(47) AP Calculus – 6.5c Notes – Integrating using Long Division & Completing the square

What would happen if we attempted to apply power rule for this problem?

$\int \frac{1}{x}dx \rightarrow \int x^{-1}dx = \cancel{\frac{x^0}{0}}$

$\int \frac{1}{x}dx = \ln|x| + C$

$\int e^u du = e^u + C$

Recall Derivative Rule:

$$\frac{d}{dx}\ln u = \frac{u'}{u}$$

$\frac{d}{dx}e^u = e^u \cdot u'$

Natural Log Integral Rule

$$\boxed{\int \frac{1}{u}du = \ln|u| + C}$$

Example 1: $\int \frac{2x}{x^2+1}dx$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$2x\,dx = du$

$dx = \frac{du}{2x}$

$\int \frac{\cancel{2x}}{u}\cdot\frac{du}{\cancel{2x}}$

$\int \frac{1}{u}du$

$\ln|u| + C$

$\boxed{\ln|x^2+1| + C}$

Example 2: $\int \frac{1}{x\ln x}dx$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$dx = x\,du$

$\int \frac{1}{\cancel{x}\cdot u}\cdot \cancel{x}\,du$

$\ln|u| + C$

$\boxed{\ln|\ln x| + C}$

Completing the Square and Long Division are two skills that help us manipulate the integrand until it becomes something we can work with. We are practicing those two skills in this lesson.

Using ~~Long Division~~ Synthetic Division to Rewrite the Integrand:

1. $\int \frac{3x^3-x^2-5x+1}{x-2}dx$

$x - 2 = 0$

$x = 2$

$$\begin{array}{c|cccc} 2 & 3 & -1 & -5 & 1 \\ & \downarrow & 6 & 10 & 10 \\ \hline & 3 & 5 & 5 & 11 \end{array}$$

$\int 3x^2 + 5x + 5 + \frac{11}{x-2}\,dx$

$\int \frac{11}{x+2}dx$

$u = x+2$

$\frac{du}{dx} = 1$

$dx = du$

$11\int \frac{1}{u}du$

$11\ln|u| + C$

$11\ln|x+2| + C$

$\boxed{\frac{\cancel{3}x^3}{\cancel{3}} + \frac{5x^2}{2} + 5x + 11\ln|x+2| + C}$

2. $\int \frac{6x^4-7x^3+x^2+2x}{3x-5}dx$

$\frac{d}{dx}\arcsin u = \frac{u'}{\sqrt{1-u^2}}$ $\frac{d}{dx}\arctan u = \frac{u'}{1+u^2}$ $\frac{d}{dx}\operatorname{arcsec} u = \frac{u'}{\lvert u\rvert\sqrt{u^2-1}}$	$\frac{d}{dx}\arccos u = -\frac{u'}{\sqrt{1-u^2}}$ $\frac{d}{dx}\operatorname{arccot} u = -\frac{u'}{1+u^2}$ $\frac{d}{dx}\operatorname{arccsc} u = -\frac{u'}{\lvert u\rvert\sqrt{u^2-1}}$	**Inverse Trig Integral Rules:** *a is a constant* 1. $\int\frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right)+C$ 2. $\int\frac{du}{a^2+u^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right)+C$ 3. $\int\frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\left(\frac{\lvert u\rvert}{a}\right)+C$

Ex. 1: $\int\frac{5}{x\sqrt{x^2-9}}dx$

$\int\frac{5}{x\sqrt{(x)^2-(3)^2}}dx$

$a=3$
$u=x$
$\frac{du}{dx}=1$
$dx=du$

$5\int\frac{du}{u\sqrt{u^2-a^2}} \to \boxed{5\cdot\frac{1}{3}\operatorname{arcsec}\left(\frac{|x|}{3}\right)+C}$

Ex. 2: $\int\frac{1}{4+(x-1)^2}dx$

$\int\frac{1}{(2)^2+(x-1)^2}dx$

$a=2$
$x-1=u$
$\frac{du}{dx}=1$
$dx=du$

$\int\frac{du}{a^2+u^2} = \boxed{\frac{1}{2}\arctan\left(\frac{x-1}{2}\right)+C}$

Completing the Square Steps:

1. Write in standard form: x^2+bx+c
2. Add spaces $x^2+bx+\_\_\_+c-\_\_\_$
3. Put $\left(\frac{b}{2}\right)^2$ into the spaces
4. Factor expression

Using Completing the Square to Rewrite the Integrand:

3. $\boxed{\int\frac{1}{x^2+6x+10}dx}$

$x^2+6x+10$ $\left(\frac{b}{2}\right)^2$

$\underbrace{x^2+6x+9}+10-9$

$(x+3)(x+3)$

$(x+3)^2+1$

$\int\frac{1}{(x+3)^2+1}dx$

$\int\frac{1}{(x+3)^2+(1)^2}dx$

$a=1$
$u=x+3$
$\frac{du}{dx}=1$
$dx=du$

$\int\frac{du}{u^2+a^2} = \frac{1}{1}\arctan\left(\frac{x+3}{1}\right)+C$

$\boxed{\arctan(x+3)+C}$

4. $\int\frac{1}{\sqrt{-x^2+8x-15}}dx$