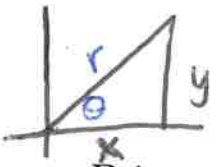
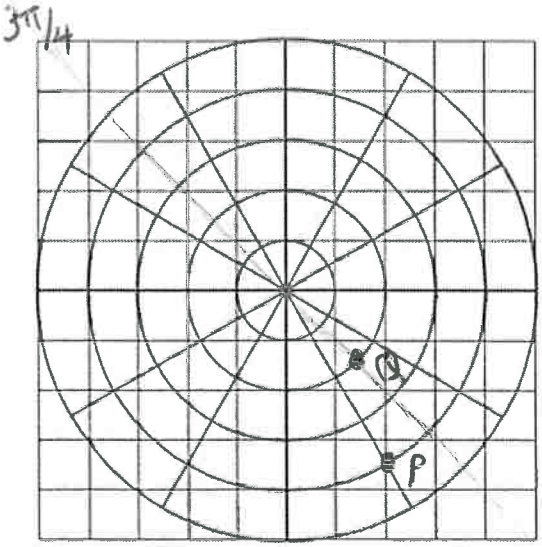


7.02 Converting Polar Coordinates Date: _____

The polar and rectangular grids do overlap so that a location can take on coordinates from either system.



If you know r and θ, how do you calculate x and y?

$\cos\theta = \frac{x}{r} \rightarrow x = r\cos\theta$

$\sin\theta = \frac{y}{r} \quad y = r\sin\theta$

If you know x and y, how do you calculate r and θ?

$x^2+y^2=r^2 \quad r=\sqrt{x^2+y^2}$

$\tan\theta = \frac{y}{x} \quad \theta = \tan^{-1}\left(\frac{y}{x}\right)$

Example: Find the rectangular coordinates for each point given in polar coordinates.

1. P (4, -60°)

r θ

$x = 4\cos(-60) \quad y = 4\sin(-60)$

$x = 4\left(\frac{1}{2}\right) = 2 \quad y = 4\left(\frac{-\sqrt{3}}{2}\right)$

$P(2, -2\sqrt{3}) \rightarrow P(2, -3.464)$

2. Q $(-2, \frac{3\pi}{4})$

r θ

$x = -2\cos\left(\frac{3\pi}{4}\right) \quad y = -2\sin\left(\frac{3\pi}{4}\right)$

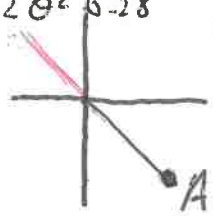
$x = -2\left(\frac{-\sqrt{2}}{2}\right) \quad y = -2\left(\frac{\sqrt{2}}{2}\right) = -\sqrt{2}$

$x = \sqrt{2} \quad Q(\sqrt{2}, -\sqrt{2}) \rightarrow (1.41, -1.41)$

Example: For each point given in rectangular coordinates, find four unique polar coordinates with $-2\pi \le \theta \le 2\pi$. $-6.28 < \theta < 6.28$

3. A (2, -5)

x y



Q4

$r = \sqrt{2^2+5^2} = \sqrt{29}$

$\theta = \tan^{-1}\left(\frac{-5}{2}\right) = -1.1902 + 2\pi$

$\theta = 5.093$

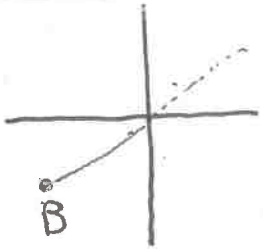
i) $(\sqrt{29}, 5.093)$

ii) $(\sqrt{29}, -1.190)$ -2π

iii) $-\sqrt{29}, 1.951)$ $-\pi$

iv) $(-\sqrt{29}, -4.331)$ -2π

4. B (-9, -4)



$r = \sqrt{9^2+4^2} = \sqrt{97}$

Q3

$\theta = \tan^{-1}\left(\frac{-4}{-9}\right) = 0.418 + \pi$

$\theta = 3.560$

i) $(\sqrt{97}, 3.560)$ $-\pi$

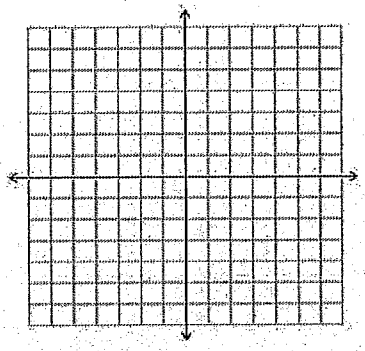
ii) $(\sqrt{97}, -2.723)$ -2π

iii) $(-\sqrt{97}, 0.418)$ -2π

iv) $(-\sqrt{97}, -5.865)$

Distance between 2 Points in the Polar Plane

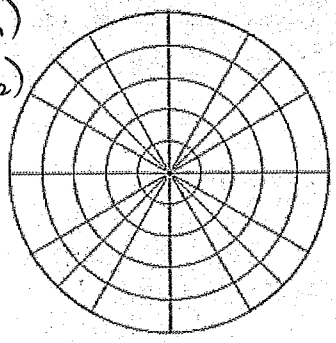
Review: How do we find the distance between two points in the Cartesian Plane?



$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

New method for distance using the Polar Plane:

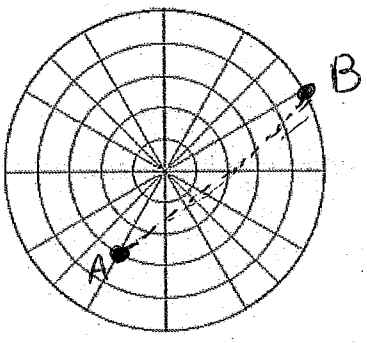
p1: (r_1, θ_1)
p2: (r_2, θ_2)



$$d = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos(\theta_2 - \theta_1)}$$

Example: Find the distance between the points $\left(-3, \frac{\pi}{3}\right)$ *and* $\left(5, -\frac{11\pi}{6}\right)$

A: r_1, θ_1 B: r_2, θ_2



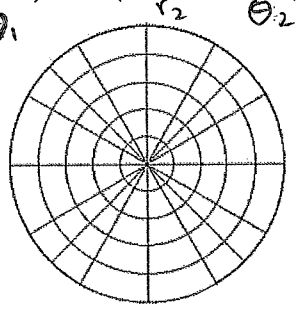
$$d = \sqrt{3^2 + 5^2 - \left[2(-3)(5)\cos\left(-\frac{11\pi}{6} - \frac{\pi}{3}\right)\right]}$$

$$d = \sqrt{34 + 30\left(\frac{\sqrt{3}}{2}\right)}$$

$$d = \sqrt{34 + 15\sqrt{3}} \approx \boxed{7.745 \text{ units}}$$

Example: A radar detects 2 plane s at the same altitude. Their polar coordinates are (5 miles, 310°) and (2 miles, 192°). How far apart are the planes?

r_1, θ_1 r_2 θ_2



$$d = \sqrt{5^2 + 2^2 - \left[2(5)(2)\cos(192-310)\right]}$$

$$\boxed{d = 6.196 \text{ mi}}$$

Classwork #1, 3, 5, 13, 15, ~~17~~, ~~23~~

7.02 Practice: Complete the odd problems. $(r\cos\theta, r\sin\theta)$

Find the rectangular coordinates for each point with the given polar coordinates. Round to the nearest thousandth if necessary. (Example 1)

1. $\left(2, \frac{\pi}{4}\right)$
2. $\left(\frac{1}{4}, \frac{\pi}{2}\right)$
3. $(5, 240°)$
4. $(2.5, 250°)$
5. $\left(-2, \frac{4\pi}{3}\right)$
6. $(-13, -70°)$
7. $\left(3, \frac{\pi}{2}\right)$
8. $\left(\frac{1}{2}, \frac{3\pi}{4}\right)$
9. $(-2, 270°)$
10. $(4, 210°)$
11. $\left(-1, -\frac{\pi}{6}\right)$
12. $\left(5, \frac{\pi}{3}\right)$

1) $(2\cos\pi/4, 2\sin\pi/4) \rightarrow \left(2\left(\frac{\sqrt{2}}{2}\right), 2\left(\frac{\sqrt{2}}{2}\right)\right)$

$(\sqrt{2}, \sqrt{2})$

3) $(5\cos 240, 5\sin 240) \rightarrow \left(\frac{-5}{2}, \frac{-5\sqrt{3}}{2}\right)$

5) $\left(-2\cos\left(\frac{4\pi}{3}\right), -2\sin\left(\frac{4\pi}{3}\right)\right)$

$\left(-2\left(-\frac{1}{2}\right), -2\left(-\frac{\sqrt{3}}{2}\right)\right)$

$(1, \sqrt{3})$

Find 4 pairs of polar coordinates for each point with the given rectangular coordinates for $[-2\pi, 2\pi]$. Round to the nearest thousandth if necessary. (Example 2)

13. $(7, 10)$
14. $(-13, 4)$
15. $(-6, -12)$
16. $(4, -12)$
17. $(2, -3)$
18. $(0, -173)$
19. $(a, 3a), a > 0$
20. $(-14, 14)$
21. $(52, -31)$
22. $(3b, -4b), b > 0$
23. $(1, -1)$
24. $(2, \sqrt{2})$

13) $r = \sqrt{7^2+10^2} = \sqrt{149}$

$\theta = \tan^{-1}\left(\frac{10}{7}\right) = 0.96$ (Q1)

Q1 i) $(\sqrt{149}, 0.960)$ $\xrightarrow{+\pi}$ Q3 iii) $(-\sqrt{149}, 4.102)$

Q1 ii) $(\sqrt{149}, -5.23)$ $\xrightarrow{-2\pi}$ Q3 iv) $(-\sqrt{149}, -2.182)$ $\xrightarrow{-2\pi}$

15) $\sqrt{6^2+12^2} = \sqrt{180}$ or $6\sqrt{5}$

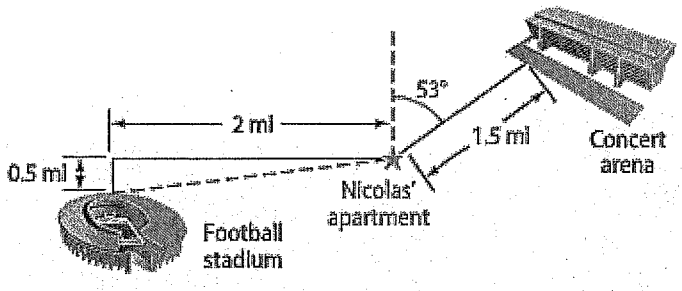
$\theta = \tan^{-1}\left(\frac{-12}{-6}\right) = 1.107 + \pi \rightarrow \theta = 4.249$

Q3 $\xrightarrow{-\pi}$ Q1

Q3 i) $(6\sqrt{5}, 4.249)$ $\xrightarrow{-2\pi}$ Q1 iii) $(-6\sqrt{5}, 1.107)$

Q3 ii) $(6\sqrt{5}, -2.034)$ $\xrightarrow{-2\pi}$ Q1 iv) $(-6\sqrt{5}, -5.176)$

25. DISTANCE Standing on top of his apartment building, Nicolas determines that a concert arena is 53° east of north. Suppose the arena is exactly 1.5 miles from Nicolas' apartment. (Example 3)



a. How many miles north and east will Nicolas have to travel to reach the arena?

b. If a football stadium is 2 miles west and 0.5 mile south of Nicolas' apartment, what are the polar coordinates of the stadium if Nicolas' apartment is at the pole?

7.02 Homework: Page 557, #1 – 25 odd

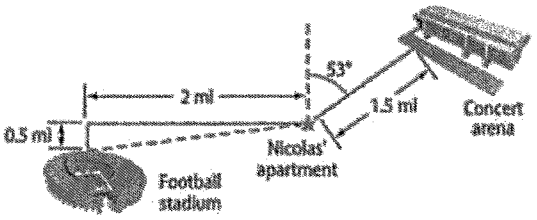
Find the rectangular coordinates for each point with the given polar coordinates. Round to the nearest thousandth if necessary. (Example 1)

1. $\left(2, \frac{\pi}{4}\right)$ $(\sqrt{2}, \sqrt{2})$
3. $(5, 240°)$ $\left(-\frac{5}{2}, -\frac{5\sqrt{3}}{2}\right)$
5. $\left(-2, \frac{4\pi}{3}\right)$ $(1, \sqrt{3})$
7. $\left(3, \frac{\pi}{2}\right)$ $(0, 3)$
9. $(-2, 270°)$ $(0, 2)$
11. $\left(-1, -\frac{\pi}{6}\right)$ $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

Find 4 pairs of polar coordinates for each point with the given rectangular coordinates for $[-2\pi, 2\pi]$. Round to the nearest thousandth if necessary. (Example 2)

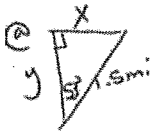
13. $(7, 10)$
15. $(-6, -12)$
17. $(2, -3)$
19. $(a, 3a), a > 0$
21. $(52, -31)$
23. $(1, -1)$

25. **DISTANCE** Standing on top of his apartment building, Nicolas determines that a concert arena is 53° east of north. Suppose the arena is exactly 1.5 miles from Nicolas' apartment. (Example 3)



a. How many miles north and east will Nicolas have to travel to reach the arena?

b. If a football stadium is 2 miles west and 0.5 mile south of Nicolas' apartment, what are the polar coordinates of the stadium if Nicolas' apartment is at the pole?

@ 

$\sin 53° = \frac{x}{1.5}$

$x = 1.5 \sin 53° = 1.198$ mi E

$\cos 53° = \frac{y}{1.5}$

$y = 1.5 \cos 53° = 0.903$ mi N

(b) $r = \sqrt{2^2 + 0.5^2}$

$r = \sqrt{4.25}$

$\theta = \tan^{-1}\left(\frac{-0.5}{-2}\right) + 180°$

$\theta = 194.036°$

$(\sqrt{4.25}, 194.036°)$

13. $(\sqrt{149}, 0.960)$
$(\sqrt{149}, -5.323)$
$(-\sqrt{149}, -2.182)$
$(-\sqrt{149}, 4.102)$

15. $(6\sqrt{5}, 4.249)$
$(6\sqrt{5}, -2.034)$
$(-6\sqrt{5}, 1.107)$
$(-6\sqrt{5}, -5.176)$

17. $(\sqrt{13}, 5.300)$
$(\sqrt{13}, -0.98)$
$(-\sqrt{13}, 2.159)$
$(-\sqrt{13}, -4.124)$

19. $r = \sqrt{a^2 + (3a)^2} = \sqrt{a^2 + 9a^2}$
$r = \sqrt{10a^2} = a\sqrt{10}$
$(a\sqrt{10}, 1.249)$
$(a\sqrt{10}, -5.034)$
$(-a\sqrt{10}, -1.893)$
$(-a\sqrt{10}, 4.391)$

21. $(\sqrt{3665}, 5.746)$
$(\sqrt{3665}, -0.538)$
$(-\sqrt{3665}, 2.604)$
$(-\sqrt{3665}, -3.679)$

23. $\left(\sqrt{2}, -\frac{\pi}{4}\right)$
$\left(\sqrt{2}, \frac{7\pi}{4}\right)$
$\left(-\sqrt{2}, \frac{3\pi}{4}\right)$
$\left(-\sqrt{2}, -\frac{5\pi}{4}\right)$