Key

6.6 AP Practice Problems (p. 473) - Integration by Parts

1. An object in rectilinear motion is moving along the x-axis. Its acceleration at any time $t > 0$ is given by $a(t) = \ln(t+1)$. If the velocity $v = v(t)$ of the object at time $t = 1$ is $v(1) = 2$, then what is its velocity v at time $t = 3$?

(A) $3\ln 4 + 8$ $\boxed{\text{(B) } 3\ln 4}$ (C) $4\ln 4 + 8$ (D) $5\ln 4$

*FFTC $v(b) = v(a) + \int_a^b a(t)\,dt$

$v(3) = v(1) + \int_1^3 \ln(t+1)\,dt$

IBP: $u = \ln(t+1)$ $dv = dt$

$du/dt = \frac{1}{t+1}$ $v = t$

$\int u\,dv = uv - \int v\,du$

$= t\ln(t+1) - \int \frac{t}{t+1}\,dt$

synthetic division

$-1 \mid 1 \quad 0$; $\downarrow \; -1$; $1 \quad -1$

$\int 1 - \frac{1}{t+1}\,dt$

$t\ln(t+1) - (t - \ln(t+1))\Big]_1^3$

$3\ln(4) - 3 + \ln(4) - (\ln 2 - 1 + \ln 2)$

$\rightarrow 4\ln 4 - 2 - 2\ln 2$

$\rightarrow 4\ln 4 - 2 - \ln 4$

$3\ln 4 - 2$

$v(3) = v(1) + \int_1^3 \ln(t+1)\,dt$

$v(3) = 2 + (3\ln 4 - 2)$

$\boxed{v(3) = 3\ln 4}$

2. If $\int x^2 e^{2x}\,dx = f(x)e^{2x} + C$, then $f(x)$ equals

(A) $x^2 + 2x + 2$ (B) $\frac{1}{2}x^2 - \frac{1}{4}x + \frac{1}{8}$

(C) $\frac{1}{2}x^2 - \frac{1}{2}x - \frac{1}{4}$ $\boxed{\text{(D)}}$ $\frac{1}{2}x^2 - \frac{1}{2}x + \frac{1}{4}$

IBP (Tab)

	u	dv
+	x^2	e^{2x}
−	$2x$	$\frac{1}{2}e^{2x}$
+	2	$\frac{1}{4}e^{2x}$
−	0	$\frac{1}{8}e^{2x}$

$\int x^2 e^{2x}\,dx = \frac{1}{2}x^2 e^{2x} - \frac{1}{2}x e^{2x} + \frac{1}{4}e^{2x} + C$

$= e^{2x}\left(\frac{1}{2}x^2 - \frac{1}{2}x + \frac{1}{4}\right) + C$

$\boxed{f(x) = \frac{1}{2}x^2 - \frac{1}{2}x + \frac{1}{4}}$

3. $\int \cos^{-1} x\,dx =$

(A) $\ln|\tan x + \sec x| + C$

(B) $x\cos^{-1} x + \sqrt{1-x^2} + C$

$\boxed{\text{(C)}}$ $x\cos^{-1} x - \sqrt{1-x^2} + C$

(D) $x\cos^{-1} x - 2\sqrt{1-x^2} + C$

*LIATE

$u = \cos^{-1}(x)$ $dv = 1$

$\frac{du}{dx} = \frac{-1}{\sqrt{1-x^2}}$ $v = x$

$\int u\,dv = uv - \int v\,du$

$= x\cos^{-1}(x) + \int \frac{x}{\sqrt{1-x^2}}\,dx$ ← (u-sub)

$u = 1 - x^2$ $dx = \frac{du}{-2x}$

$\frac{du}{dx} = -2x$

$\int \frac{x}{u^{1/2}} \cdot \frac{du}{-2x}$

$-\frac{1}{2}\int u^{-1/2}\,du \rightarrow \frac{-1}{2} \cdot \frac{u^{1/2}}{1/2} + C$

$= x\cos^{-1}(x) + \left(-(1-x^2)^{1/2}\right) + C$

$= \boxed{x\cos^{-1}(x) - \sqrt{1-x^2} + C}$

4. $\int_0^2 xe^{-x}\,dx =$

(A) $1 - 3e^{-2}$ (B) $-1 - e$

(C) $1 + e^{-2}$ (D) $3e^{-2}$

u	dv
$+\ x$	e^{-x}
$-\ 1$	$-e^{-x}$
$+\ 0$	e^{-x}

$$-xe^{-x} - e^{-x}\Big]_0^2 = -2e^{-2} - e^{-2} - \left(0e^0 - e^0\right)$$

$$-3e^{-2} - 0 + 1 = \boxed{1 - 3e^{-2}}$$

5. $\int (3x^2 + 2)\sin x\,dx =$

(A) $-6x\cos x + 6\int x\cos x\,dx$

(B) $-(3x^2 + 2)\cos x + 6\int x\cos x\,dx$

(C) $-6x\cos x - \int (3x^2 + 2)\sin x\,dx$

(D) $(3x^2 + 2)\cos x - 6\int x\cos x\,dx$

IBP

$u = 3x^2 + 2$ $dv = \sin x$

$\frac{du}{dx} = 6x$ $v = -\cos x$

$$\int u\,dv = uv - \int v\,du$$

$$= -(3x^2+2)\cos x - \int -6x\cos x\,dx$$

$$= \boxed{-(3x^2+2)\cos x + \int 6x\cos x\,dx}$$

u	dv
$+\ 3x^2+2$	$\sin x$
$-\ 6x$	$-\cos x$
$+\ 6$	$-\sin x$
$-\ 0$	$\cos x$

$$-(3x^2+2)\cos x + 6x\sin x + C$$

* Tab Method does not match answer choice!

6. $\int xf'(x)\,dx =$

(A) $f(x) + C$ (B) $xf(x) - f(x) + C$

(C) $xf(x) - \int f(x)\,dx + C$ (D) $\frac{x^2}{2}f(x) + C$

IBP:

$u = x$ $dv = f'(x)\,dx$

$\frac{du}{dx} = 1$ $v = f(x)$

$$\int u\,dv = uv - \int v\,du$$

$$= \boxed{xf(x) - \int f(x)\,dx + C}$$