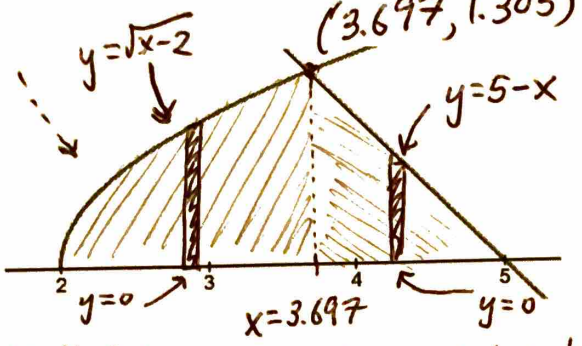
Key

A.P. Calculus AB Chapter 7.-7.2 Area & Volume Unit Review WS #1

1) Given the region below enclosed by $f(x) = \sqrt{x-2}$, $g(x) = 5 - x$, and the x-axis.

a) Find the area of the below region. (Write the integral notation(s) as well as the numeric approximation rounded to 3 decimal places)

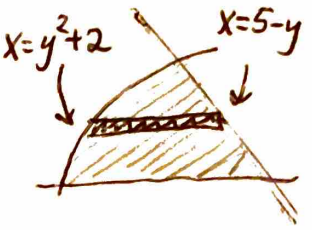


Method 2: Right-Left

$y = \sqrt{x-2}$ | $y = 5-x$

$y^2 = x-2$ | $x = 5-y$

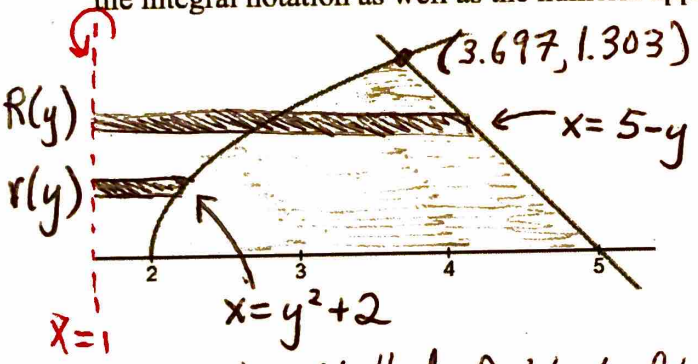
$y^2+2 = x$



$$\text{Area} = \int_0^{1.303} 5-y-(y^2+2)\,dy = \boxed{2.323}$$

Method 1: Top-Bottom, split into 2 regions: $\text{Area} = \int_2^{3.697} \sqrt{x-2} - 0\,dx + \int_{3.697}^{5} 5-x-0\,dx = \boxed{2.323}$

b) Find the Volume of solid generated when the enclosed region is revolved about the line x = 1 (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



* Washer Method, Right-Left

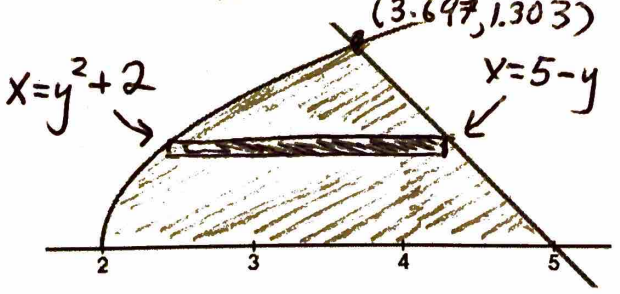
$R(y) = 5-y-(1) = 4-y$

$r(y) = y^2+2-(1) = y^2+1$

$$V = \pi \int_0^{1.303} [4-y]^2 - [y^2+1]^2\,dy$$

$$\boxed{V = 11.265\pi \text{ units}^3}$$

c) The enclosed region is the base of a solid. The cross section of the solid taken <u>perpendicular to the y-axis</u> is an equilateral triangle. Find the volume of the given solid. (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



$\text{base} = 5-y-(y^2+2) = 5-y-y^2-2$

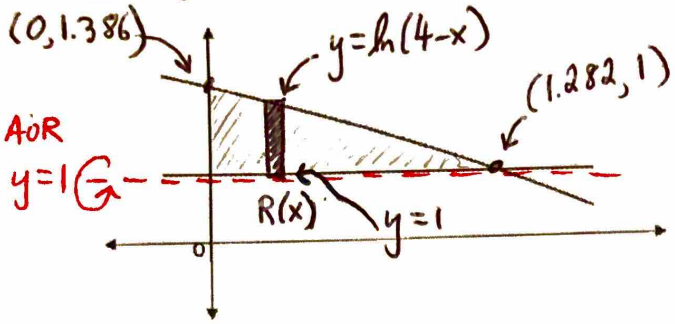
$= 3-y^2-y$

$\text{Area} = \frac{\sqrt{3}}{4}(\text{base})^2 \rightarrow \frac{\sqrt{3}}{4}(3-y^2-y)^2$

$$V = \int_0^{1.303} \frac{\sqrt{3}}{4}(3-y^2-y)^2\,dy = \boxed{2.225 \text{ units}^3}$$

2) Given the region below enclosed by $f(x) = \ln(4 - x)$, the line y = 1 , and the y-axis.

a) Find the Volume of solid generated when the enclosed region is revolved about the line y = 1 (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



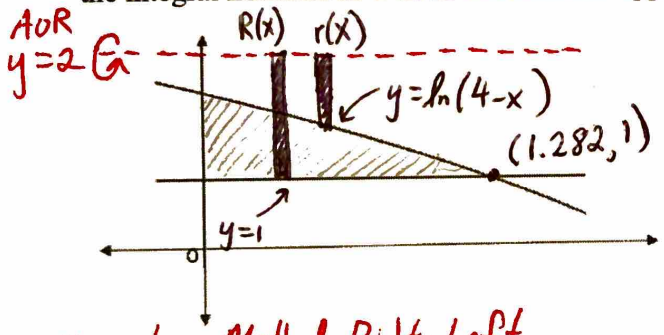
$R(x) = \ln(4-x) - (1)$

$V = \pi \int_0^{1.282} [\ln(4-x) - 1]^2 \, dx$

$V = 0.069\pi$ units^3

* Disc Method, Top-Bottom

b) Find the Volume of solid generated when the enclosed region is revolved about the line y = 2 (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



$R(x) = 2 - (1) = 1$

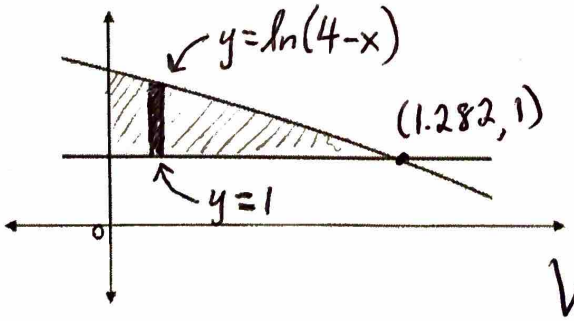
$r(x) = 2 - (\ln(4-x))$

$V = \pi \int_0^{1.282} [1]^2 - [2 - \ln(4-x)]^2 \, dx$

$V = 0.457\pi$ units^3

* washer Method, Right-Left

c) The enclosed region is the base of a solid. The cross section of the solid taken <u>perpendicular to the x-axis</u> is a rectangle whose height is twice the base. Find the volume of the given solid. (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



$height = 2[\ln(4-x) - 1]$

$base = \ln(4-x) - (1)$

$Area = (base)(height)$

$Area = 2[\ln(4-x) - 1]^2$

$V = \int_0^{1.282} 2[\ln(4-x) - 1]^2 \, dx$

$V = 0.139$ units^3