Key

**7.1-7.2 Help Session #2   Test Review Problems**

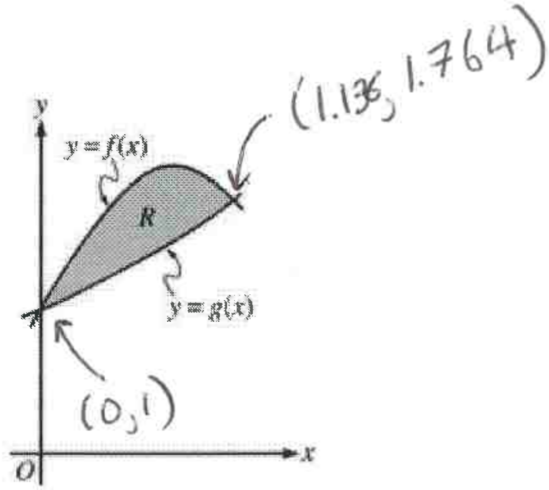
1.

Let $f$ and $g$ be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let $R$ be the shaded region in the first quadrant enclosed by the graphs of $f$ and $g$ as shown in the figure above.

(a) Find the area of $R$.

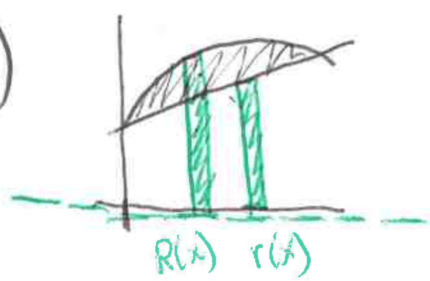
(b) Find the volume of the solid generated when $R$ is revolved about the $x$-axis.

(c) The region $R$ is the base of a solid. For this solid, the cross sections perpendicular to the $x$-axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.




a) $\text{Area} = \int_0^{1.136} 1+\sin(2x) - e^{x/2}\,dx = \boxed{0.429}$

b)




$R(x) = 1+\sin(2x) - 0$

$r(x) = e^{x/2} - 0$

$V = \pi \int_0^{1.136} [1+\sin(2x)]^2 - [e^{x/2}]^2\,dx$

$\boxed{V = 1.358\pi}$

c)

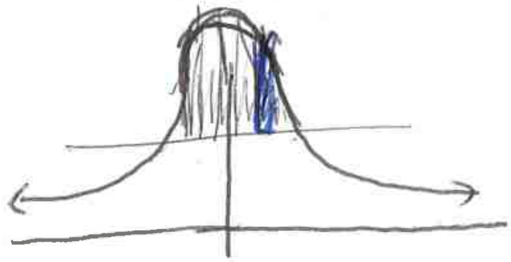
$\text{base} = 1+\sin(2x) - e^{x/2}$

$A = \frac{\pi}{8}(\text{base})^2$

$A = \frac{\pi}{8}\left(1+\sin(2x) - e^{x/2}\right)^2$

$V = \frac{\pi}{8}\int_0^{1.136} \left[1+\sin(2x) - e^{x/2}\right]^2 dx \rightarrow \boxed{\frac{\pi}{8}(0.19775) \text{ or } 0.077}$

2.



Let $R$ be the region in the first and second quadrants bounded above by the graph of $y = \frac{20}{1+x^2}$ and below by the horizontal line $y = 2$.

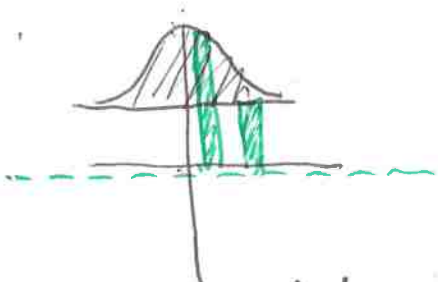
(a) Find the area of $R$.

(b) Find the volume of the solid generated when $R$ is rotated about the $x$-axis.

(c) The region $R$ is the base of a solid. For this solid, the cross sections perpendicular to the $x$-axis are semicircles. Find the volume of this solid.

a) $\text{Area} = \int_{-3}^{3} \frac{20}{1+x^2} - 2\,dx = \boxed{37.961 \text{ units}^2}$

b)



$R(x) = \frac{20}{1+x^2} - 0$

$r(x) = 2 - 0 = 2$

$V = \pi \int_{-3}^{3} \left[\frac{20}{1+x^2}\right]^2 - 2^2\,dx = \boxed{595.618\pi}$

c)

$\text{base} = \frac{20}{1+x^2} - 2$

$\text{Area} = \frac{\pi}{8}(\text{base})^2 \rightarrow \frac{\pi}{8}\left(\frac{20}{1+x^2} - 2\right)^2$

$V = \frac{\pi}{8} \int_{-3}^{3} \left[\frac{20}{1+x^2} - 2\right]^2 dx = \boxed{174.268 \text{ or } 55.471\pi}$

3.

Let $R$ be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$.

(a) Find the area of $R$.

(b) Find the volume of the solid generated when $R$ is rotated about the vertical line $x = -1$.

(c) The region $R$ is the base of a solid. For this solid, the cross sections perpendicular to the $y$-axis are squares. Find the volume of this solid.

*Intersection: $(\sqrt{x})^2 = \left(\frac{x}{3}\right)^2$

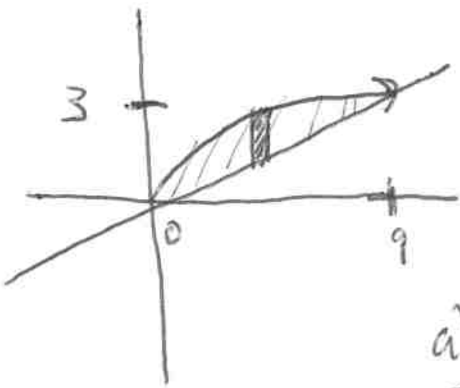
$x = \frac{x^2}{9}$

$9x = x^2$

$9x - x^2 = 0$

$x(9-x) = 0$

$x = 0, 9$




a) $A = \int_0^9 \left(\sqrt{x} - \frac{x}{3}\right) dx = \boxed{4.5}$

$y = \sqrt{x}$

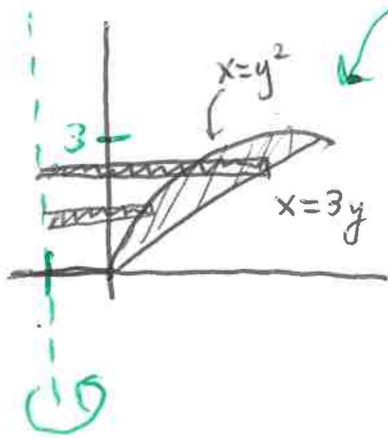
$y^2 = x$

$\boxed{x = y^2}$

$y = \frac{x}{3}$

$\boxed{x = 3y}$

b)

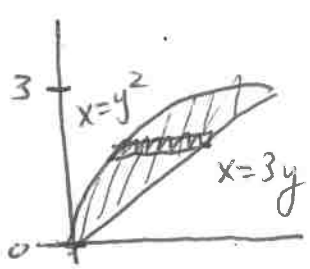



$R(y) = 3y - (-1) = 3y + 1$

$r(y) = y^2 - (-1) = y^2 + 1$

$V = \pi \int_{-3}^{3} [3y+1]^2 - [y^2+1]^2 \, dy = \frac{207}{5}\pi$ or $\boxed{41.4\pi}$

c)

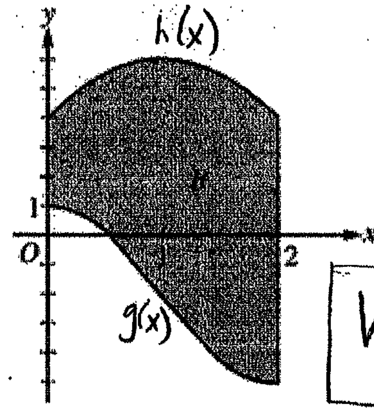


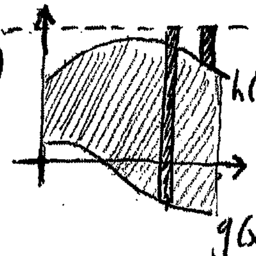

base $= 3y - y^2$

Area $= (\text{base})^2$

$A = (3y - y^2)^2$

$V = \int_0^3 [3y - y^2]^2 \, dy = \boxed{8.1}$




c) 


AOR $y=6$

* washer method:

$R(x) = 6 - g(x) \qquad r(x) = 6 - h(x)$

$$\boxed{V = \pi \int_0^2 [6-g(x)]^2 - [6-h(x)]^2 \, dx}$$

(Non-calculator)

5. Let $R$ be the region enclosed by the graphs of $g(x) = -2 + 3\cos\left(\frac{\pi}{2}x\right)$ and $h(x) = 6 - 2(x-1)^2$, the $y$-axis, and the vertical line $x = 2$, as shown in the figure above.

$\boxed{4}$ (a) Find the area of $R$.

$\boxed{2}$ (b) Region $R$ is the base of a solid. For the solid, at each $x$ the cross section perpendicular to the $x$-axis has area $A(x) = \frac{1}{x+3}$. Find the volume of the solid. 2 dx

$\boxed{3}$ (c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when $R$ is rotated about the horizontal line $y = 6$.

a) $* \text{Area} = \int_{x_1}^{x_2} (\text{Top} - \text{Bottom})\, dx$

$\text{Area} = \int_0^2 6 - 2(x-1)^2 - \left(-2 + 3\cos\left(\frac{\pi}{2}x\right)\right) dx$

$\text{Area} = \int 6 - 2(x-1)^2 + 2 - 3\cos\left(\frac{\pi}{2}x\right) dx$

$= \int_0^2 8 - 2(x-1)^2 - 3\cos\left(\frac{\pi}{2}x\right) dx$

$\int 8\, dx - 2\int (x-1)^2 dx - 3\int \cos\left(\frac{\pi}{2}x\right) dx$

$u = x-1 \qquad \frac{du}{dx} = 1 \qquad 2\int u^2 du \qquad -2\left(\frac{u^3}{3}\right) \qquad -\frac{2}{3}(x-1)^3$

$u = \frac{\pi}{2}x \qquad \frac{du}{dx} = \frac{\pi}{2} \qquad \pi dx = 2du \qquad dx = \frac{2}{\pi} du$

$3\int \cos u \cdot \frac{2}{\pi} du \qquad \frac{6}{\pi}\int \cos u\, du$

$$8x - \frac{2}{3}(x-1)^3 + \frac{6}{\pi}\sin\left(\frac{\pi}{2}x\right)\Big]_0^2$$

$= 8(2) - \frac{2}{3}(1)^3 + \frac{6}{\pi}\sin(\pi) - \left[0 - \frac{2}{3}(-1) + \frac{6}{\pi}\sin 0\right]$

$= 16 - \frac{2}{3} + 0 - \frac{2}{3} = 16 - \frac{4}{3} = \boxed{\frac{44}{3}}$

b) $\text{Volume of Cross Section} = \int_{x_1}^{x_2} [\text{Area}]\, dx$

$\int_0^2 \frac{1}{x+3} dx \qquad u = x+3 \qquad \frac{du}{dx} = 1$

$\int \frac{1}{u} du = \ln u$

$\ln|x+3| \Big]_0^2 = \boxed{\ln 5 - \ln 3 = \ln\left(\frac{5}{3}\right)}$

c) (see top of page)