Key

6.3 AP Practice Problems (p.423-424)

$\frac{d}{dx}\tan u = \sec^2 u \cdot u'$

1. $\int_0^{\pi/4} \sec^2 x \, dx =$

(A) $-\frac{1}{2}$ (B) $\sqrt{2}$ $\boxed{\text{(C)}}$ 1 (D) $\frac{1}{2}$

$$\int_0^{\pi/4} \sec^2 x \, dx = \tan x \Big]_0^{\pi/4}$$
$$= \tan(\pi/4) - \tan 0$$
$$= 1 - 0 = \boxed{1}$$

2. If $F(x) = \int_0^x \sin t \, dt$ then $F'\left(\frac{\pi}{2}\right)$ is

(A) 0 (B) $\frac{1}{2}$ $\boxed{\text{(C)}}$ 1 (D) -1

*SFTC

$$\frac{d}{dx}\int_a^{p(x)} f(t)\,dt = f(p(x)) \cdot p'(x)$$
$$F'(x) = \frac{d}{dx}\int_0^x \sin t \, dt = \sin(x) \cdot 1$$
$$F'(\pi/2) = \sin(\pi/2) = \boxed{1}$$

3. $\int_0^x e^t dt =$

(A) e^x (B) $e^x - e$ $\boxed{\text{(C)}}$ $e^x - 1$ (D) $\frac{e^x - 1}{\ln|x|}$

$$\int e^t dt = e^t \Big]_0^x = e^x - e^0$$
$$= \boxed{e^x - 1}$$

4. $\int_1^e \frac{1}{x^3} dx =$

(A) $\frac{e-1}{2e}$ (B) $\frac{e^2-1}{2}$ (C) $\frac{1-e^2}{2}$ $\boxed{\text{(D)}}$ $\frac{e^2-1}{2e^2}$

$$\int x^{-3} dx = \frac{x^{-2}}{-2}$$
$$\rightarrow \frac{-1}{2x^2}\Big]_1^e = \frac{-1}{2e^2} - \left(\frac{-1}{2(1)}\right)$$
$$= \frac{-1}{2e^2} + \frac{e^2}{2e^2} = \boxed{\frac{e^2-1}{2e^2}}$$

5. If P is a polynomial function of degree n, what is the degree of the function $Q(x) = \int_0^x P(t)\,dt$?

(A) $n-1$ (B) n $\boxed{\text{(C)}}$ $n+1$ (D) $2n$

* Antiderivative will increase the degree of polynomial by +1

6. If $F(x) = \int_1^{x^3+1} \sqrt{t^2 - 1}\, dt$, then $F'(x)$ equals

(A) $\sqrt{(x^3+1)^2 - 1}$ (B) $3x^2\sqrt{x^3}$

(C) x^3 $\boxed{\text{(D)}}$ $3x^2\sqrt{(x^3+1)^2 - 1}$

SFTC: $\frac{d}{dx}\int_a^{p(x)} f(t)\,dt = f(p(x)) \cdot p'(x)$

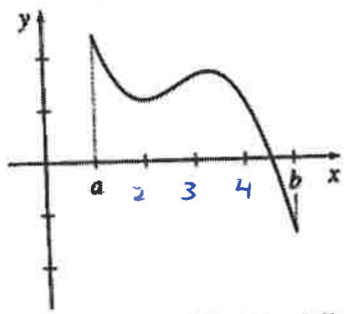
$$\frac{d}{dx}\int_1^{x^3+1} \sqrt{t^2-1}\,dt = \sqrt{(x^3+1)^2 - 1} \cdot 3x^2$$
$$\boxed{F'(x) = 3x^2\sqrt{(x^3+1)^2 - 1}}$$

7. If $x > 1$, then $\frac{d}{dx}\int_e^x \frac{1}{t}\,dt =$

$\frac{d}{dx}\int_e^x \frac{1}{t}\,dt = \frac{1}{x}\cdot(1) = \boxed{\frac{1}{x}}$

$\boxed{\text{(A)}}$ $\frac{1}{x}$ (B) $\ln x$ (C) $\ln x - 1$ (D) $\frac{1}{x} - \frac{1}{e}$

8. The graph of a function $y = g(x)$, where $a \le x \le b$, is shown below.

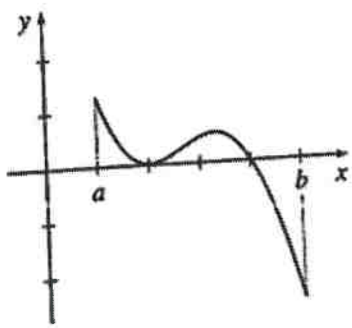


$g(x) = \int_a^x f(t)\,dt$

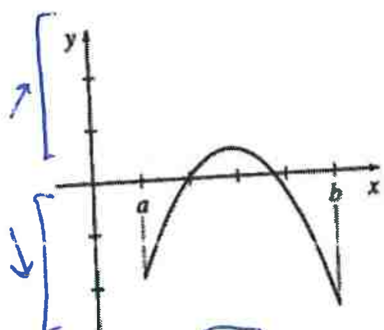
$g'(x) = f(x)$

*In another words, f(x) graph is the derivative graph of g(x).

If $g(x) = \int_a^x f(t)\,dt$, then which of the following could be the graph of f on $[a, b]$?



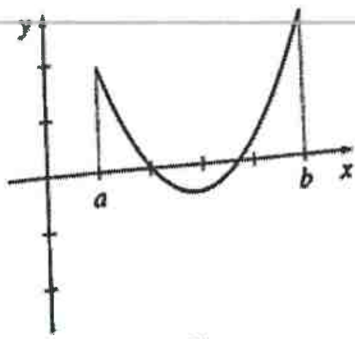
(A)



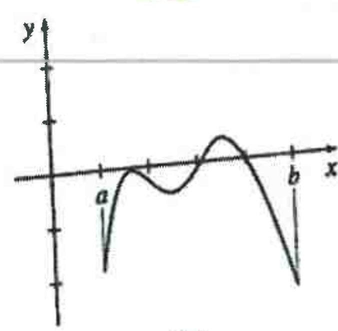
$\boxed{\text{(B)}}$

$g'(x)$: ↘ ↗ ↘ at a, 2, 3, b

$g''(x)$: ∪ + | ∩ − at 2.5



(C)



(D)

9. If $f(x) = \int_1^{3x^2} \sqrt{t^2+1}\,dt$, then $f'(-2)$ equals

$\boxed{\text{(A)}}$ $-12\sqrt{145}$ (B) $-12\sqrt{37}$

(C) $12\sqrt{37}$ (D) $\sqrt{145}$

$$f'(x) = \frac{d}{dx}\int_1^{3x^2} \sqrt{t^2+1}\,dt$$

$$f'(x) = \sqrt{(3x^2)^2+1}\cdot 6x$$

$$f'(-2) = \sqrt{12^2+1}\cdot 6(-2)$$

$$\boxed{f'(-2) = -12\sqrt{145}}$$

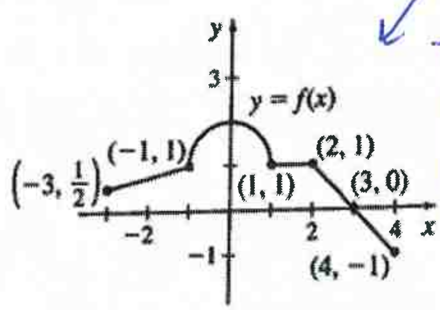
10. The graph of a function f is shown below. If

$$g(t) = \int_{-3}^{t} f(x)\,dx$$

for what number t is $g(t)$ the greatest?

$g'(t) = f(t)dt$

$*f(x)$ is the derivative graph of $g(x)$



$g'(x)$ + 3 −

Relative max (and absolute max) at $x=3$

(A) 0 (B) 2 $\boxed{\text{(C)}}$ 3 (D) 4

$$\int_0^{k} \sin x\,dx = 0.2$$

$$-\cos x \Big]_0^{k} = -\cos k + \cos 0 = 0.2$$

$$-\cos k = -0.8$$

$$\cos k = 0.8$$

$$k = \cos^{-1}(0.8)$$

$$\boxed{k = 0.644}$$

11. The area under the graph of $f(x) = \sin x$ from 0 to k, $0 \le k \le \frac{\pi}{2}$, is 0.2. Then k equals

(A) 1.2 $\quad$ $\boxed{\text{(B)}}$ 0.644 $\quad$ (C) 0.356 $\quad$ (D) 0.314

12. Suppose $g(x) = \int_0^x \sin\left(t - \frac{\pi}{2}\right) dt$ for $0 \le x \le \frac{3\pi}{2}$.

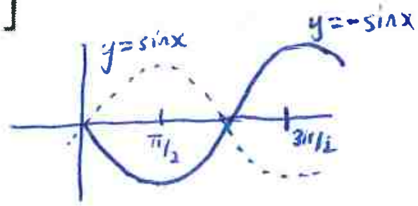
On which interval is g increasing?

(A) $\left[0, \frac{\pi}{2}\right]$ $\quad$ (B) $[0, \pi]$

(C) $\left[\frac{\pi}{2}, \pi\right]$ $\quad$ $\boxed{\text{(D)}}$ $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

$$g'(x) = -\cos\left(x - \pi/2\right)$$

$$g'(x) = -\sin x$$



13. A biologist is growing bacteria for a lab experiment. There are 10 mg of bacteria in a controlled environment when she changes the temperature. The amount P of bacteria then grows at a rate of $20(0.95)^t$ per hour, where t is the number of hours since the temperature changed.

(a) Write an integral that models the amount of bacteria at time $t \ge 0$.

(b) Find the amount after 24 hours.

a) $P(t) = P(0) + \int_0^t 20(0.95)^x dx$

b) 286.064 mg

a) * Final Amount = Initial Amount + Amount Added

$$P(t) = P(0) + \int_0^t 20(0.95)^x dx$$

b) $$P(24) = 10 + \int_0^{24} 20(0.95)^x dx$$

$$= 10 + 276.064$$

$$\boxed{P(24) = 286.064}$$