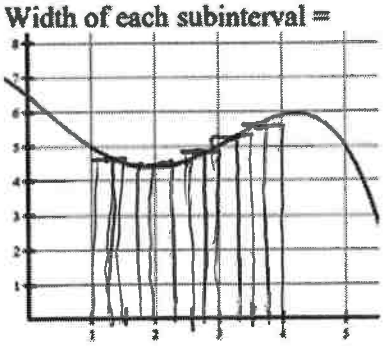
$\frac{1}{2}(3)[7+13]+\frac{1}{2}(3)[13+18]+\frac{1}{2}(3)[18+23]+\frac{1}{2}(3[23+27])$

$\boxed{213 \text{ gallons}}$

Sketch the following rectangular approximations. Find the width of each subinterval.

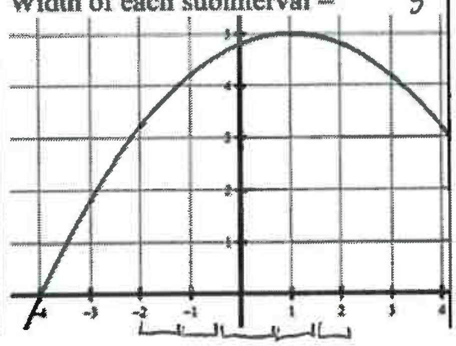
3. Midpoint on the interval [1,4] with $n = 6$ subintervals

Width of each subinterval = $\frac{4-1}{6} = \frac{3}{6} = \frac{1}{2}$



4. Right Endpoint on [-2,2] with $n = 5$ subintervals

Width of each subinterval = $\frac{2-(-2)}{5} = \frac{4}{5}$



5. Left Endpoint on [-2,4] with $n = 10$ subintervals

Width of each subinterval = $\frac{4-(-2)}{10} = \frac{6}{10} = \frac{3}{5}$