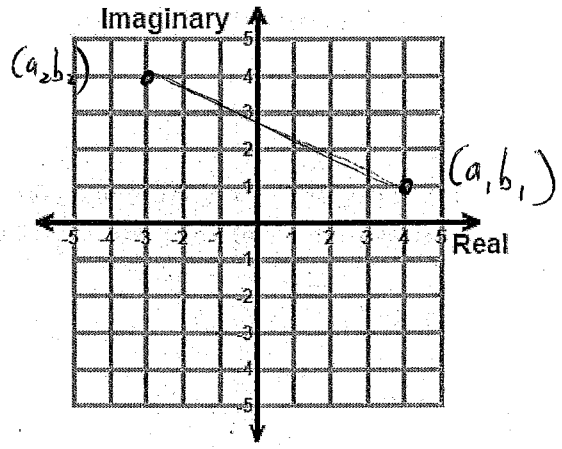
Distance & Midpoint between Complex Numbers

Investigation: Find the distance between complex numbers $z_1 = 3 + i$ and $z_2 = -4 + 3i$.

First, a visual usually helps, so plot the complex numbers.



How would you find the distance between those two points?

$$d = \sqrt{(a_2 - a_1)^2 + (b_2 - b_1)^2}$$

Formula: The distance between two complex numbers is

$$d = |z_1 - z_2| \quad \text{or} \quad d = \sqrt{(a_2 - a_1)^2 + (b_2 - b_1)^2}$$

Examples: Find the distance between the two complex numbers.

1) $z_1 = 5 - 3i$ and $z_2 = -1 - 8i$

$(5 - -1) + (-3 - -8)i$

$|6 + 5i| = \sqrt{6^2 + 5^2}$

$= \sqrt{36 + 25} = \sqrt{61}$

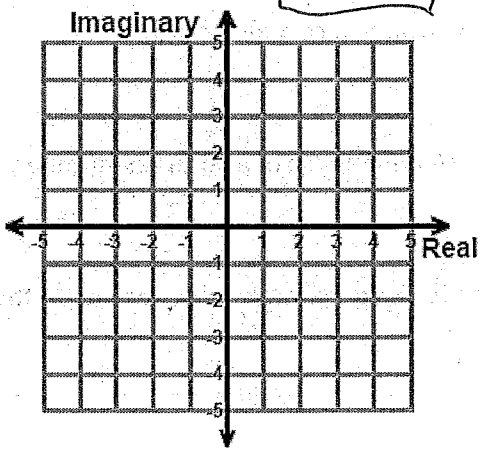
2) $z_1 = -8 + 4i$ and $z_2 = 1 + 7i$

$-8 - 1 + (4 - 7)i$

$|-9 - 3i| = \sqrt{9^2 + 3^2} = \sqrt{90}$ or $3\sqrt{10}$

Investigation: Find the midpoint between complex numbers $z_1 = 3 + i$ and $z_2 = -4 + 3i$.

Again, plot the complex numbers so that you can "see" this.



How would you find the midpoint between the two points?

Midpoint Formula: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Formula: The midpoint between two complex numbers is

$$M = \frac{z_1 + z_2}{2}$$

Example: Find the midpoint between the two complex numbers

3) $z_1 = 5 - 3i$ and $z_2 = -1 - 8i$

$\frac{5-1}{2} + \frac{-3-8}{2}i \rightarrow 2 - \frac{11}{2}i$

4. $z_1 = -8 + 4i$ and $z_2 = 1 + 7i$

$\frac{-8+1}{2} + \frac{4+7}{2}i = -\frac{7}{2} + \frac{11}{2}i$

Classwork p. 15
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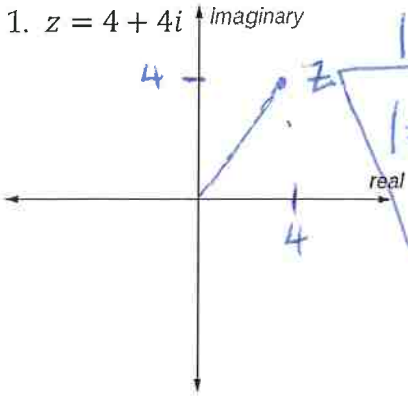
7.05 Homework: Rectangular Form of Complex Numbers

Plot each complex number and find its modulus.

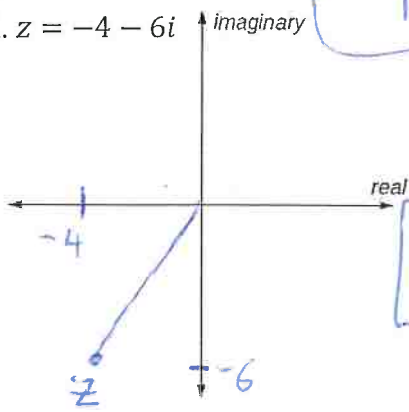
1. $z = 4 + 4i$



$|z| = \sqrt{4^2+4^2}$

$|z| = \sqrt{32}$ or $4\sqrt{2}$

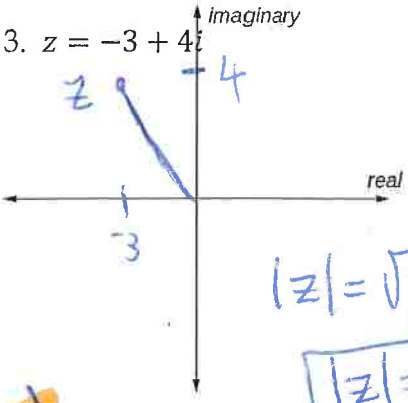
2. $z = -4 - 6i$



$|z| = \sqrt{4^2+6^2}$

$|z| = \sqrt{52} = 2\sqrt{13}$

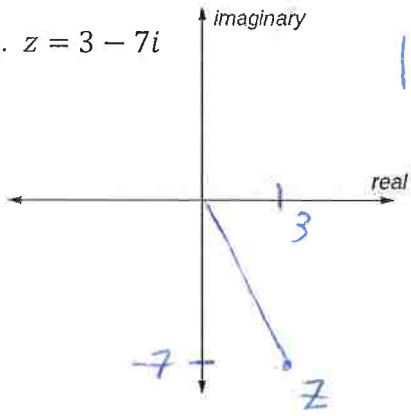
3. $z = -3 + 4i$



$|z| = \sqrt{3^2+4^2}$

$|z| = 5$

4. $z = 3 - 7i$



$|z| = \sqrt{3^2+7^2}$

$|z| = \sqrt{58}$

$|z_1 - z_2|$

Find the distance between the points in the complex plane.

5. $1 + 2i, -1 + 4i$

$1-(-1)+(2-4)i$

$|2 - 2i| = \sqrt{2^2+2^2} = \sqrt{8} = 2\sqrt{2}$

6. $-5 + i, -2 + 5i$

$-5-(-2)+(1-5)i$

$|-3 - 4i| = \sqrt{3^2+4^2} = 5$

7. $6i, 3 - 4i$

$0+6i, 3-4i$

$0-3+(6--4)i$

$|-3+10i|$

$\sqrt{3^2+10^2} = \sqrt{109}$

8. $-7 - 3i, 3 + 5i$

$-7-3+(-3-5)i$

$|-10 - 8i| = \sqrt{10^2+8^2} = \sqrt{164} = 2\sqrt{41}$

$\left|\frac{z_1+z_2}{2}\right|$

Find the midpoint of the segment connecting the points in the complex plane.

9. $2 + i, 6 + 5i$

$\frac{2+6}{2} + \frac{1+5}{2}i$

$\frac{8}{2} + \frac{6}{2}i$

$4 + 3i$

10. $-3 + 4i, 1 - 2i$

$\frac{-3+1}{2} + \frac{4-2}{2}i$

$\frac{-2}{2} + \frac{2}{2}i$

$-1 + 1i$

11. $7i, 9 - 10i$

$0+7i, 9-10i$

$\frac{0+9}{2} + \frac{7-10}{2}i$

$\frac{9}{2} - \frac{3}{2}i$

12. $-1 + \frac{1}{2}i, \frac{1}{2} + \frac{1}{4}i$

$\frac{-1+\frac{1}{2}}{2} + \frac{\frac{1}{2}+\frac{1}{4}}{2}i$

$-\frac{1}{4} + \frac{3}{8}i$

7.06 Adding & Subtracting Complex Numbers Geometrically **Date:** _____

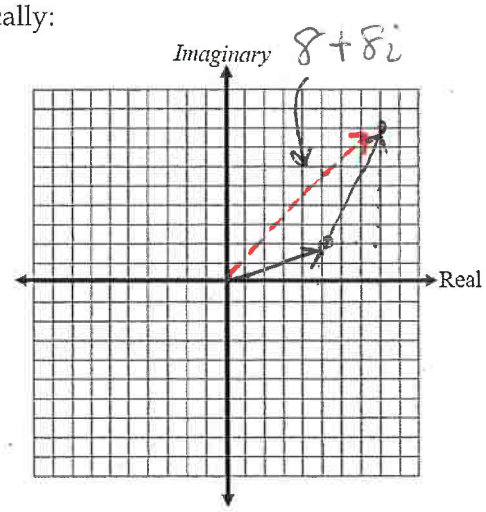
Recall that complex numbers take the form a + b*i*.
When adding or subtracting complex numbers algebraically, real parts are added together or subtracted then imaginary parts are added together or subtracted - similar to combining like terms.

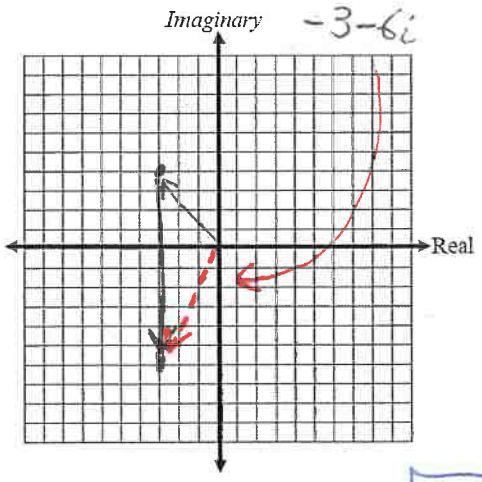
Complex numbers can also be added or subtracted geometrically/graphically by plotting the points in the complex plane and creating vectors with them. Then using geometric vector addition or subtraction.

Examples: 1. $(5 + 2i) + (3 + 6i) =$ 2. $(-3 + 4i) + (-10i) =$ $-3-6i$

Algebraically: $8+8i$

Geometrically:





3. $(5 - i) - (6 - 5i) =$

Algebraically: $5-6 \quad -1i+5i$

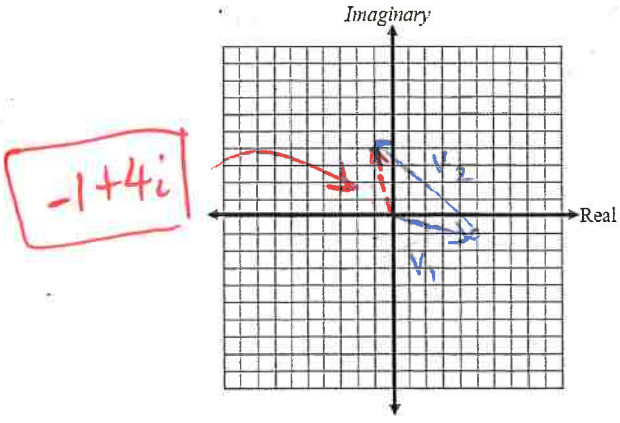
$-1+4i$

Geometrically: $(5-i)+(-6+5i)$

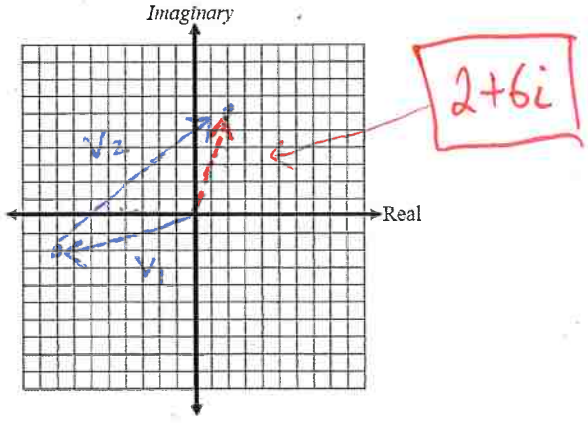
4. $(-8 - 2i) - (-10 - 8i) =$

$-8-2i+10+8i$ → $2+6i$

$-8-2i+(10+8i)$



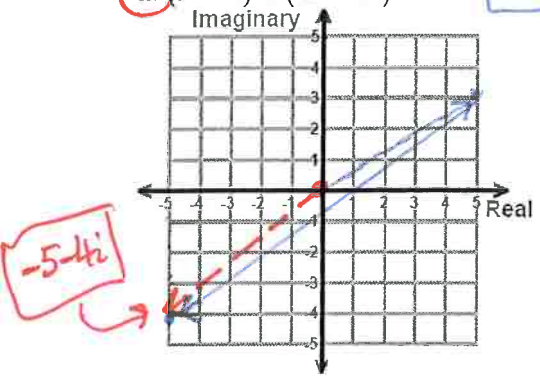
$-1+4i$



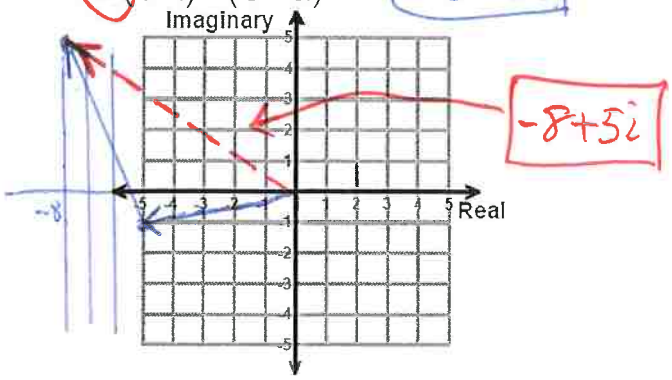
$2+6i$

7.06 Practice: Evaluate each sum or difference geometrically, then verify your answer using algebra.

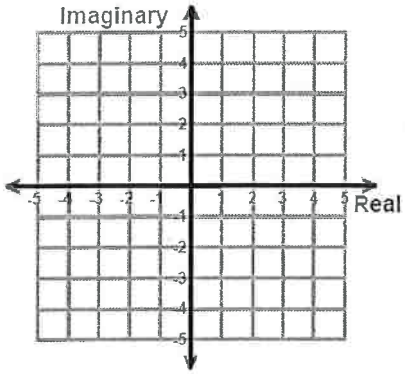
1. $(5 + 3i) + (-10 - 7i)$ $-5-4i$



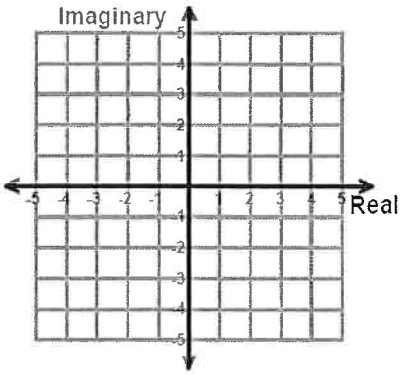
2. $(-5 - i) + (-3 + 6i)$ $-8+5i$



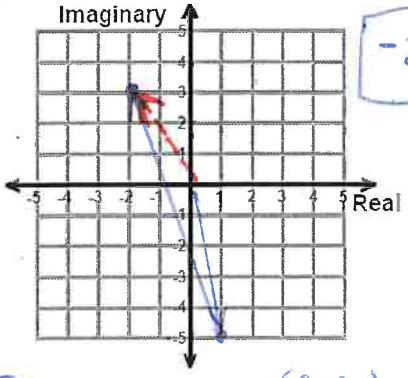
3. $(4 - 2i) + (-6 - 2i)$



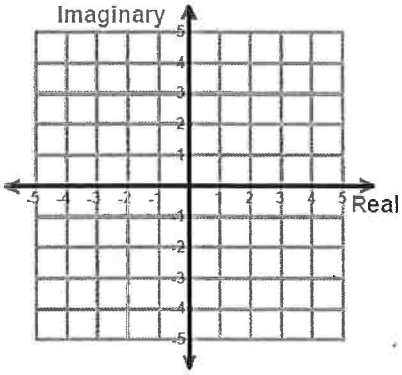
4. $-4i + (3 - i)$



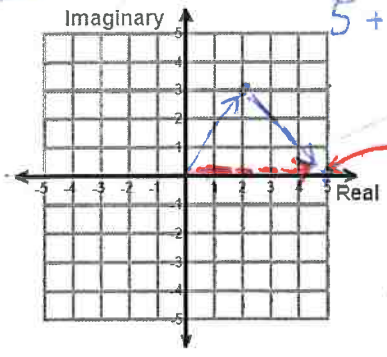
5. $(1 - 5i) - (3 - 8i)$ $(1-5i)+(-3+8i)$



6. $4i - (4 + i)$



7. $(2 + 3i) - (-3 + 3i)$ $(2+3i)+(3-3i)$ $5+0i=5$



8. $(-5 - 5i) - (-4 - 2i)$