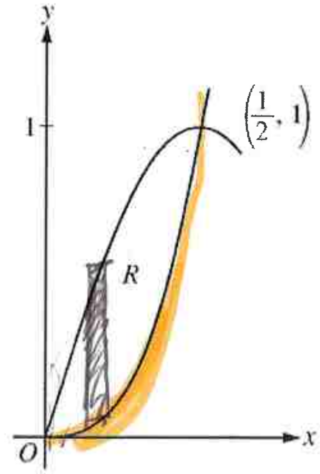
Help session #1
Ch. 7.1–7.2 WS

1



1. Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.

(a) Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.

(b) Find the area of R.

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

a) $f\left(\frac{1}{2}\right) = 8\left(\frac{1}{2}\right)^3 = 8\left(\frac{1}{8}\right) = 1$

$f'(x) = 24x^2$

$f'(1/2) = 24\left(\frac{1}{2}\right)^2 = 24\left(\frac{1}{4}\right) = 6$

point: $\left(\frac{1}{2}, 1\right)$

slope: $m = 6$

$y - y_1 = m(x - x_1)$

$\boxed{y - 1 = 6(x - 1/2)}$

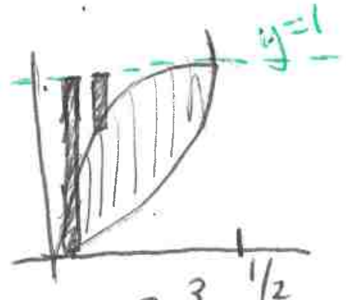
b) $\text{Area} = \int_0^{1/2} \sin(\pi x) - 8x^3 \, dx$

$-\frac{1}{\pi}\cos(\pi x) - \frac{8x^4}{4}\Big]_0^{1/2} = -\frac{1}{\pi}\cancel{\cos(\pi/2)} - 2(1/2)^4 - \left(-\frac{1}{\pi}\cos 0 - 0\right)$

0

$-\frac{2}{16} + \frac{1}{\pi} \rightarrow \boxed{-\frac{1}{8} + \frac{1}{\pi}}$

c)



*washer method

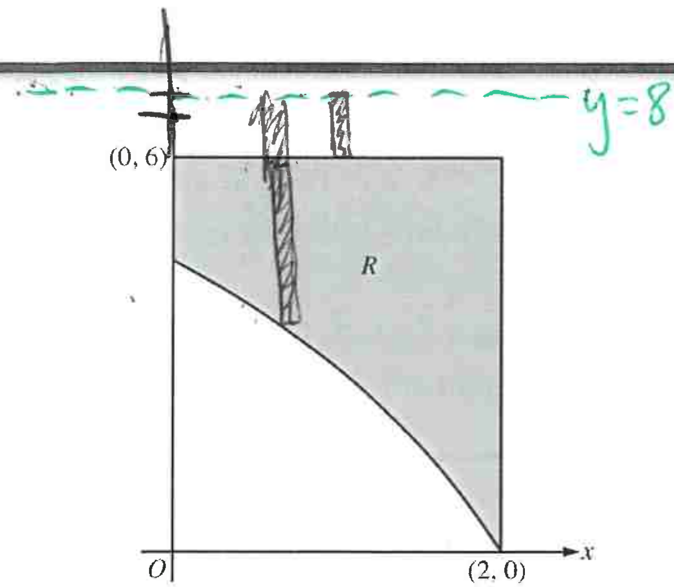
$\boxed{V = \pi \int_0^{1/2} [1 - 8x^3]^2 - [1 - \sin(\pi x)]^2 \, dx}$

$R(x) = 1 - 8x^3$

$r(x) = 1 - \sin(\pi x)$

2

2



In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.

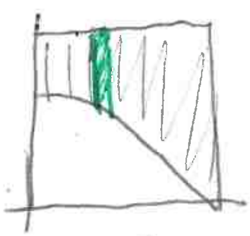
(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.

a) $\text{Area} = \int_0^2 6 - 4\ln(3-x)\,dx = \boxed{6.816}$

b) $R(x) = 8 - 4\ln(3-x)$ $\quad r(x) = 8 - 6 = 2$

*washer method

$V = \pi \int_0^2 [8 - 4\ln(3-x)]^2 - [2]^2\,dx = \boxed{53.533\pi \text{ units}^3}$

c) 

base $= 6 - 4\ln(3-x)$

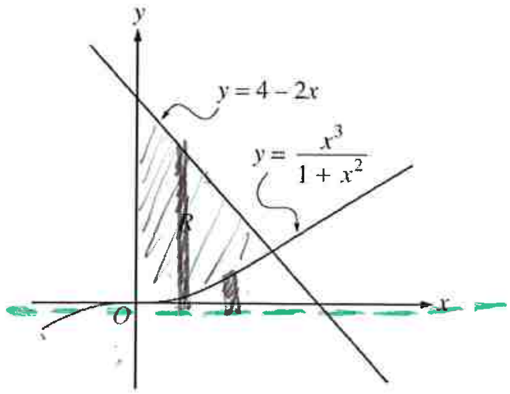
$\downarrow$

$\text{Area} = (\text{base})^2$

$\text{Area} = [6 - 4\ln(3-x)]^2$

$V = \int_0^2 [6 - 4\ln(3-x)]^2\,dx$

$\boxed{V = 26.266}$ units^3



a) Area $= \int_0^{1.487} 4-2x-\frac{x^3}{1+x^2}\,dx$ 3

$\boxed{\text{Area} = 3.214}$

Let R be the region bounded by the y-axis and the graphs of $y = \frac{x^3}{1+x^2}$ and $y = 4 - 2x$, as shown in the figure above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

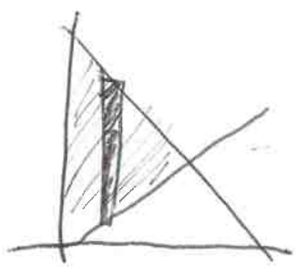
b) *washer Method

$R(x) = 4-2x-0$

$r(x) = \frac{x^3}{1+x^2} - 0$

$V = \pi \int_0^{1.487} [4-2x]^2 - \left[\frac{x^3}{1+x^2}\right]^2 dx$

$\boxed{V = 10.149\pi}$

c) 

base $= 4-2x-\frac{x^3}{1+x^2}$

Area $= (\text{base})^2$

Area $= \left[4-2x-\frac{x^3}{1+x^2}\right]^2$

$\boxed{V = \int_0^{1.487} \left[4-2x-\frac{x^3}{1+x^2}\right]^2 dx}$ $\boxed{= 8.997}$

4

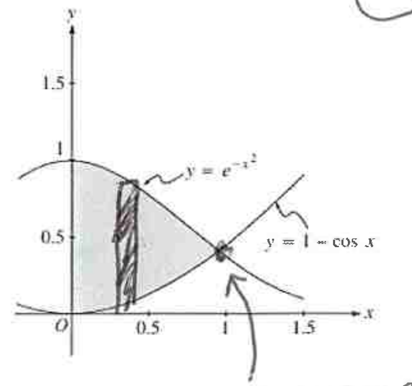
4

Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.

(a) Find the area of the region R.

(b) Find the volume of the solid generated when the region R is revolved about the x-axis.

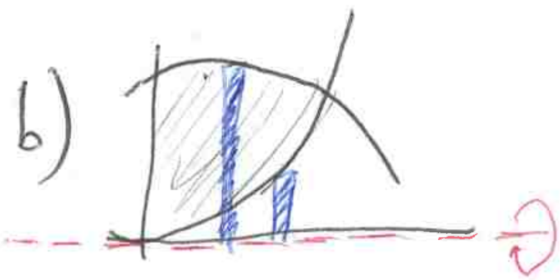
(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.



$X = 0.9419$

a) $A = \int_0^{0.9419} e^{-x^2} - (1 - \cos x)\,dx = \boxed{0.591}$ units^2

b)



* washer Method

$R(x) = e^{-x^2} - 0 \qquad r(x) = 1 - \cos x - (0)$

$V = \pi \int [e^{-x^2}]^2 - [1 - \cos x]^2\,dx$

$\boxed{= 0.556\pi}$

c) $\text{base} = e^{-x^2} - (1 - \cos x)$

$A = (\text{base})^2 \rightarrow (e^{-x^2} - 1 + \cos x)^2$

$V = \int_0^{0.9419} [e^{-x^2} - 1 + \cos x]^2\,dx = \boxed{0.461}$