

7. $(-1 + \sqrt{3}i)^5$

Convert each complex number into polar form. Then find all distinct roots of the complex number. Answer in polar form where $0 \leq \theta \leq 2\pi$.

8. Sixth roots of i

$0 + 1i$ \uparrow

$r = 1$ $z = 1 \left[\cos\left(\frac{\pi}{2}\right) + i \sin\left(\frac{\pi}{2}\right) \right]$

$\theta = \frac{\pi}{2}$ $z^{1/6} = \sqrt[6]{1} \left[\cos\left(\frac{1}{6} \cdot \frac{\pi}{2}\right) + i \sin\left(\frac{1}{6} \cdot \frac{\pi}{2}\right) \right]$

i) $z^{1/6} = \sqrt[6]{1} \left[\cos\left(\frac{\pi}{12}\right) + i \sin\left(\frac{\pi}{12}\right) \right]$

ii) $\sqrt[6]{1} \left[\cos\left(\frac{5\pi}{12}\right) + i \sin\left(\frac{5\pi}{12}\right) \right]$

iii) $\sqrt[6]{1} \left[\cos\left(\frac{3\pi}{4}\right) + i \sin\left(\frac{3\pi}{4}\right) \right]$

iv) $\sqrt[6]{1} \left[\cos\left(\frac{13\pi}{12}\right) + i \sin\left(\frac{13\pi}{12}\right) \right]$

v) $\sqrt[6]{1} \left[\cos\left(\frac{17\pi}{12}\right) + i \sin\left(\frac{17\pi}{12}\right) \right]$

vi) $\sqrt[6]{1} \left[\cos\left(\frac{7\pi}{4}\right) + i \sin\left(\frac{7\pi}{4}\right) \right]$

$\frac{2\pi}{6} = \frac{\pi}{3}$

* keep adding $\frac{\pi}{3}$ to get to next solution!

9. Fourth roots of $4\sqrt{3} - 4i$

$r = \sqrt{(4\sqrt{3})^2 + (-4)^2} = \sqrt{64} = 8$

$\theta = \tan^{-1}\left(\frac{-4}{4\sqrt{3}}\right) = -30^\circ + 360^\circ$

$\theta = 330 \cdot \frac{\pi}{180}$

$\theta = \frac{11\pi}{6}$

$z = 8 \left[\cos\left(\frac{11\pi}{6}\right) + i \sin\left(\frac{11\pi}{6}\right) \right]$

$z^{1/4} = \sqrt[4]{8} \left[\cos\left(\frac{1}{4} \cdot \frac{11\pi}{6}\right) + i \sin\left(\frac{1}{4} \cdot \frac{11\pi}{6}\right) \right]$

i) $\sqrt[4]{8} \left[\cos\left(\frac{11\pi}{24}\right) + i \sin\left(\frac{11\pi}{24}\right) \right]$

ii) $\sqrt[4]{8} \left[\cos\left(\frac{23\pi}{24}\right) + i \sin\left(\frac{23\pi}{24}\right) \right]$

iii) $\sqrt[4]{8} \left[\cos\left(\frac{35\pi}{24}\right) + i \sin\left(\frac{35\pi}{24}\right) \right]$

iv) $\sqrt[4]{8} \left[\cos\left(\frac{47\pi}{24}\right) + i \sin\left(\frac{47\pi}{24}\right) \right]$

$\frac{2\pi}{4} = \frac{\pi}{2}$

10. Fifth roots of unity (1)

$1 + 0i$ \rightarrow

$r = \sqrt{1^2 + 0^2} = 1$

$\theta = 0$

$z = 1 \left[\cos 0 + i \sin 0 \right]$

$z^{1/5} = \sqrt[5]{1} \left[\cos\left(\frac{0}{5}\right) + i \sin\left(\frac{0}{5}\right) \right]$

i) $\sqrt[5]{1} \left[\cos 0 + i \sin 0 \right]$

ii) $\sqrt[5]{1} \left[\cos\left(\frac{2\pi}{5}\right) + i \sin\left(\frac{2\pi}{5}\right) \right]$

iii) $\sqrt[5]{1} \text{ cis } \left(\frac{4\pi}{5}\right)$

iv) $\sqrt[5]{1} \text{ cis } \left(\frac{6\pi}{5}\right)$

v) $\sqrt[5]{1} \text{ cis } \left(\frac{8\pi}{5}\right)$

$\frac{2\pi}{5}$

$\downarrow + \frac{2\pi}{5}$

7.10 More Practice with Operations of Complex Numbers

Find the product $z_1 \cdot z_2$ and the quotient $\frac{z_1}{z_2}$. Express answers in both polar and rectangular form.

Match angle measurement units to the problem, where $0^\circ \leq \theta \leq 360^\circ$ or $0 \leq \theta \leq 2\pi$.

product
 $z_1 \cdot z_2$

1. Let $z_1 = 7\left(\cos\frac{9\pi}{8} + i\sin\frac{9\pi}{8}\right)$ and $z_2 = 2\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)$

$z_1 \cdot z_2 = 7 \cdot 2 \left[\cos\left(\frac{9\pi}{8} + \frac{\pi}{8}\right) + i\sin\left(\frac{9\pi}{8} + \frac{\pi}{8}\right) \right]$

$= 14 \left[\cos\left(\frac{5\pi}{4}\right) + i\sin\left(\frac{5\pi}{4}\right) \right]$ polar form

$14\cos\left(\frac{5\pi}{4}\right) + 14i\sin\left(\frac{5\pi}{4}\right)$

$14\left(\frac{-\sqrt{2}}{2}\right) + 14\left(\frac{-\sqrt{2}}{2}\right)i$

$-7\sqrt{2} - 7\sqrt{2}i$ Rectangular Form

quotient
 $\frac{z_1}{z_2}$

2. Let $z_1 = 4(\cos 200^\circ + i\sin 200^\circ)$ and $z_2 = 25(\cos 150^\circ + i\sin 150^\circ)$

$\frac{z_1}{z_2} = \frac{4}{25} \left[\cos(200 - 150) + i\sin(200 - 150) \right]$

$\frac{4}{25} [\cos 50 + i\sin 50]$ Polar Form

$0.103 + 0.123i$ Rectangular Form

Convert each complex number into polar form. Then find each power. Answer in polar form where $0 \leq \theta \leq 2\pi$.

3. $(1 - \sqrt{3}i)^4$ $\theta = 300 \cdot \frac{\pi}{180} = \frac{5\pi}{3}$

$z = 2 \left[\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3} \right]$

$z^4 = 2^4 \left[\cos\left(4 \cdot \frac{5\pi}{3}\right) + i\sin\left(4 \cdot \frac{5\pi}{3}\right) \right]$

$z^4 = 16 \left[\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right) \right]$

4. $(-\sqrt{2} + \sqrt{2}i)^5$

$r = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2}$

$r = \sqrt{4} = 2$

$\theta = \tan^{-1}\left(\frac{\sqrt{2}}{-\sqrt{2}}\right)$

$\theta = -45 + 180 = 135 = \frac{3\pi}{4}$

$z = 2 \left[\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right) \right]$

$z^5 = 2^5 \left[\cos\left(5 \cdot \frac{3\pi}{4}\right) + i\sin\left(5 \cdot \frac{3\pi}{4}\right) \right]$

$z^5 = 32 \left[\cos\frac{15\pi}{4} + i\sin\frac{15\pi}{4} \right]$

$32 \text{ cis } \frac{7\pi}{4}$

Convert each complex number into polar form. Then find all distinct roots of the complex number. Answer in polar form where $0 \leq \theta \leq 2\pi$.

5. Fifth roots of 32

$32 + 0i$

② $2 \text{ cis } \left(\frac{2\pi}{5}\right)$

③ $2 \text{ cis } \left(\frac{4\pi}{5}\right)$

④ $2 \text{ cis } \left(\frac{6\pi}{5}\right)$

⑤ $2 \text{ cis } \left(\frac{8\pi}{5}\right)$

6. Fourth roots of $-81i$

$r = \sqrt{8^2 + 0^2} = 81$

$\theta = 270$ or $\frac{3\pi}{2}$

$z = 81 \left[\cos\left(\frac{3\pi}{2}\right) + i\sin\left(\frac{3\pi}{2}\right) \right]$

$z^{1/4} = \sqrt[4]{81} \left[\cos\left(\frac{1}{4} \cdot \frac{3\pi}{2}\right) + i\sin\left(\frac{1}{4} \cdot \frac{3\pi}{2}\right) \right]$

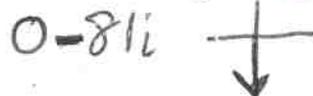
① $= 3 \left[\cos\frac{3\pi}{8} + i\sin\frac{3\pi}{8} \right]$

② $3 \left[\cos\frac{7\pi}{8} + i\sin\frac{7\pi}{8} \right]$

③ $3 \text{ cis } \frac{11\pi}{8}$

④ $3 \text{ cis } \frac{15\pi}{8}$

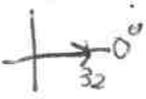
Add $\frac{2\pi}{n} \rightarrow \frac{2\pi}{4} \rightarrow \frac{\pi}{2}$



$r = \sqrt{1^2 + \sqrt{3}^2}$

$r = 2$

$\theta = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = -60 + 360$



$r = \sqrt{32^2 + 0^2} = 32$

$\theta = 0$

$z = 32 \left[\cos 0 + i\sin 0 \right]$

$z^{1/5} = 32^{1/5} \left[\cos\left(\frac{0}{5}\right) + i\sin\left(\frac{0}{5}\right) \right]$

$z^{1/5} = 2 \left[\cos 0 + i\sin 0 \right]$

$\frac{2\pi}{5}$