6.3 HW p. 420-424 #5-35 odd, 39, 59-65 odd
~~71-74 all~~

Second Fundamental Theorem of Calculus

$$\frac{d}{dx}\int_a^{p(x)} f(t)\,dt = f(p(x))\cdot p'(x)$$

5) $\frac{d}{dx}\int_1^x \sqrt{t^2+1}\,dt = \sqrt{x^2+1}\,(1) = \boxed{\sqrt{x^2+1}}$

7) $\frac{d}{dx}\int_0^t (3+x^2)^{3/2}\,dx = (3+t^2)^{3/2}(1) = \boxed{(3+t^2)^{3/2}}$

9) $\frac{d}{dx}\int_1^x \ln u\,du = \ln x \cdot 1 = \boxed{\ln x}$

11) $\frac{d}{dx}\int_1^{2x^3} \sqrt{t^2+1}\,dt = \sqrt{(2x^3)^2+1}\cdot 6x^2 = \boxed{6x^2\sqrt{4x^6+1}}$

13) $\frac{d}{dx}\int_2^{x^3} \sec t\,dt = \sec(x^3)\cdot 3x^2 = \boxed{3x^2\sec(x^3)}$

15) $\frac{d}{dx}\int_x^5 \sin(t^2)\,dt = \frac{d}{dx}\int_5^x -\sin(t^2)\,dt = \boxed{-\sin(x^2)}$

17) $\frac{d}{dx}\int_{5x^2}^5 (6t)^{2/3}\,dt = \frac{d}{dx}\int_5^{5x^2} -(6t)^{2/3}\,dt = -(6\cdot 5x^2)^{2/3} = -(30x^2)^{2/3}\cdot 10x$

$$= \boxed{-10x(30x^2)^{2/3}}$$

$\frac{d}{dx}\int_5^{5x^2} -(6t)^{2/3}\,dt$

$\int -6^{2/3}t^{2/3}\,dt$

$-6^{2/3}\cdot\frac{t^{5/3}}{5/3}$

$-\frac{3}{5}(6)^{2/3}t^{5/3}\Big]_5^{5x^2} = -\frac{3}{5}(6)^{2/3}\cdot(5x^2)^{5/3} - -\frac{3}{5}(6)^{2/3}(5)^{5/3}$

$\frac{d}{dx} \to -\frac{3}{5}\cdot 6^{2/3}\cdot\frac{5}{3}(5x^2)^{2/3}\cdot 10x - 0$

1st Fundamental Theorem of Calculus

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

19) $\int_{-2}^{3} 1\,dx \rightarrow 1x\Big]_{-2}^{3} = 1(3) - 1(-2) = \boxed{5}$

21) $\int_{-1}^{-2} x^3\,dx \rightarrow \frac{x^4}{4}\Big]_{-1}^{-2} = \frac{(-2)^4}{4} - \frac{(-1)^4}{4} = \boxed{\frac{15}{4}}$

23) $\int_0^1 \sqrt{u}\,du \rightarrow \int u^{1/2}\,du \rightarrow \frac{u^{3/2}}{3/2} \rightarrow \frac{2}{3}u^{3/2}\Big]_0^1 = \frac{2}{3}(1)^{3/2} - \frac{2}{3}(0)^{3/2} = \boxed{\frac{2}{3}}$

25) $\int_{\pi/6}^{\pi/2} \csc^2 x\,dx \rightarrow -\cot x\Big]_{\pi/6}^{\pi/2} \rightarrow -\cot(\pi/2) - (-\cot(\pi/6))$

$\rightarrow 0 + \sqrt{3} = \boxed{\sqrt{3}}$

27) $\int_0^{\pi/4} \sec x \tan x\,dx \rightarrow \sec x\Big]_0^{\pi/4} \rightarrow \sec(\pi/4) - \sec(0)$

$\frac{2}{\sqrt{2}} - 1 = \boxed{\frac{2-\sqrt{2}}{\sqrt{2}} \text{ or } \sqrt{2}-1}$

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29) $\int_{-1}^{0} e^x\,dx = e^x\Big]_{-1}^{0} = e^0 - e^{-1} = \boxed{1 - \frac{1}{e}}$

31) $\int_1^e \frac{1}{x}\,dx = \ln|x|\Big]_1^e = \ln e - \ln 1 = \boxed{1}$

33) $\int_0^1 \frac{1}{1+x^2}\,dx \rightarrow \tan^{-1}(x)\Big]_0^1 = \tan^{-1}(1) - \tan^{-1}(0) = \frac{\pi}{4} - 0 = \boxed{\pi/4}$

35) $\int_{-1}^{8} x^{2/3} \rightarrow \frac{x^{5/3}}{5/3}\Big]_{-1}^{8}$

$\frac{3}{5}(8)^{5/3} - \frac{3}{5}(-1)^{5/3}$

$\frac{3}{5}\cdot 2^5 - \frac{3}{5}(-1)$

$\frac{96}{5} + \frac{3}{5} = \boxed{\frac{99}{5}}$

6.3

39) $\int_{-1}^{2} x^3 \to \frac{x^4}{4}\Big]_{-1}^{2} = \frac{2^4}{4} - \frac{1^4}{4} = \boxed{\frac{15}{4}}$

59) $\int_0^2 R(t)\,dt = 23$ means 23 million dollars in sales were made between $t=0$ and $t=2$ years

61) $\int_a^b H(t)\,dt$ means amount of helium leaked out from the balloon over a time interval

c) $\int_0^{300} H(t)\,dt = -100$ means the balloon lost 100 cm^3 of helium.

63) a) $\int_0^6 B'(t)\,dt = 44.740g.$

b) $\int_0^6 B'(t)\,dt$ means the projected increase in (# of) population of bacteria

c) $B(t) = B(0) + \int_0^t B'(t)\,dt$

$B(6) = B(0) + \int_0^6 B'(t)\,dt$

$B(6) = 15 + \int_0^6 B'(t)\,dt \to 15 + 44.74g = \boxed{59.74g}$

65) a) $\int_a^b R'(x)\,dx$ means the change in the amount of Revenue (in hundreds of dollars)

b) $\int_{40}^{50} \sqrt[3]{x}\,dx = \int x^{1/3} \to \frac{x^{4/3}}{4/3}\Big]_{40}^{50} = \frac{3}{4}(50)^{4/3} - \frac{3}{4}(40)^{4/3} = 35.55$ hundred dollars

$= \boxed{\$3553.30}$