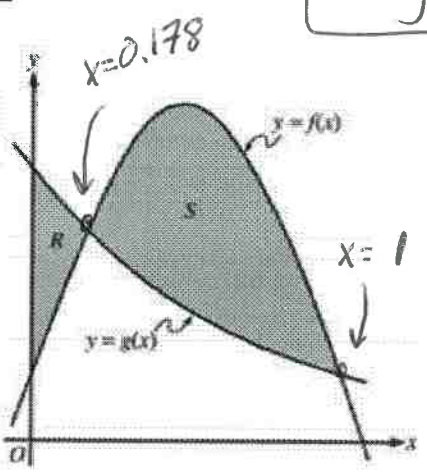
Key

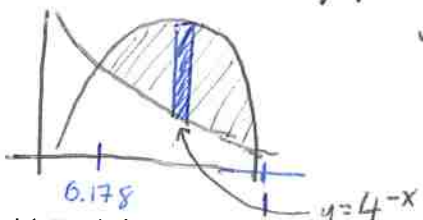
A.P. Calculus AB Chapter 7.-7.2 Area & Volume Unit Review WS #2

1)

Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of f and g, and let S be the shaded region in the first quadrant enclosed by the graphs of f and g, as shown in the figure above.



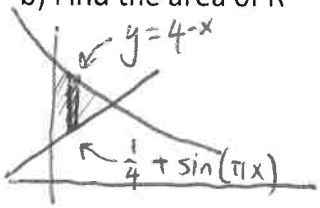
a) Find the area of S



$$A = \int_{0.178}^{1} \frac{1}{4} + \sin(\pi x) - (4^{-x})\,dx$$

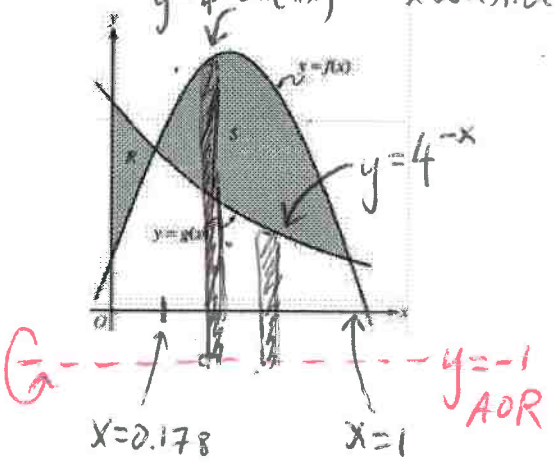
$$A = 0.410 \text{ units}^2$$

b) Find the area of R



$$Area = \int_{0}^{0.178} 4^{-x} - \left(\frac{1}{4} + \sin(\pi x)\right) dx = 0.0648 \text{ units}^2$$

c) Find the volume of the solid generated when S is revolved about the horizontal line y = - 1.



*Washer Method

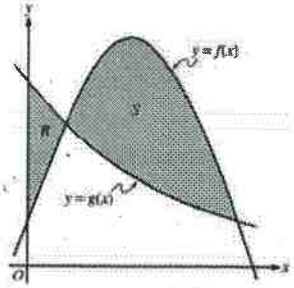
$$R(x) = \frac{1}{4} + \sin(\pi x) - (-1) = \frac{5}{4} + \sin(\pi x)$$

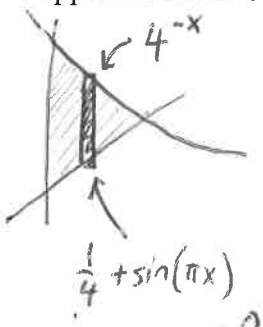
$$r(x) = 4^{-x} - (-1) = 4^{-x} + 1$$

$$V = \pi \int_{x_1}^{x_2} R(x)^2 - r(x)^2\,dx$$

$$V = \pi \int_{0.178}^{1} \left[\frac{5}{4} + \sin(\pi x)\right]^2 - \left[4^{-x} + 1\right]^2 dx = 1.451\pi \text{ units}^3$$

d) The enclosed region R is the base of a solid. The cross section of the solid taken parallel to the y-axis is a isosceles right triangle with leg on base. Find the volume of the given solid. (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)





$$base = 4^{-x} - \left(\frac{1}{4} + \sin(\pi x)\right)$$

$$base = 4^{-x} - \frac{1}{4} - \sin(\pi x)$$

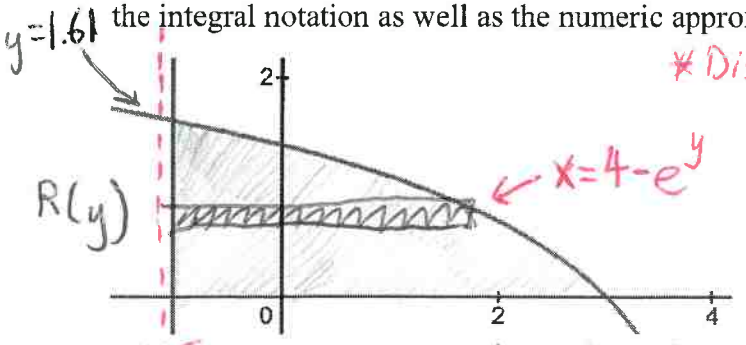
$$Area = \frac{1}{2}(base)^2 \rightarrow \frac{1}{2}\left[4^{-x} - \frac{1}{4} - \sin(\pi x)\right]^2$$

$$V = \int_{x_1}^{x_2} [Area]\,dx \rightarrow V = \int_{0}^{0.178} \frac{1}{2}\left[4^{-x} - \frac{1}{4} - \sin(\pi x)\right]^2 dx = 0.016 \text{ units}^3$$

2) Given the region below enclosed by $f(x) = \ln(4-x)$, the line x = -1, and the x-axis.

AOR $x=-1$

a) Find the Volume of solid generated when the enclosed region is revolved about the line x = -1 (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



(-1,5)
$x=-1$
AOR

* Disc Method

$y = \ln(4-x)$ | $e^y = 4-x$

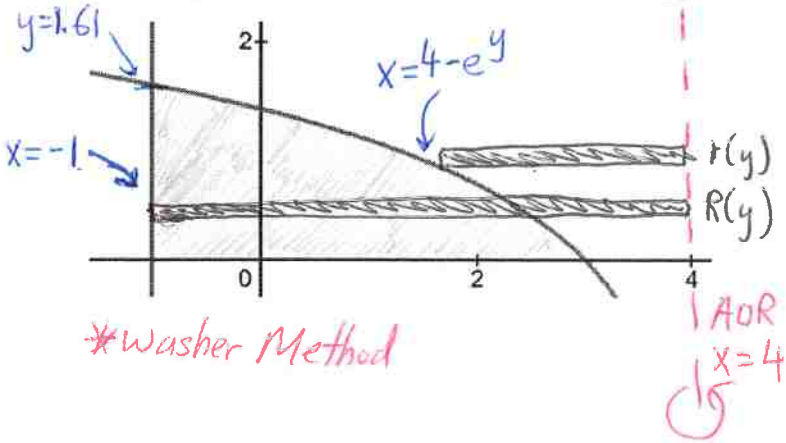
$e^y = e^{\ln(4-x)}$ | $x = 4-e^y$

$e^y = (4-x)$

$R(y) = 4-e^y-(-1)$ | $V = \pi\int_{y_1}^{y_2} R[y]^2\,dy$

$R(y) = 5-e^y$ | $V = \pi\int_0^{1.61}[5-e^y]^2\,dy = \boxed{12.236\pi \text{ units}^3}$

b) Find the Volume of solid generated when the enclosed region is revolved about the line x = 4 (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



* Washer Method

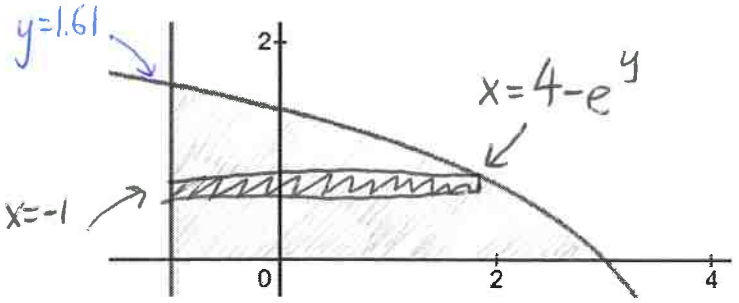
$R(y) = 4-(-1) = 5$

$r(y) = 4-(4-e^y) = e^y$

$V = \pi\int_{y_1}^{y_2} R(y)^2 - r(y)^2\,dy$

$V = \pi\int_0^{1.61}[5]^2-[e^y]^2\,dy = \boxed{28.236\pi \text{ units}^3}$

c) The enclosed region is the base of a solid. The cross section of the solid taken parallel to the x-axis is a rectangle whose height is 4. Find the volume of the given solid. (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



$base = 4-e^y-(-1) = 5-e^y$

$Area = (base)(height)$ ← height = 4

$A = (5-e^y)(4)$

$V = \int_{y_1}^{y_2}[Area]\,dy \rightarrow \int_0^{1.61} 4(5-e^y)\,dy = \boxed{16.189 \text{ units}^3}$