7. $(-1+\sqrt{3}i)^5$

Convert each complex number into polar form. Then find all distinct roots of the complex number. Answer in polar form where $0 \le \theta \le 2\pi$.

$\boxed{\frac{2\pi}{6} = \frac{\pi}{3}}$ * keep adding $\frac{\pi}{3}$ to get to next solution!

8. Sixth roots of i $0+1i$

$r = 1$

$\theta = \frac{\pi}{2}$

$z = 1\left[\cos\left(\frac{\pi}{2}\right) + i\sin\left(\pi/2\right)\right]$

$z^{1/6} = \sqrt[6]{1}\left[\cos\left(\frac{1}{6}\cdot\frac{\pi}{2}\right) + i\sin\left(\frac{1}{6}\cdot\frac{\pi}{2}\right)\right]$

i) $z^{1/6} = \sqrt[6]{1}\left[\cos\left(\frac{\pi}{12}\right) + i\sin\left(\frac{\pi}{12}\right)\right]$

ii) $\sqrt[6]{1}\left[\cos\left(\frac{5\pi}{12}\right) + i\sin\left(\frac{5\pi}{12}\right)\right]$

iii) $\sqrt[6]{1}\left[\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right]$

iv) $\sqrt[6]{1}\left[\cos\left(\frac{13\pi}{12}\right) + i\sin\left(\frac{13\pi}{12}\right)\right]$

v) $\sqrt[6]{1}\left[\cos\left(\frac{17\pi}{12}\right) + i\sin\left(\frac{17\pi}{12}\right)\right]$

vi) $\sqrt[6]{1}\left[\cos\left(\frac{7\pi}{4}\right) + i\sin\left(\frac{7\pi}{4}\right)\right]$

9. Fourth roots of $4\sqrt{3} - 4i$

$r = \sqrt{(4\sqrt{3})^2 + (4)^2} = \sqrt{64} = 8$

$\theta = \tan^{-1}\left(\frac{-4}{4\sqrt{3}}\right) = -30$ +360

$\theta = 330 \cdot \frac{\pi}{180}$

$\theta = \frac{11\pi}{6}$

$z = 8\left[\cos\left(\frac{11\pi}{6}\right) + i\sin\left(\frac{11\pi}{6}\right)\right]$

$z^{1/4} = \sqrt[4]{8}\left[\cos\left(\frac{1}{4}\cdot\frac{11\pi}{6}\right) + i\sin\left(\frac{1}{4}\cdot\frac{11\pi}{6}\right)\right]$

i) $\sqrt[4]{8}\left[\cos\left(\frac{11\pi}{24}\right) + i\sin\left(\frac{11\pi}{24}\right)\right]$

$\frac{2\pi}{4} = \boxed{\frac{\pi}{2}}$

ii) $\sqrt[4]{8}\left[\cos\frac{23\pi}{24} + i\sin\left(\frac{23\pi}{24}\right)\right]$

iii) $\sqrt[4]{8}\left[\cos\frac{35\pi}{24} + i\sin\left(\frac{35\pi}{24}\right)\right]$

iv) $\sqrt[4]{8}\left[\cos\left(\frac{47\pi}{24}\right) + i\sin\left(\frac{47\pi}{24}\right)\right]$

$n=5$

10. Fifth roots of unity (1) $1+0i$

$r = \sqrt{1^2+0^2} = 1$

$\theta = 0$

$z = 1\left[\cos 0 + i\sin 0\right]$

$z^{1/5} = \sqrt[5]{1}\left[\cos\left(\frac{0}{5}\right) + i\sin\left(\frac{0}{5}\right)\right]$

$\frac{2\pi}{5}$

i) $\sqrt[5]{1}\left[\cos 0 + i\sin 0\right]$

ii) $\sqrt[5]{1}\left[\cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}\right]$ $+\frac{2\pi}{5}$

iii) $\sqrt[5]{1}\ \mathrm{cis}\left(\frac{4\pi}{5}\right)$

iv) $\sqrt[5]{1}\ \mathrm{cis}\left(\frac{6\pi}{5}\right)$

v) $\sqrt[5]{1}\ \mathrm{cis}\left(\frac{8\pi}{5}\right)$

7.10 More Practice with Operations of Complex Numbers

Find the product $z_1 \cdot z_2$ and the quotient $\frac{z_1}{z_2}$. Express answers in both polar and rectangular form. Match angle measurement units to the problem, where $0° \leq \theta \leq 360°$ or $0 \leq \theta \leq 2\pi$.

product $z_1 \cdot z_2$

1. Let $z_1 = 7\left(\cos\frac{9\pi}{8} + i\sin\frac{9\pi}{8}\right)$ and $z_2 = 2\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)$

$z_1 \cdot z_2 = 7 \cdot 2\left[\cos\left(\frac{9\pi}{8}+\frac{\pi}{8}\right) + i\sin\left(\frac{9\pi}{8}+\frac{\pi}{8}\right)\right]$

$= \boxed{14\left[\cos\left(\frac{5\pi}{4}\right) + i\sin\left(\frac{5\pi}{4}\right)\right]}$ polar form

$14\cos\left(\frac{5\pi}{4}\right) + 14\sin\left(\frac{5\pi}{4}\right)i$

$14\left(-\frac{\sqrt{2}}{2}\right) + 14\left(-\frac{\sqrt{2}}{2}\right)i$

$\boxed{-7\sqrt{2} - 7\sqrt{2}i}$ Rectangular Form

quotient $\frac{z_1}{z_2}$

2. Let $z_1 = 4(\cos 200° + i\sin 200°)$ and $z_2 = 25(\cos 150° + i\sin 150°)$

$\frac{z_1}{z_2} = \frac{4}{25}\left[\cos(200-150) + i\sin(200-150)\right]$

$\boxed{\frac{4}{25}\left[\cos 50 + i\sin 50\right]}$ Polar Form

$\boxed{0.103 + 0.123i}$ Rectangular Form

Convert each complex number into polar form. Then find each power. Answer in polar form where $0 \leq \theta \leq 2\pi$.

3. $(1-\sqrt{3}i)^4$

$r = \sqrt{1^2 + \sqrt{3}^2}$

$r = 2$ Q4

$\theta = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = -60 + 360$

$\theta = 300 \cdot \frac{\pi}{180} = \frac{5\pi}{3}$

$z = 2\left[\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right]$

$z^4 = 2^4\left[\cos\left(4\cdot\frac{5\pi}{3}\right) + i\sin\left(4\cdot\frac{5\pi}{3}\right)\right]$

$z^4 = 16\left[\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right]$

4. $(-\sqrt{2} + \sqrt{2}i)^5$

$r = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2}$

$r = \sqrt{4} = 2$

$\theta = \tan^{-1}\left(\frac{\sqrt{2}}{-\sqrt{2}}\right)$

$\theta = -45 + 180 = 135 = \frac{3\pi}{4}$

$z = 2\left[\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right]$

$z^5 = 2^5\left[\cos\left(5\cdot\frac{3\pi}{4}\right) + i\sin\left(5\cdot 3\pi/4\right)\right]$

$z^5 = 32\left[\cos\frac{15\pi}{4} + i\sin\frac{15\pi}{4}\right]$

$\boxed{32 \text{ cis } \frac{7\pi}{4}}$

Convert each complex number into polar form. Then find all distinct roots of the complex number. Answer in polar form where $0 \leq \theta \leq 2\pi$.

5. Fifth roots of 32

0°, 32

$32 + 0i$

$r = \sqrt{32^2 + 0^2} = 32$

$\theta = 0$

$z = 32[\cos 0 + i\sin 0]$

$z^{1/5} = 32^{1/5}\left[\cos\left(\frac{0}{5}\right) + i\sin\left(\frac{0}{5}\right)\right]$

1 $z^{1/5} = 2[\cos 0 + i\sin 0]$

* $\frac{2\pi}{5}$

2 $2\text{cis}\left(\frac{2\pi}{5}\right)$

3 $2\text{cis}(4\pi/5)$

4 $2\text{cis}(6\pi/5)$

5 $2\text{cis}(8\pi/5)$

6. Fourth roots of $-81i$

$0 - 81i$

$r = \sqrt{81^2 + 0^2} = 81$

$\theta = 270$ or $\frac{3\pi}{2}$

$z = 81\left[\cos\left(\frac{3\pi}{2}\right) + i\sin\left(\frac{3\pi}{2}\right)\right]$

$z^{1/4} = \sqrt[4]{81}\left[\cos\left(\frac{1}{4}\cdot\frac{3\pi}{2}\right) + i\sin\left(\frac{1}{4}\cdot\frac{3\pi}{2}\right)\right]$

1 $= 3\left[\cos\frac{3\pi}{8} + i\sin\frac{3\pi}{8}\right]$

Add $\frac{2\pi}{n} \to \frac{2\pi}{4} \to \frac{\pi}{2}$

$+\pi/2$

2 $3\left[\cos\frac{7\pi}{8} + i\sin\frac{7\pi}{8}\right]$

3 $3\text{cis}\frac{11\pi}{8}$

4 $3\text{cis}\frac{15\pi}{8}$