6.5 HW p. 449–454 #9–19 odd, 27–51 odd, 61, 63, 79, 87

9) $\int x^{2/3}\,dx \to \dfrac{x^{5/3}}{5/3}+c \to \boxed{\frac{3}{5}x^{5/3}+C}$

11) $\int \dfrac{1}{\sqrt{1-x^2}}\,dx$

$* \int \dfrac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right)+c$

$\to \arcsin\left(\frac{x}{1}\right)+c \to \boxed{\arcsin(x)+C}$

13) $\int \dfrac{5x^2+2xe^x-1}{x}\,dx \to \int \dfrac{5x^2}{x}+\dfrac{2xe^x}{x}-\dfrac{1}{x}\,dx$

$\int 5x+2e^x-\frac{1}{x}\,dx \to \boxed{\frac{5x^2}{2}+2e^x-\ln|x|+C}$

15) $\int \dfrac{\tan x}{\cos x}\,dx \to \cancel{\int \dfrac{\frac{\sin x}{\cos x}}{\cos x}\,dx \to \int \dfrac{\sin x}{(\cos x)^2}\,dx}$

$\int \tan x \cdot \dfrac{1}{\cos x}\,dx \to \int \tan x \sec x\,dx = \boxed{\sec x + C}$

17) $\int \dfrac{2}{5\sqrt{1-x^2}}\,dx \quad \dfrac{2}{5}\int \dfrac{dx}{\sqrt{1-x^2}}\,dx \to \dfrac{2}{5}\arcsin\left(\frac{x}{1}\right)+c$

$\boxed{\frac{2}{5}\arcsin(x)+C}$

19) $\int \dfrac{e^t+e^{-t}}{2}\,dt$

$\int \frac{1}{2}e^t\,dt + \int \frac{1}{2}e^{-t}\,dt$

$\frac{1}{2}e^t - \frac{1}{2}e^{-t}+c$

$\boxed{\frac{1}{2}e^t - \frac{1}{2e^t}+C}$

27) $\int \sin(3x)dx$ $\quad u=3x \quad \frac{du}{dx}=3 \quad \Big| \quad dx=\frac{du}{3}$

$$\int \sin(3x)dx \rightarrow \int \sin(u)\cdot\frac{du}{3} \rightarrow \frac{1}{3}\int \sin u\,du$$

$$\rightarrow -\frac{1}{3}\cos u + c \rightarrow \boxed{-\frac{1}{3}\cos(3x)+c}$$

29) $\int \sin x \cos^2 x\,dx$

$\int \sin x\,(\cos x)^2 dx$

$u=\cos x \quad \frac{du}{dx}=-\sin x \quad dx=\frac{du}{-\sin x}$

$\int \cancel{\sin x}\cdot u^2\cdot\frac{du}{-\cancel{\sin x}}$

$-\int u^2 du$

$-\frac{u^3}{3}+c$

$\boxed{-\frac{1}{3}(\cos x)^3+c}$

31) $\int \frac{e^{1/x}}{x^2}dx$ $\quad \int \frac{e^{x^{-1}}}{x^2}dx$ $\quad u=x^{-1} \quad \frac{du}{dx}=-1x^{-2} \quad \Big| \quad \frac{du}{dx}=\frac{-1}{x^2} \quad dx=-x^2du$

$$\int \frac{e^u}{\cancel{x^2}}\cdot -\cancel{x^2}du \rightarrow -\int e^u du = -e^u+c \rightarrow \boxed{-e^{\frac{1}{x}}+c}$$

33) $\int \frac{x\,dx}{x^2-1}$ $\quad \int \frac{x}{x^2-1}dx$ $\quad u=x^2-1 \quad \frac{du}{dx}=2x \quad \Big| \quad dx=\frac{du}{2x}$

$$\int \frac{\cancel{x}}{u}\cdot\frac{du}{2\cancel{x}} \rightarrow \frac{1}{2}\int \frac{1}{u}du \rightarrow \frac{1}{2}\ln|u|+c \rightarrow \boxed{\frac{1}{2}\ln|x^2-1|+c}$$

35) $\int \frac{e^x}{\sqrt{1+e^x}}dx$

$\int \frac{e^x}{(1+e^x)^{1/2}}dx$

$\int \frac{\cancel{e^x}}{u^{1/2}}\cdot\frac{du}{\cancel{e^x}}$ $\quad u=1+e^x \quad \frac{du}{dx}=e^x \quad dx=\frac{du}{e^x}$

$$\int u^{-1/2}du = \frac{u^{1/2}}{1/2}+c \rightarrow \boxed{2(1+e^x)^{1/2}+c}$$

37) $\int \frac{1}{\sqrt{x}\,(1+\sqrt{x})^4}\,dx$

$u = 1 + x^{1/2}$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$dx = 2\sqrt{x}\,du$

$$\int \frac{1}{\cancel{\sqrt{x}}\cdot u^4}\cdot 2\cancel{\sqrt{x}}\,du \rightarrow 2\int u^{-4}\,du \rightarrow 2\,\frac{u^{-3}}{-3} + C$$

$$-\frac{2}{3}\cdot\frac{1}{(1+\sqrt{x})^3} + C \rightarrow \boxed{\frac{-2}{3(1+\sqrt{x})^3} + C}$$

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39) $\int \frac{3e^x}{\sqrt[4]{e^x - 1}}\,dx$

$\int \frac{3e^x}{(e^x-1)^{1/4}}\,dx$

$u = e^x - 1$

$\frac{du}{dx} = e^x$

$dx = \frac{du}{e^x}$

$\int \frac{3\cancel{e^x}}{u^{1/4}}\cdot\frac{du}{\cancel{e^x}}$

$\int 3u^{-1/4}\,du \rightarrow 3\cdot\frac{u^{3/4}}{3/4} + C$

$$3\cdot\frac{4}{3}u^{3/4} + C \rightarrow 4u^{3/4} + C \rightarrow \boxed{4(e^x - 1)^{3/4} + C}$$

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41) $\int \frac{\cos x\,dx}{2\sin x - 1}\,dx$

$u = 2\sin x - 1$

$\frac{du}{dx} = 2\cos x$

$dx = \frac{du}{2\cos x}$

$$\int \frac{\cancel{\cos x}}{u}\cdot\frac{du}{2\cancel{\cos x}} \rightarrow \frac{1}{2}\int \frac{1}{u}\,du \rightarrow \frac{1}{2}\ln|u| + C \rightarrow \boxed{\frac{1}{2}\ln|2\sin x - 1| + C}$$

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43) $\int \sec(5x)\,dx$

$u = 5x$

$\frac{du}{dx} = 5$

$dx = \frac{du}{5}$

$\boxed{* \int \sec u\,du = \ln|\sec u + \tan u| + C}$

$\int \sec(u)\cdot\frac{du}{5}$

$\frac{1}{5}\int \sec u\,du = \frac{1}{5}\ln|\sec u + \tan u| + C$

$$\boxed{\frac{1}{5}\ln|\sec(5x) + \tan(5x)| + C}$$

45) $\int \sqrt{\tan x} \cdot \sec^2 x \, dx$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$dx = \frac{du}{\sec^2 x}$

$\int (\tan x)^{1/2} \cdot \sec^2 x \, dx$

$\int u^{1/2} \cdot \cancel{\sec^2 x} \cdot \frac{du}{\cancel{\sec^2 x}} \rightarrow \int u^{1/2} du \rightarrow \frac{u^{3/2}}{3/2} + C$

$\boxed{\frac{2}{3} (\tan x)^{3/2} + C}$

47) $\int \frac{\sin x}{\cos^2 x} dx \rightarrow \int \frac{\sin x}{(\cos x)^2} dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{du}{-\sin x}$

$\int \frac{\cancel{\sin x}}{u^2} \cdot \frac{du}{-\cancel{\sin x}} \rightarrow -\int u^{-2} du \rightarrow \frac{-u^{-1}}{-1} + C \rightarrow \frac{1}{u} + C$

$\rightarrow \boxed{\frac{1}{\cos x} + C}$ OR

$\rightarrow \boxed{\sec x + C}$

49) $\int \sin x \cdot e^{\cos x} dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{du}{-\sin x}$

$\int \sin x \cdot e^u \cdot \frac{du}{-\sin x}$

$-\int e^u du \rightarrow -e^u + C$

$= \boxed{-e^{\cos x} + C}$

51) $\int x \sqrt{x + 3} \, dx$

$\int x (x+3)^{1/2} dx$

$u = x + 3$

$\frac{du}{dx} = 1 \rightarrow dx = du$

$x = u - 3$

$\int x \cdot u^{1/2} \cdot du$

$\int (u-3) u^{1/2} du$

$\int u^{3/2} - 3u^{1/2} du$

$\frac{u^{5/2}}{5/2} - \frac{3u^{3/2}}{3/2} + C$

$\frac{2}{5}(x+3)^{5/2} - \frac{2}{3} \cdot 3 (x+3)^{3/2} + C$

$\boxed{\frac{2}{5}(x+3)^{5/2} - 2(x+3)^{3/2} + C}$

61) $\int_{-2}^{0} \frac{x}{(x^2+3)^2}\,dx$

$u = x^2+3$ | $dx = \frac{du}{2x}$

$\frac{du}{dx} = 2x$

$\int \frac{\cancel{x}}{u^2}\cdot\frac{du}{2\cancel{x}}$

$\frac{1}{2}\int u^{-2}\,du$

$\frac{1}{2}\cdot\frac{u^{-1}}{-1}$

$\left.\frac{-1}{2(x^2+3)}\right]_{-2}^{0}$

$\frac{-1}{2(0+3)} - \frac{-1}{2(7)}$

$= \frac{-1}{6} + \frac{1}{14} = \boxed{\frac{-2}{21}}$

63) $\int_{0}^{1} x^2 \cdot e^{x^3+1}\,dx$

$u = x^3+1$ | $dx = \frac{du}{3x^2}$

$\frac{du}{dx} = 3x^2$

$\int \cancel{x^2} e^u \cdot \frac{du}{3\cancel{x^2}}$

$\frac{1}{3}\int e^u\,du$

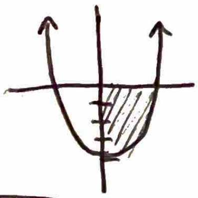
$\frac{1}{3}e^u \rightarrow$

$\left.\frac{1}{3}e^{x^3+1}\right]_{0}^{1} = \frac{1}{3}e^2 - \frac{1}{3}e^{0+1}$

$\boxed{\frac{1}{3}(e^2-e)}$

79) $\int_{-2}^{2} (x^2-4)\,dx$

*use properties of even and odd functions



*even function

$\int_{-2}^{2}(x^2-4)\,dx = 2\int_{0}^{2} x^2-4\,dx \rightarrow \left.\frac{x^3}{3}-4x\right]_{0}^{2}$

$2\left[\frac{8}{3}-8-(0-0)\right] = \boxed{\frac{-32}{3}}$

87) f is an odd function.

$\int_{0}^{3} f(x)\,dx = 6$ and $\int_{3}^{5} f(x)\,dx = 2$

$\int_{-3}^{5} f(x)\,dx = -6+6+2 = \boxed{2}$