BC Calculus - 6.6 Notes – Integration by Parts

Integration by Parts (I.B.P.) -a method of integration useful for problems involving the product of two different types of functions . (example: logs and polynomial)

Integration by parts is typically used for the integration of the product of two functions.

$$\int f(x)g'(x) =$$

Integration by parts is based on the product rule:
$$[fg]' = f'g + fg'$$

Basic rule for choosing *f* and *g'*:
1. For f: choose something that becomes simpler when you differentiate.
2. For g': choose something that can easily be integrated.

Steps:

1. Determine the u-value by using the acronym L.I.P.E.T.
   a. LIPET shows the priority order for determining u-value
   b. **L**ogs **I**nverse Trig **P**olynomial **E**xponential function **T**rigonometric function
2. Let dv be other function
3. Find u, dv, v, and dv
4. Plug into formula and integrate

Tabular Integration: Differentiate to 0 for the chosen $f(x)$. Integrate your chosen $g'(x)$ the same number of times. Follow the sign convention, which is plus/minus repeating.

2. $\int x^4 \sin x \, dx$

$$f(x) \qquad\qquad g'(x)$$

Practice Problems:

Integrate the following.

1. $\int x \cos(x)\,dx$

2. $\int 2x \cos(3x + 1)\,dx$

3. $\int x^2 \sin(x)\,dx$

4. $\int 4xe^{3x+1}\,dx$

5. $\int_1^{e^2} x^4 \ln x\,dx$

6. $\int \ln x\,dx$

7. $\int_1^2 (3x^2 - 2x + 1) \ln x \, dx$

8. $\int x^3 e^x \, dx$

9. The table gives values of f, f', g, and g' for selected values of x. If $\int_0^3 f'(x)g(x) \, dx = 6$, then
$\int_0^3 f(x)g'(x) \, dx = ?$

x	0	3
$f(x)$	1	5
$f'(x)$	5	-3
$g(x)$	-4	3
$g'(x)$	3	2

10. Let f be a twice-differentiable function with selected values of f and its derivatives shown in the table. What is the value of $\int_0^3 xf''(x) \, dx$?

x	$f(x)$	$f'(x)$	$f''(x)$
0	2	-2	5
3	5	7	-2

11. $\int x \cos 2x \, dx$

(A) $\frac{1}{2}x^2 \sin(2x) + C$

(B) $\frac{1}{2}x^2 \cos(2x) + \frac{1}{2}\sin(2x) + C$

(C) $\frac{1}{2}x \sin(2x) - \frac{1}{4}\cos(2x) + C$

(D) $\frac{1}{2}x \sin(2x) + \frac{1}{4}\cos(2x) + C$

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12. $\int_1^e x^4 \ln x \, dx$

A) $\frac{6e^5-1}{25}$ (B) $\frac{4e^5+1}{25}$ (C) $\frac{1-e^3}{3}$ (D) e^4

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13. Let f be a differentiable function such that $\int f(x) \cos x \, dx = f(x) \sin x - \int \frac{1}{2}x^3 \sin x \, dx$. Which of the following could be $f(x)$.

A) $\frac{1}{2}\sin x$ (B) $\frac{1}{2}\cos x$ (C) $\frac{1}{8}x^4$ (D) $\frac{1}{2}x^3$