**7.07 Complex Numbers in Polar Form** formal **Date:** _____

*Polar Form of Complex Numbers*, also know as *Trigonometric Form*:

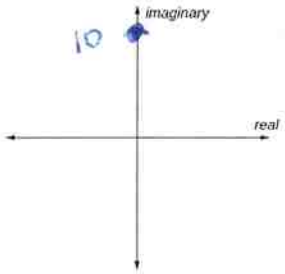
$z = r(\cos\theta + i\sin\theta)$ → or $r\,\text{cis}\,\theta$ (shorthand)

$a = r\cos\theta$
$b = r\sin\theta$
$a + bi$

$r = \sqrt{a^2 + b^2}$
$\tan\theta = \frac{b}{a}$
$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

Examples: Graph each complex number on the rectangular plane. Then, find its polar form, where $0 \le \theta \le 2\pi$. Be exact.

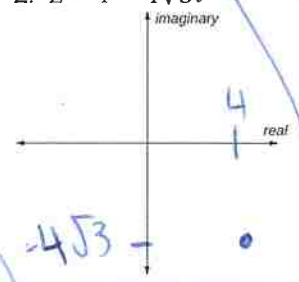
1. $z = 10i$ $z = 0 + 10i$




$r = 10$
$\theta = \pi/2$

$z = 10\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) \rightarrow 10\,\text{cis}\,\frac{\pi}{2}$

2. $z = 4 - 4\sqrt{3}i$




$r = \sqrt{4^2 + (4\sqrt{3})^2} = 8$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$\theta = \tan^{-1}\left(\frac{-4\sqrt{3}}{4}\right) = -60 + 360$

$\theta = 300° \cdot \frac{\pi}{180}$ $\theta = \frac{5\pi}{3}$

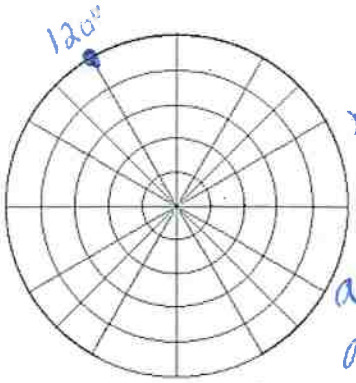
$z = 8\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

$z = 8\,\text{cis}\,\frac{5\pi}{3}$

Examples: Graph each complex number on the polar plane. Then, find its rectangular form. Be exact.

→ $a + bi$

3. $5(\cos 120° + i\sin 120°)$




$r = 5$
$\theta = 120$

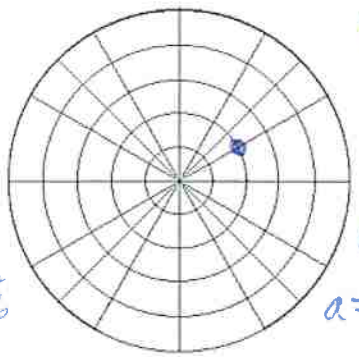
$a = r\cos\theta$
$b = r\sin\theta$

$a = 5\cos 120$
$a = 5\left(-\frac{1}{2}\right)$

$b = 5\sin 120$
$b = 5\left(\frac{\sqrt{3}}{2}\right)$

$-\frac{5}{2} + \frac{5\sqrt{3}}{2}i$

4. $-2\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$




$r = -2$
$\theta = \frac{7\pi}{6}$

$a = r\cos\theta$
$a = -2\cos\frac{7\pi}{6}$
$a = -2\left(-\frac{\sqrt{3}}{2}\right)$

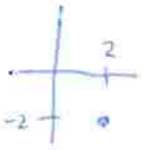
$b = r\sin\theta$
$b = -2\sin\left(\frac{7\pi}{6}\right)$
$b = -2\left(-\frac{1}{2}\right)$

$\sqrt{3} + 1i$

**7.07 Homework: Directions: Be exact. Work these problems without using a calculator!**

Write the polar form of each complex number where $0 \le \theta \le 2\pi$.

1. $z = 2 - 2i$



$r = \sqrt{2^2+2^2} = \sqrt{8} = 2\sqrt{2}$ (Q4)

$\theta = \tan^{-1}\left(\frac{-2}{2}\right) = -45^\circ + 360 = 315^\circ$

$\theta = \frac{7\pi}{4}$

$z = 2\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

$z = 2\sqrt{2}\,\text{cis}\,\frac{7\pi}{4}$

2. $z = 3 + 3i$

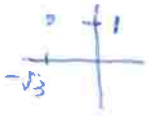
$r = \sqrt{3^2+3^2} = \sqrt{18} = 3\sqrt{2}$

$\theta = \tan^{-1}\left(\frac{3}{3}\right) = 45^\circ = \frac{\pi}{4}$ (Q1)

$z = 3\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$z = 3\sqrt{2}\,\text{cis}\,\frac{\pi}{4}$

3. $z = -\sqrt{3} + i$



Q2

$r = \sqrt{\sqrt{3}^2+1}$

$r = 2$

$\theta = \tan^{-1}\left(\frac{1}{-\sqrt{3}}\right) = -30 + 180$

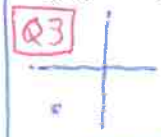
$= 150^\circ$

$\theta = \frac{5\pi}{6}$

$z = 2\,\text{cis}\left(\frac{5\pi}{6}\right)$

4. $z = -5 - 5\sqrt{3}i$

Q3



$r = \sqrt{5^2+(5\sqrt{3})^2}$

$r = 10$

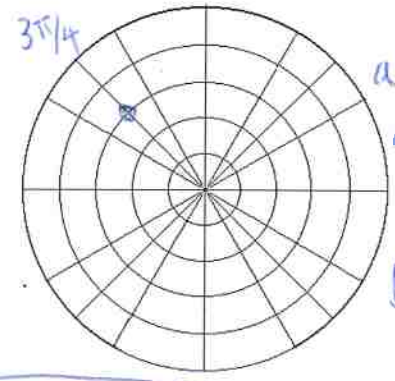
$\theta = \tan^{-1}\left(\frac{-5\sqrt{3}}{-5}\right) = 60 + 180$

$\theta = 240^\circ = \frac{4\pi}{3}$

$z = 10\,\text{cis}\,\frac{4\pi}{3}$

Graph each number on a polar grid. Then express it in rectangular form.

5. $z = 3\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$



$3\pi/4$

$r = 3$

$\theta = \frac{3\pi}{4}$

$a = r\cos\theta$

$a = 3\cos\left(\frac{3\pi}{4}\right)$

$a = 3\left(\frac{-\sqrt{2}}{2}\right)$

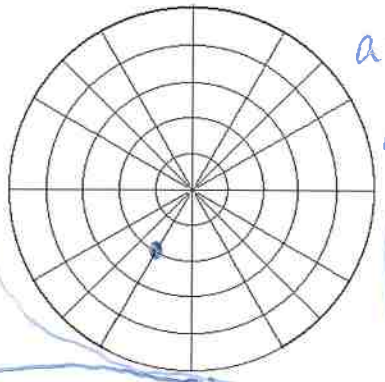
$b = r\sin\theta$

$b = 3\sin\left(\frac{3\pi}{4}\right)$

$b = 3\left(\frac{\sqrt{2}}{2}\right)$

$-\frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i$

6. $z = 2\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$



$r = 2$

$\theta = \frac{4\pi}{3}$

$a = 2\cos\left(\frac{4\pi}{3}\right)$

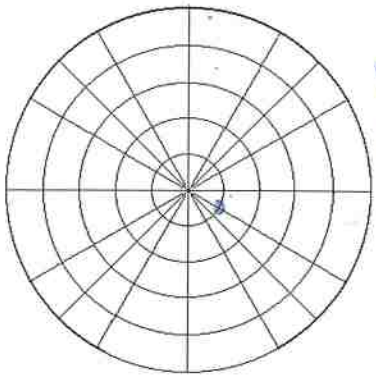
$a = 2\left(\frac{-1}{2}\right)$

$b = 2\sin\left(\frac{4\pi}{3}\right)$

$b = 2\left(\frac{-\sqrt{3}}{2}\right)$

$-1 - \sqrt{3}i$

7. $z = \left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$



$r = 1$

$\theta = 11\pi/6$

$a = 1\cos\left(\frac{11\pi}{6}\right)$

$a = 1\left(\frac{\sqrt{3}}{2}\right)$

$b = 1\sin\left(\frac{11\pi}{6}\right)$

$b = 1\left(\frac{-1}{2}\right)$

$\rightarrow \frac{\sqrt{3}}{2} - \frac{1}{2}i$