* Divide exponent by 4, find ~~Remainder~~ Remainder

R1 → 0.25 → i

R2 → 0.50 → i^2

R3 → 0.75 → i^3

No Remainder → i^4

7.04 Complex Numbers Review

Recall that the imaginary number i is defined such that $i^2 = -1$.

1. $i = \sqrt{-1}$
2. $i^2 = i \cdot i =$ -1
3. $i^3 = i^2 \cdot i =$ $-i$
4. $i^4 = i^2 \cdot i^2 =$ $+1$
5. $i^{11} =$ $-i$ ($11 \div 4 = 2$ R3)
6. $i^{18} =$ -1 ($18 \div 4 = 4$ R2)
7. $i^{67} =$ $-i$ ($67 \div 4 = 16$ R3)
8. $i^{724} =$ 1 ($724 \div 4 = 181$ R0)

A complex number has two parts: the real part and the imaginary part.

9. The standard form for complex number is $a + bi$.

Perform the given operation. Write your answer in standard form of a complex number.

10. $(-4 + 7i) + (2 - 3i) =$ __________

11. $(7 - 12i) - (4 + 9i) =$ $3 - 21i$

$7 - 12i - 4 - 9i$

12. $(5 + 8i) \cdot (2 - 10i) =$ __________

13. $(3 + 4i) \cdot (-8 + 2i) =$ $-32 - 26i$

$-24 + 6i - 32i + 8i^2$

$8(-1)$

-24 $\quad -8$

14. $(3 - 5i) \cdot (3 - 5i) =$ __________

15. $(3 - 5i) \cdot (3 + 5i) =$ $\boxed{34}$

$9 + 15i - 15i - 25i^2$ | $9 - 25(-1)$

$9 + 25$

#14: two factors that are the exact same multiplied together (just like a binomial squared). #15: two factors that only have the sign in the middle changed. They are conjugates.

Which product resulted in an entirely real value having no imaginary part? #15

Generalize it as a formula by simplifying: $(a + bi) \cdot (a - bi) =$ $\boxed{a^2 + b^2}$

$a^2 - \cancel{abi} + \cancel{abi} - b^2i^2$

$a^2 - b^2(-1)$

State the conjugate $(a - bi)$ of the given complex number $(a + bi)$.

16. $9 + 4i$ $9-4i$ 17. $5 - 2i$ $5+2i$ 18. $-3 + 7i$ $-3-7i$

Use the conjugate of the denominator to rationalize the following fractions.

19. $\frac{1+i}{5-2i} =$ $\frac{3}{29} + \frac{7}{29}i$

$$\frac{(1+i)}{(5-2i)} \cdot \frac{(5+2i)}{(5+2i)} = \frac{5+2i+5i+2i^2}{25+4}$$

$5 + 7i + 2(-1)$

$$\frac{3+7i}{29} \rightarrow \boxed{\frac{3}{29} + \frac{7}{29}i}$$

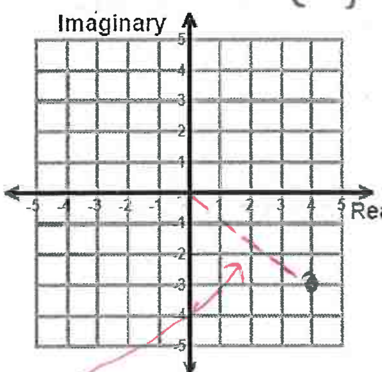
20. $\frac{5-6i}{-3+7i} =$ $-\frac{57}{58} - \frac{17}{58}i$

$$\frac{(5-6i)}{-3+7i} \frac{(-3-7i)}{-3-7i} \rightarrow \frac{-15-35i+18i+42i^2}{9+49}$$

$$\rightarrow \frac{-57-17i}{58} \rightarrow \boxed{\frac{-57}{58} - \frac{17}{58}i}$$

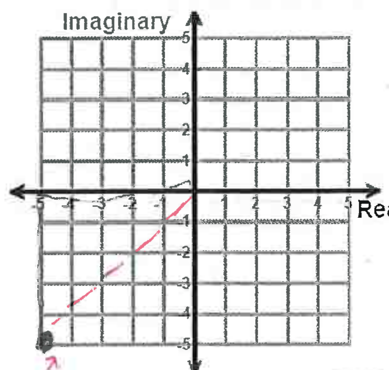
Graph the number on the complex plane and find its absolute value (distance from zero).

21. $4 - 3i$ $(4, -3)$



$|4 - 3i| =$ $\sqrt{3^2+4^2} = 5$

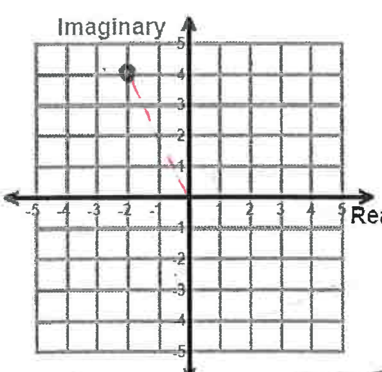
22. $-5 - 5i$



$\sqrt{(-5)^2 + (-5)^2}$

$|-5 - 5i| =$ $\sqrt{5^2+5^2} = \boxed{5\sqrt{2}}$

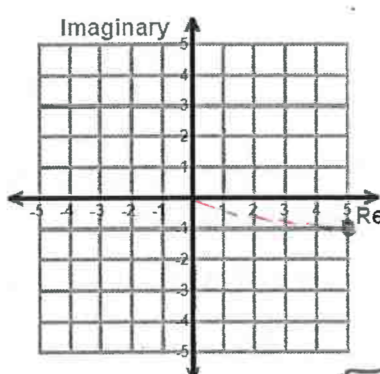
23. $-2 + 4i$



$|-2 + 4i| =$ $\sqrt{2^2+4^2}$

$\sqrt{20} = \boxed{2\sqrt{5}}$

24. $5 - i$

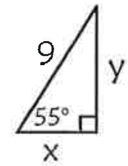


$|5 - i| =$ $\sqrt{5^2+1^2} = \boxed{\sqrt{26}}$

7.05 Complex Numbers in Rectangular Form **Date:** ____________________

Opener: where we have been this year?

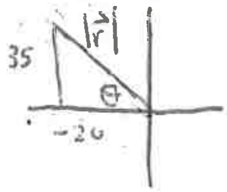
1. From <u>right triangle trigonometry</u>: In the triangle to the right, find x and y.



$\cos(55) = \frac{x}{9}$ | $\sin(55) = \frac{y}{9}$

$\boxed{x = 9\cos 55}$ $\boxed{y = 9\sin(55)}$

2. From <u>vectors</u>: For a bird flying 20m West and 35m North, find the resulting magnitude and direction (measured in standard position) of its flight.



$|\vec{r}| = \sqrt{20^2 + 35^2} = \sqrt{1625} = \boxed{5\sqrt{65}}$

$\theta = \tan^{-1}\left(\frac{35}{-20}\right) = -60.255^\circ + 180 = \boxed{119.745^\circ}$

Q2

3. From <u>polar coordinates</u>: convert (-2, -2) from rectangular form into polar form. Q3

$r = \sqrt{2^2 + 2^2} = \sqrt{8}$

$\boxed{r = 2\sqrt{2}}$

x y

$\theta = \tan^{-1}\left(\frac{-2}{-2}\right) = 45^\circ + 180$

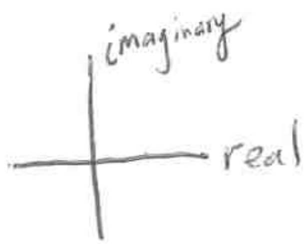
$\boxed{\theta = 225^\circ}$

Complex Numbers:

Rectangular Form, also known as *Standard Form*:

$a + bi$

Graphing a complex number:



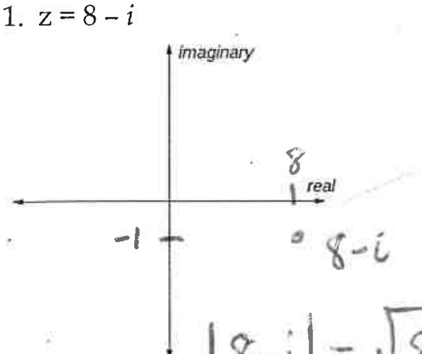
Absolute Value of a complex number, also known as the *modulus*:

$|a + bi| = \sqrt{a^2 + b^2}$

distance (use pythagorean theorem)

Examples: Graph each number in the complex plane and find its modulus.

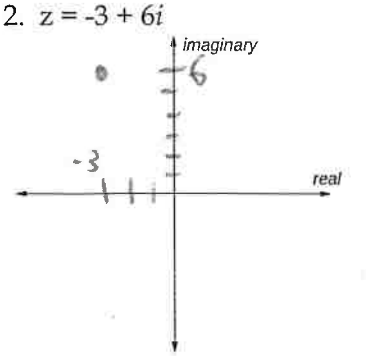
1. z = 8 – *i*



$|8 - i| = \sqrt{8^2 + 1^2}$

$= \boxed{\sqrt{65}}$

2. z = -3 + 6*i*



$|-3 + 6i| = \sqrt{3^2 + 6^2}$

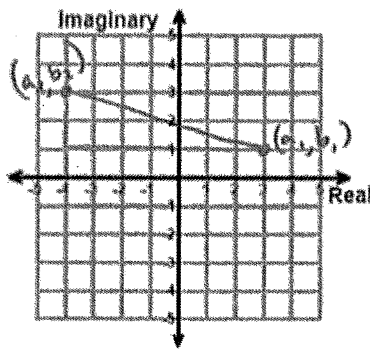
$= \sqrt{45}$

$= \boxed{3\sqrt{5}}$

Distance & Midpoint between Complex Numbers

Investigation: Find the distance between complex numbers $z_1 = 3 + i$ and $z_2 = -4 + 3i$.

First, a visual usually helps, so plot the complex numbers.



How would you find the distance between those two points?

$d = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}$

same

$z_1 - z_2 = a_1 - a_2 + b_1 - b_2$

$|z_1 - z_2| = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}$

Formula: The distance between two complex numbers is

$$d = |z_1 - z_2| \quad \text{or} \quad d = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}$$

Examples: Find the distance between the two complex numbers.

1) $z_1 = 5 - 3i$ and $z_2 = -1 - 8i$

$z_1 - z_2 = 5 - (-1) + (-3i) - (-8i)$

$6 + 5i$

$|z_1 - z_2| = \sqrt{6^2 + 5^2} = \sqrt{61}$

2) $z_1 = -8 + 4i$ and $z_2 = 1 + 7i$

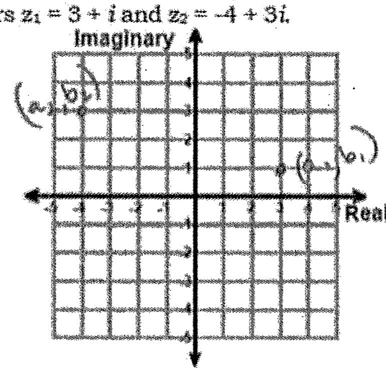
$z_1 - z_2 = -9 - 3i$

$|z_1 - z_2| = \sqrt{(-9)^2 + (-3)^2} = \sqrt{90}$

$3\sqrt{10}$

Investigation: Find the midpoint between complex numbers $z_1 = 3 + i$ and $z_2 = -4 + 3i$.

Again, plot the complex numbers so that you can "see" this.



How would you find the midpoint between the two points?

rectangular points: midpoint: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

complex #'s: midpoint: $\frac{a_1 + a_2}{2} + \frac{b_1 + b_2}{2} i$

$\frac{z_1 + z_2}{2} = \frac{a_1 + a_2 + (b_1 + b_2)i}{2}$

Formula: The midpoint between two complex numbers is

$$m = \frac{z_1 + z_2}{2} \qquad m = \frac{a_1 + a_2}{2} + \frac{b_1 + b_2}{2} i$$

Example: Find the midpoint between the two complex numbers

3) $z_1 = 5 - 3i$ and $z_2 = -1 - 8i$

$m = \frac{z_1 + z_2}{2} = \frac{5 + (-1) + (-3i) + (-8i)}{2}$

$\frac{4 - 11i}{2} = \boxed{2 - \frac{11}{2} i}$

4. $z_1 = -8 + 4i$ and $z_2 = 1 + 7i$

$\frac{z_1 + z_2}{2} = \frac{-7 + 11i}{2}$

$= \boxed{\frac{-7}{2} + \frac{11}{2} i}$