Key

6.4 AP Practice Problems (p. 437)

$* \int_0^5 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^5 f(x)\,dx$

$7 = -3 + \int_2^5 f(x)\,dx$

$10 = \int_2^5 f(x)\,dx$

1. If $\int_0^2 f(x)\,dx = -3$ and $\int_0^5 f(x)\,dx = 7$, then $\int_2^5 [4f(x) - 1]\,dx$ equals

(A) 13 (B) 15 $\boxed{\text{(C)}}$ 37 (D) 39

$\int_2^5 4f(x) - 1\,dx = 4\int_2^5 f(x)\,dx - \int_2^5 1\,dx$

$= 4(10) - x\Big]_2^5$

$= 40 - (5-2)$

$= 40 - 3$

$= \boxed{37}$

2. An object in rectilinear (linear) motion moves along the x-axis with velocity $v = v(t)$ meters per second. If $v(t) = 3t^2 + t - 2$, what is the average velocity of the object during the interval $0 \le t \le 6$?

$\boxed{\text{(A)}}$ 37 m/s (B) 38 m/s (C) 41 m/s (D) 222 m/s

Avg. value theorem: $f(c) = \frac{1}{b-a}\int f(x)\,dx$

Avg. velocity $= \frac{1}{b-a}\int_a^b v(t)\,dt \rightarrow \frac{1}{6-0}\int_0^6 3t^2+t-2\,dt = \boxed{37\ m/s}$

3. If $g(x) = 2f(x) - 4$ on the interval $[-2, 8]$, then $\int_{-2}^8 [f(x) + g(x)]\,dx$ equals

(A) $3\int_{-2}^8 f(x)\,dx - 24$ $\boxed{\text{(B)}}$ $3\int_{-2}^8 f(x)\,dx - 40$

(C) $3\int_{-2}^8 f(x)\,dx - 4$ (D) $\int_{-2}^8 f(x)\,dx - 40$

$\int_{-2}^8 f(x)\,dx + \int_{-2}^8 g(x)\,dx$

$\int_{-2}^8 f(x)\,dx + \int_{-2}^8 2f(x) - 4\,dx$

$3\int_{-2}^8 f(x)\,dx - \int_{-2}^8 4\,dx$

$-4x\Big]_{-2}^8 = -4(8) - (-4(-2))$

$= -32 - 8 = -40$

$\boxed{3\int_{-2}^8 f(x)\,dx - 40}$

4. $\int_1^e \frac{3x^2+1}{x}\,dx =$

$*$ power Rule

(A) $\frac{3e^2-1}{2}$ (B) $\frac{3e^2+1}{2}$

(C) $e^3 + 1$ (D) $3e^2 - 1$

$\int \frac{3x^2}{x} + \frac{1}{x}\,dx$

$\int 3x + \frac{1}{x}\,dx$

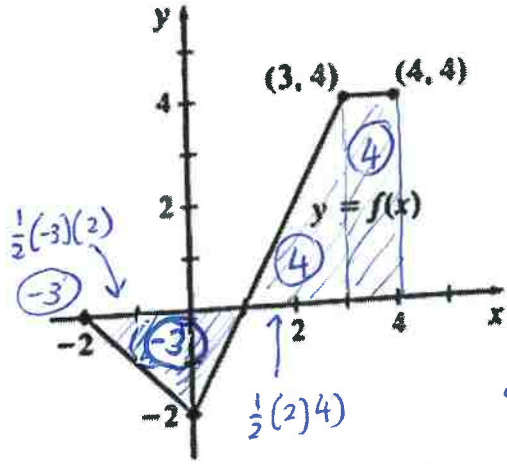
$\frac{3x^2}{2} + \ln x\Big]_1^e = \frac{3e^2}{2} + \ln e - \left(\frac{3}{2} + \ln 1\right)$

$\frac{3}{2}e^2 + 1 - \frac{3}{2} + 0$

$\frac{3}{2}e^2 - \frac{1}{2}$

$= \boxed{\frac{3e^2-1}{2}}$

5. The graph of the piecewise function f is below.
What is $\int_{-2}^{4} f(x)\,dx$?



$8-3=\boxed{5}$

(A) 2 $\boxed{\text{(B)}}$ 5 (C) $\frac{17}{2}$ (D) 9

6. $\int_1^4 \sqrt{x}\left(x - \frac{1}{x}\right) dx =$

(A) $\frac{32}{5}$ (B) $\frac{44}{5}$ $\boxed{\text{(C)}}$ $\frac{52}{5}$ (D) $\frac{56}{15}$

$\frac{2}{5}x^{5/2} - 2x^{1/2}\Big]_1^4$

$\frac{2}{5}(4)^{5/2} - 2(4)^{1/2} - \left(\frac{2}{5} - 2\right)$

$\frac{2}{5}(2)^5 - 2(2) - \frac{2}{5} + 2$

$\int x\sqrt{x} - \frac{\sqrt{x}}{x}\,dx \to \int x^{3/2} - x^{-1/2}\,dx \to \frac{x^{5/2}}{5/2} - \frac{x^{1/2}}{1/2}$

$\frac{64}{5} - 4 - \frac{2}{5} + 2 = 10.4 = \boxed{\frac{52}{5}}$

7. The average value of $f(x) = \sin x$ on the interval $\left[-\frac{\pi}{3}, \frac{\pi}{2}\right]$ is

(A) $-\frac{3}{5\pi}$ $\boxed{\text{(B)}}$ $\frac{3}{5\pi}$ (C) $\frac{1}{2}$ (D) $\frac{5\pi}{12}$

Avg. value $= \frac{1}{\frac{\pi}{2} - \left(-\frac{\pi}{3}\right)} \int_{-\pi/3}^{\pi/2} \sin x\,dx$

$= \frac{1}{\frac{5\pi}{6}} \cdot -\cos x\Big]_{-\pi/3}^{\pi/2} = -\cos(\pi/2) + \cos($

$\frac{6}{5\pi}\left(0 + \frac{1}{2}\right) = \boxed{\frac{3}{5\pi}}$

8. If $f(x) = \begin{cases} x^3 \text{ if } x \le 1 \\ \frac{1}{x} \text{ if } x > 1 \end{cases}$ then $\int_0^e f(x)\,dx$ equals

(A) 0 $\boxed{\text{(B)}}$ $\frac{5}{4}$ (C) $\frac{1}{4}+e$ (D) $\frac{e^4}{4}$

$$\int_0^e f(x)dx = \int_0^1 x^3 dx + \int_1^e \frac{1}{x}dx$$

$$= \frac{x^4}{4}\Big]_0^1 + \ln x\Big]_1^e$$

$$= \frac{1}{4} - \frac{0}{4} + \ln e - \ln 1 = \frac{1}{4} - 0 + 1 - 0$$

$$= \frac{1}{4} + 1 = \boxed{\frac{5}{4}}$$

9. What is the average value of $f(x) = \frac{1}{x}$ on the closed interval $[1, 4]$?

(A) $\ln\frac{4}{3}$ (B) $\frac{\ln 3}{3}$ $\boxed{\text{(C)}}$ $\frac{\ln 4}{3}$ (D) $\ln 4$

$$\text{Avg. value} = \frac{1}{4-1}\int_1^4 \frac{1}{x}dx$$

$$= \frac{1}{3}\ln x\Big]_1^4 = \frac{1}{3}\ln(4) - \frac{1}{3}\ln(1) = \boxed{\frac{1}{3}\ln 4}$$

10. The area under the graph of $f(x) = x^2(3-x)$ from 0 to 3 is

(A) 6 $\boxed{\text{(B)}}$ $\frac{27}{4}$ (C) 7 (D) 7.5

$$\int_0^3 x^2(3-x)dx \rightarrow \int_0^3 3x^2 - x^3 dx \rightarrow \frac{3x^3}{3} - \frac{x^4}{4}\Big]_0^3$$

$$= 3^3 - \frac{3^4}{4} = 27 - \frac{81}{4} = \boxed{\frac{27}{4}}$$

11. If $\int_1^8 f(x)\,dx = 5$ and $\int_8^4 f(x)\,dx = 9$, then $\int_1^4 f(x)\,dx$ equals

(A) -4 (B) 4 (C) 8 (D) 14

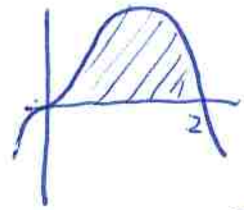
$$\int_1^8 f(x)dx = \int_1^4 f(x)dx + \int_4^8 f(x)dx$$

$$5 = \int_1^4 f(x)dx + (-9)$$

$$14 = \int_1^4 f(x)dx$$

$$\int_1^4 f(x)dx = \boxed{14}$$

12. What is the average value of the part of the graph of $f(x) = x^3(2-x)$ that lies in the first quadrant?



$\boxed{\text{(A)}}$ $\frac{4}{5}$ (B) 1 (C) $\frac{8}{5}$ (D) $\frac{12}{5}$

$$\text{Avg. value} = \frac{1}{2-0}\int_0^2 x^3(2-x)dx$$

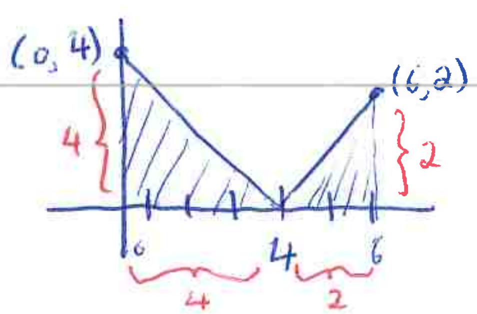
$$= \frac{1}{2}\int 2x^3 - x^4 dx$$

$$\int x^3 - \frac{1}{2}x^4 dx \rightarrow \left.\frac{x^4}{4} - \frac{x^5}{10}\right]_0^2$$

$$\frac{16}{4} - \frac{2^5}{10} = \boxed{\frac{4}{5}}$$

13. $\int_0^6 |x-4|\,dx =$

(A) 6 $\boxed{\text{(B)}}$ 10 (C) 22 (D) 42



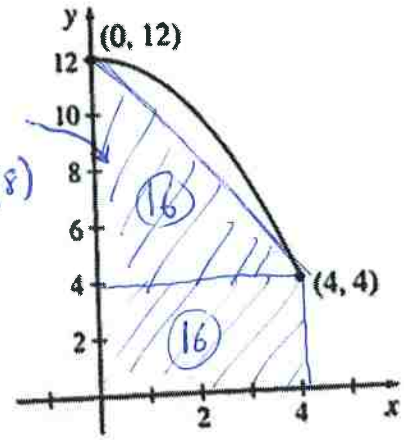
* $A = \frac{1}{2}bh$

$$\int_0^6 |x-4|dx = \frac{1}{2}(4)(4) + \frac{1}{2}(2)(2)$$

$$= 8 + 2 = \boxed{10}$$

* Add the areas of the 2 triangles

14. The graph of f is shown below.



$\frac{1}{2}bh$

$\frac{1}{2}(4)(8)$

Area is greater than 32

Then $\int_0^4 f(x)\,dx$ must be between

(A) 4 and 12 (B) 20 and 32 $\boxed{\text{(C)}}$ 16 and 48 (D) 40 and 60

15. Find $\int_{-2}^{10} f(x)\,dx$ where $f(x) = \begin{cases} x+3 & \text{if } -2 \le x < 2 \\ 3x & \text{if } \ \ 2 \le x \le 10 \end{cases}$

(A) 144 (B) 150 $\boxed{\text{(C)}}$ 156 (D) 306

$$\int_{-2}^{10} f(x)dx = \int_{-2}^{2} x+3\,dx + \int_{2}^{10} 3x\,dx$$

$$= 12 + 144 = \boxed{156}$$

16. A car's velocity $v = v(t)$ (in ft/s) is measured each second t for $t = 0$ to $t = 8$ and posted in the table. Use a Right Riemann sum with four subintervals of equal length to approximate the car's average velocity over the interval from 0 to 8 seconds.

t	0	1	2	3	4	5	6	7	8
$v(t)$	0	2	4	6	7	7	8	6	2

(A) 42 ft/s (B) 7 ft/s (C) 5 ft/s $\boxed{\text{(D)}}$ 5.25 ft/s

$$\text{avg. velocity} = \frac{1}{b-a}\int_a^b v(t)dt$$

$$= \frac{1}{8-0}\int_0^8 v(t)dt$$

$$\frac{1}{8}\cdot\left[2+4+6+7+7+8+6+2\right]$$

$$= \frac{1}{8}(42)$$

$$= \boxed{5.25 \text{ ft/s}}$$

17. An object in rectilinear motion is moving along a horizontal line with velocity $v(t) = 3t^2 - 6t$, $1 \leq t \leq 4$ (in meters per second).

(a) Find the total distance the object moves from $t = 1$ to $t = 4$.

(b) For what time(s) is the object at rest?

(c) If at time $t = 1$, the object is 2 m from the origin, what is its position at $t = 4$?

(d) Find the average velocity of the object from $t = 1$ to $t = 4$.

a) 22 meters

b) $t = 2$ secs.

c) 20 meters

d) 6 m/sec

a) $v(t) = 3t^2 - 6t$

$0 = 3t(t-2)$

$t = 0,\ t = 2$

$v(t)$: sign chart on $[1, 4]$: $-$ between 1 and 2, $+$ between 2 and 4

* change in direction at $t = 2$

Total distance $\rightarrow \int_1^2 3t^2 - 6t\,dt + \int_2^4 3t^2 - 6t\,dt$

$2 + 20 = \boxed{22 \text{ meters}}$

b) object is at rest at $t = 2$ secs since $v(t) = 0$

c) $x(b) = x(a) + \int_a^b v(t)\,dt$

$x(4) = x(1) + \int_1^4 v(t)\,dt$

$x(4) = 2 + 18 = \boxed{20 \text{ meters}}$

d) Avg. velocity $= \frac{1}{b-a}\int_a^b v(t)\,dt$

$\frac{1}{4-1}\int_1^4 v(t)\,dt = \frac{1}{3}(18) = \boxed{6 \text{ m/sec}}$