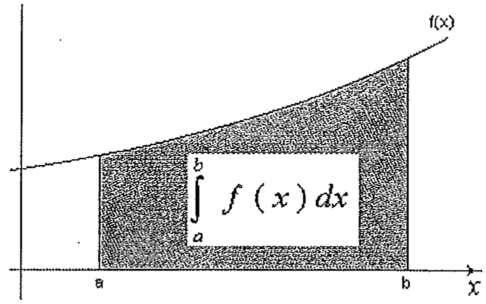
$$\int_a^b f(x)dx = F(b) - F(a)$$

where F is the antiderivative of f.



Recall:

*The general derivative is a **slope-finding function** or formula : (ex. f '(x) = 2x + 1)

*The specific derivative is the **actual slope** at a point (ex: f '(3) = 7)

Likewise…

The indefinite integral is an **Area-Finding Function** or formula (Ex: $\int 2x dx = x^2 + C$)

The definite integral is the **Actual Area** of the region for an interval (Ex: $\int_1^3 2x dx = 8$)

*If a function is **continuous** on a closed interval, then the function is able to be integrated on that interval

Class Examples:

1. Evaluate $\int_1^4 (3x^2 + 4x - 1)dx$

**NOTE: For definite integrals, we don't need to worry about the constant of integration " +C ". It will always wash out.

2. Evaluate $\int_{-2}^1 2x dx$

Integral Properties:

1) $\int_a^a f(x)dx = 0$

2) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

3) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ (given that c is between a and b)

Example 3: If $\int_0^3 f(x)dx = 4$ and $\int_3^6 f(x)dx = -1$, find the below:

a) $\int_0^6 f(x)dx$

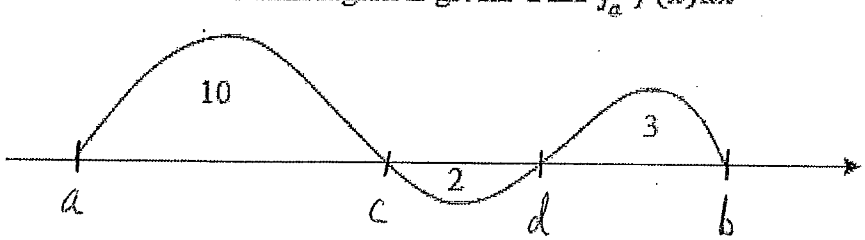
b) $\int_6^3 f(x)dx$

c) $\int_3^3 f(x)dx$

d) $\int_3^6 (-5f(x)+3)dx$

Ex. 4: If $\int_3^8 f'(x)dx = 10$ and $f(8) = 6$, find $f(3)$.
*Reminder that the FFTC can be used as an equation solving tool to find the value of an antiderivative at a specific point. Recall that $\int_a^b f'(x)dx = f(b) - f(a)$

Ex. 5: The area for each region is given. Find $\int_a^b f(x)dx$



6.3b Notes – 2nd Fundamental Theorem of Calculus and Average Value Theorem

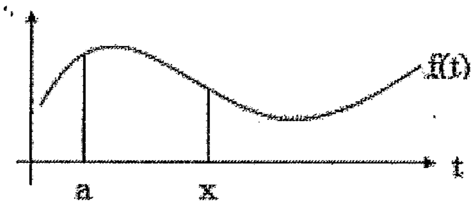
Definite Integral as a Function

To recap, we've covered:

1) Indefinite General Integrals (Area-finding functions)
2) Definite Integrals (Finds Area between 2 x-values)

There is also now a function that is the integral itself. Instead of going from a constant to another constant, we are going from a constant to a moving value of x.

Consider: $f(x) = \int_a^x f(t)dt$



2nd Fundamental Theorem of Calculus **Very Important**

Applies the concept that derivative and integrals are inverse operations of each other.

1) $\frac{d}{dx}\left[\int_a^{p(x)} f(t)dt\right] = f(p(x)) \cdot p'(x)$ (a is a constant)

2) $\frac{d}{dx}\left[\int_{q(x)}^{p(x)} f(t)dt\right] = f(p(x)) \cdot p'(x) - f(q(x)) \cdot q'(x)$

Example 1:

$$\frac{d}{dx}\left[\int_1^{x^2} (2t-3)dt\right]$$

1) $\frac{d}{dx}\left[\int_{a}^{p(x)} f(t)\,dt\right] = f(p(x))\cdot p'(x)$ (a is a constant)

2) $\frac{d}{dx}\left[\int_{q(x)}^{p(x)} f(t)\,dt\right] = f(p(x))\cdot p'(x) - f(q(x))\cdot q'(x)$

Example 2:

a) $\frac{d}{dx}\left[\int_{-3}^{x} \sqrt{t^2+4}\,dt\right] =$

b) $\frac{d}{dx}\left[\int_{3}^{x^2} \sqrt{t-1}\,dt\right] =$

c) $\frac{d}{dx}\left[\int_{10}^{x^2} \sqrt{t-1}\,dt\right] =$

d) $\frac{d}{dx}\left[\int_{3x}^{0} \frac{1}{t+2}\,dt\right] =$

e) $\frac{d}{dx}\left[\int_{x}^{x^2} (2t+3)\,dt\right] =$

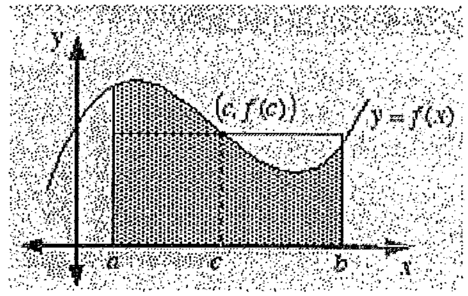
Ex. 2 Find $\frac{d}{dx}\left[\int_{2x^3}^{5} \frac{2t}{5-t^2}\,dt\right]$

* $\frac{d}{dx}\int_{a}^{p(x)} f(t)\,dt = f(p(x))\,p'(x)$

6.3b Average Value Theorem

If function f is integrable on the closed interval [a, b], then the average value of f on the interval is

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx$$



*There exists a rectangle such that the area of the rectangle is the same as the area under the curve (shaded region). f (c) is the height of the rectangle

Example 1: a) Find the average value of $f(x) = x^2 + 1$ on [2, 5]. b) find the c value

Practice Problem:

Avg. value theorem: $f(c) = \frac{1}{b-a}\int_a^b f(x)dx$

#47) $f(x) = 4 - x^2$ [-2, 2] a) Find avg. value
b) find c-value

Practice Problems: Average Value Theorem, First Theorem of Calculus, 2^{nd} Theorem of Calculus

For each problem, find the values of c that satisfy the Mean Value Theorem for Integrals.

1) $f(x) = 2x^2 - 4x - 4$; $[0, 3]$

For each problem, find the average value of the function over the given interval.

2) $f(x) = -x^2 + 2x + 1$; $[1, 4]$

1. $\frac{d}{dx}\int_2^x \ln t \, dt =$

(A) $\ln x$ (B) $\ln 2$ (C) $\frac{1}{x}$

(D) $\frac{1}{2}$ (E) $\ln x - \ln 2$

2. If $g(x) = \int_\pi^{\pi x} \cos(t^2)\, dt$, then $g'(x) =$

(A) $\sin(\pi^2 x^2)$ (B) $\pi x \sin(\pi^2 x^2)$ (C) $\pi x \cos(\pi^2 x^2)$

(D) $\cos(\pi^2 x^2)$ (E) $\pi \cos(\pi^2 x^2)$

3. $\frac{d}{dx}\int_{\sin x}^4 \sqrt{1+t^2}\, dt =$

(A) $\sqrt{1+\sin^2 x}$ (B) $-\cos x\sqrt{1+\sin^2 x}$ (C) $-\sqrt{1+\sin^2 x}$

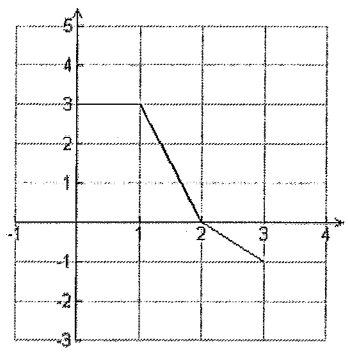
(D) $\cos x\sqrt{1+\sin^2 x}$ (E) $\sqrt{1+\cos^2 x}$

4. If f has two continuous derivatives on [5, 10], then $\int_5^{10} f''(t)\,dt =$

(A) $f'''(10) - f'''(5)$ (B) $f(10) - f(5)$ (C) $f'(10) - f'(5)$

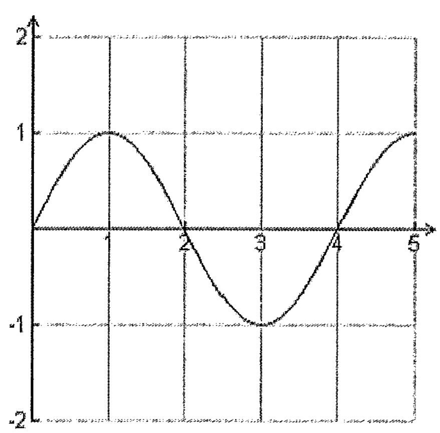
(D) $f''(10) - f''(5)$ (E) $f''(5) - f''(10)$

5. The graph of f is given, and g is an antiderivative of f. If $g(3) = 6$, find $g(0)$.



(A) 1 (B) 2 (C) 4 (D) 5 (E) 10

6. The graph of f is given. $F(x) = \int_0^x f(t)\,dt$



Which of the following statements is true?

(A) F decreases on (1, 2).
(B) F has a relative minimum at $x = 2$
(C) F decreases on (2, 4)
(D) F has a relative maximum at $x = 1$.
(E) F has a point of inflection at $x = 4$.

7. $\frac{d}{dx}\int_{x}^{x^2} \tan(t)\,dt =$

(A) $\tan\left(x^2\right)-\tan x$

(B) $\tan x-\tan\left(x^2\right)$

(C) $\tan x-2x\tan\left(x^2\right)$

(D) $2x\tan\left(x^2\right)-\tan x$

(E) $\sec^2\left(x^2\right)-\sec^2 x$

8. $\int_{1}^{e}\left(x-\frac{5}{x}\right)dx =$

(A) $\frac{1}{2}e^2-\frac{11}{2}$

(B) $\frac{1}{2}e^2-\frac{9}{2}$

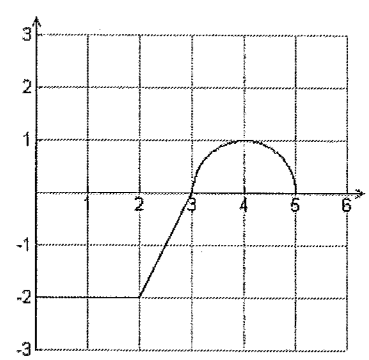
(C) $e^2-\frac{11}{2}$

(D) $\frac{1}{2}e^2-\frac{3}{2}$

(E) $\frac{11}{2}-\frac{1}{2}e^2$

The graph of f is given. It consists of two line segments and a semi-circle.

$g(x)=\int_{1}^{x} f(t)\,dt$



(a) Find $g(0)$, $g(1)$, and $g(5)$.

(b) Find $g'(2)$, $g''(2)$, and $g'''(4)$ or state that it does not exist.

(c) For what value(s) of x does the graph of g have a point of inflection? Justify your answer.

(d) Find the absolute maximum and absolute minimum values of g on [0, 5]. Justify your answer.