Non-AP Calculus 7.1-7.2 Area/Volume Review Worksheet #1

$$\text{'rea} = \int_{x_1}^{x_2} (\textbf{Top graph} - \textbf{Bottom graph})\, dx$$

$Radius\ [R(x)] = distance\ from\ the\ AOR\ (Axis\ of\ Revolution)\ to\ the\ graph\ curve$

$$\textbf{Disc Method}: Volume = \pi \int_{x_1}^{x_2} [R(x)]^2 dx$$

$Radius\ [R(x)] = distance\ from\ the\ AOR\ (Axis\ of\ Revolution)\ to\ the\ \textbf{further}\ graph\ curve$

$radius\ [r(x)] = distance\ from\ the\ AOR\ (Axis\ of\ Revolution)\ to\ the\ \textbf{closer}\ graph\ curve$

$$\textbf{Washer Method}: Volume = \pi \int_{x_1}^{x_2} [R(x)]^2 - [r(x)]^2 dx$$

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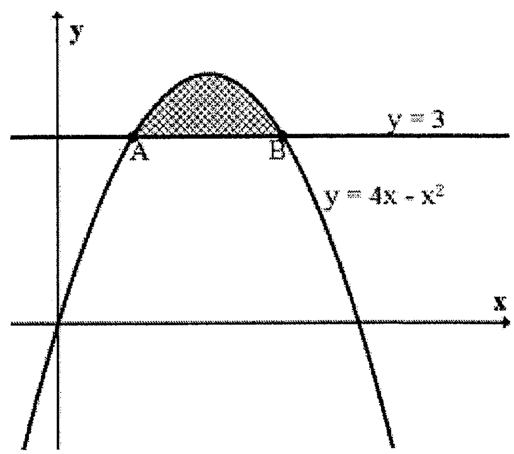
1)

The diagram shows the curve $y = 4x - x^2$ and the line $y = 3$.

a) Calculate the coordinates of A and B

b) Calculate the shaded area. (Show work!)



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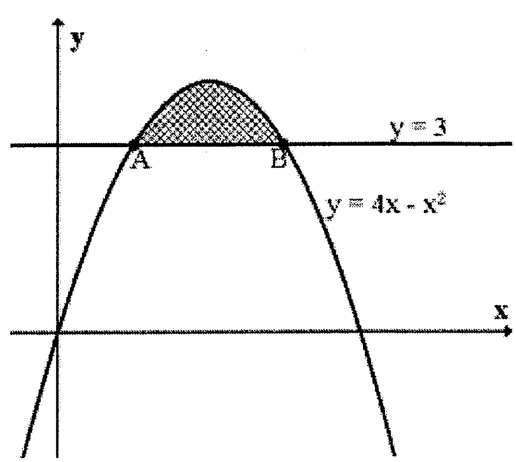
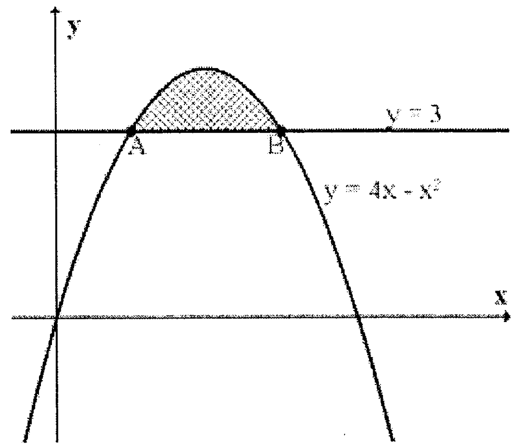
2)

Find the volume of the solid created by revolving the function $y = 4x - x^2$ bounded by the line $y = 3$ about the line $y = 3$ (Show work!)



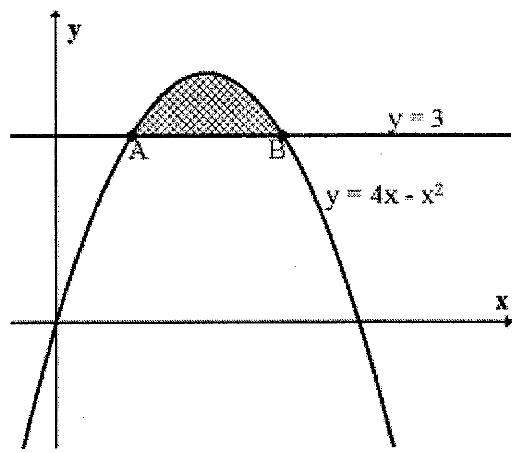
3)

Find the volume of the solid created by revolving the function $y = 4x - x^2$ bounded by the line $y = 3$ about the line $y = -1$.



4)

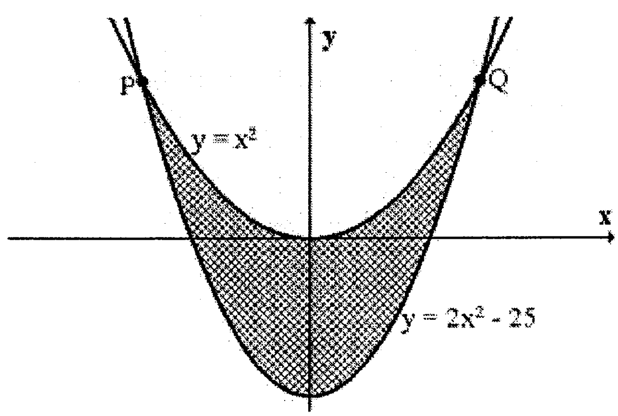
Find the volume of the solid created by revolving the function $y = 4x - x^2$ bounded by the line $y = 3$ about the line $y = 6$.



5)

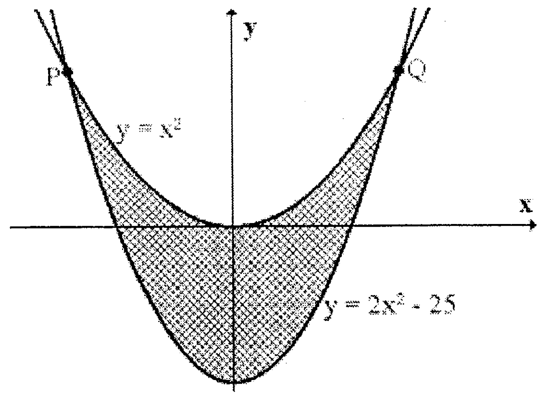
The curves with equations $y = x^2$ and $y = 2x^2 - 25$ intersect at P and Q.

Calculate the area enclosed between the curves.



6)

Find the volume of the solid created by revolving the function $y = 2x^2 - 25$ bounded by the curve $y = x^2$ about the line $y = -30$.



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7)

The diagram shows the curve $y = 3x$ and the curve $y = 7x - 2x^2$

a) Calculate the coordinates of A and B

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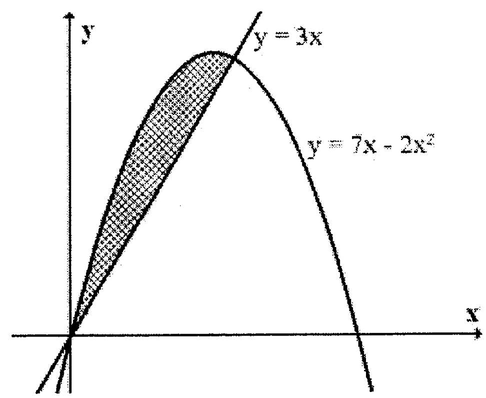

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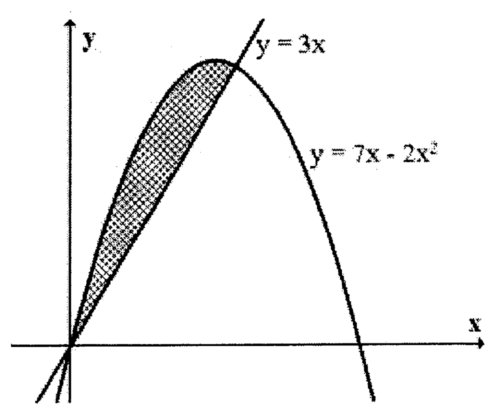
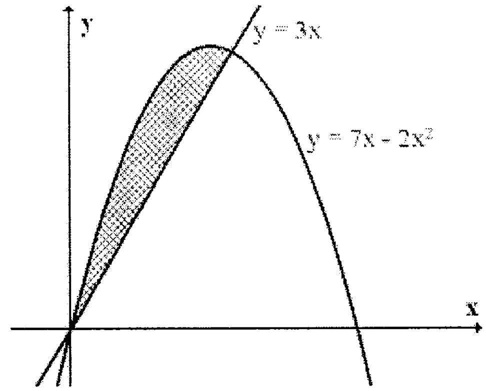
8)

Find the volume of the solid created by revolving the curve $y = 3x$ and the curve $y = 7x - 2x^2$ about the line $y = -3$



9)

Find the volume of the solid created by revolving the curve y = 3x and the curve y = 7x – 2x² about the line y = 6



10) Find the area of the region bounded by the following graphs.
$y = \sqrt{x}$, $x = 0$, $y = -1$, $and\ x = 4$ (Show Work!)

11) Find the volume of the solid created by revolving region created by
$y = \sqrt{x}$, $x = 0$, $y = -1$, $and\ x = 4$ about the line y = -1 (Show Work!)

Non-AP Calculus 7.1-7.2 Area/Volume Review Worksheet #1

Key

$$Area = \int_{x_1}^{x_2} (Top\ graph - Bottom\ graph)\, dx$$

$Radius\ [R(x)] = distance\ from\ the\ AOR\ (Axis\ of\ Revolution)\ to\ the\ graph\ curve$

$$\textbf{Disc Method}: Volume = \pi \int_{x_1}^{x_2} [R(x)]^2 dx$$

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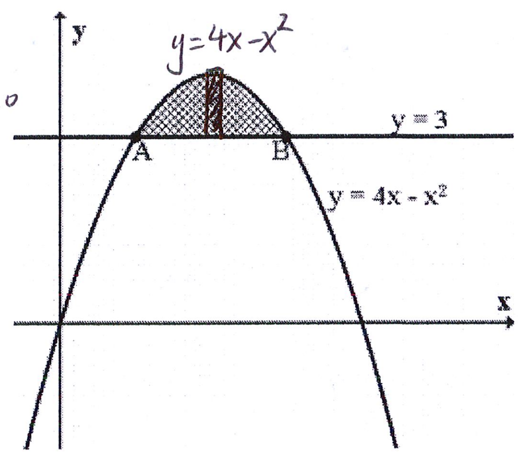
1)

The diagram shows the curve $y = 4x - x^2$ and the line $y = 3$.

a) Calculate the coordinates of A and B

b) Calculate the shaded area. (Show work!)



a) *find intersection: $3 = 4x - x^2$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1)=0$

$\boxed{x=1, x=3}$

b) $Area = \int_1^3 4x - x^2 - (3)\,dx = \int_1^3 -x^2 + 4x - 3\,dx$

$\left. -\frac{x^3}{3} + \frac{4x^2}{2} - 3x \right]_1^3 = \frac{-3^3}{3} + 2(3)^2 - 9 - \left(\frac{-1}{3} + 2 - 3\right)$

$Area = \boxed{\frac{4}{3}}$ units^2

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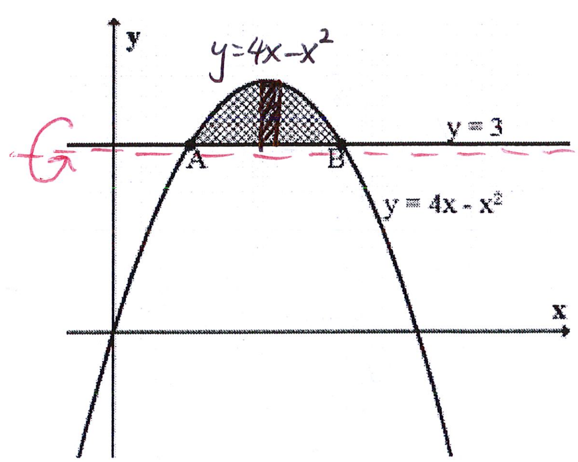
2)

Find the volume of the solid created by revolving the function $y = 4x - x^2$ bounded by the line $y = 3$ about the line $y = 3$ (Show work!)



intersection:

$4x - x^2 = 3$

$0 = x^2 - 4x + 3$

$(x-3)(x-1)$

$x = 1, 3$

*Disc Method

$R(x) = 4x - x^2 - 3$

$V = \pi \int_1^3 [4x - x^2 - 3]^2 dx$

$\boxed{V = \frac{16}{15}\pi \text{ units}^3}$

3)

Find the volume of the solid created by revolving the function $y = 4x - x^2$ bounded by the line $y = 3$ about the line $y = -1$.

intersections: $x=1, x=3$

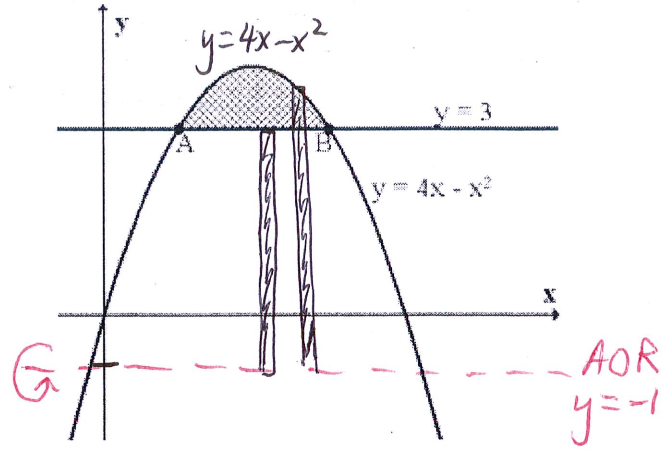
*washer method:

$R(x) = 4x - x^2 - (-1) = 4x - x^2 + 1$

$r(x) = 3 - (-1) = 4$

$$V = \pi \int_1^3 [4x - x^2 + 1]^2 - [4]^2 \, dx$$

$$\boxed{V = \frac{176}{15}\pi \text{ units}^3}$$



4)

Find the volume of the solid created by revolving the function $y = 4x - x^2$ bounded by the line $y = 3$ about the line $y = 6$.

*washer method

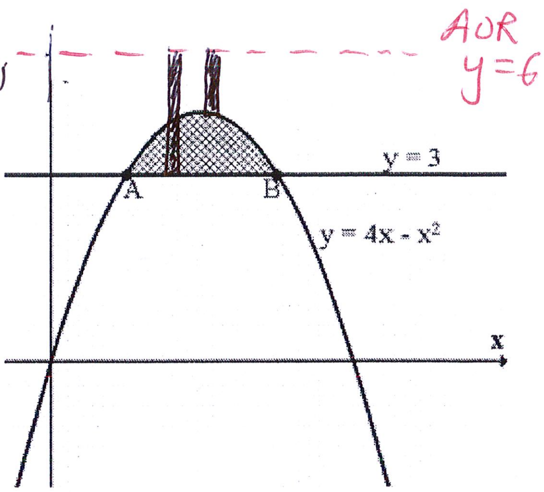
*intersections $x=1, x=3$

$R(x) = 6 - 3 = 3$

$r(x) = 6 - (4x - x^2) = 6 - 4x + x^2$

$$V = \pi \int_1^3 [3]^2 - [x^2 - 4x + 6]^2 \, dx$$

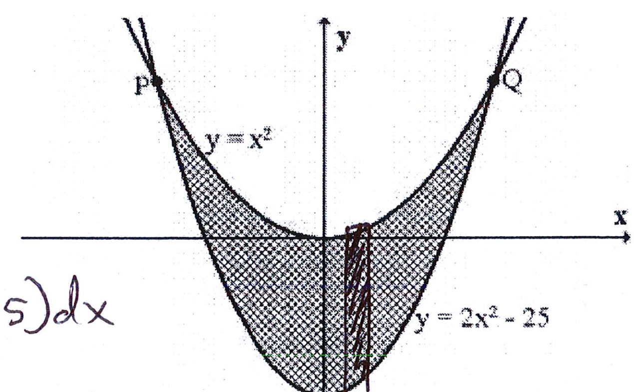
$$\boxed{V = \frac{104}{15}\pi \text{ units}^3}$$



5)

The curves with equations $y = x^2$ and $y = 2x^2 - 25$ intersect at P and Q.

Calculate the area enclosed between the curves.



*intersections:

$2x^2 - 25 = x^2$

$x^2 - 25 = 0$

$(x-5)(x+5) = 0$

$x = 5, -5$

$$A = \int_{-5}^{5} x^2 - (2x^2 - 25) \, dx$$

$$\int x^2 - 2x^2 + 25 \, dx$$

$$A = \int_{-5}^{5} -x^2 + 25 \, dx$$

$$\left. \frac{-x^3}{3} + 25x \right]_{-5}^{5} = \frac{-5^3}{3} + 25(5) - \left(\frac{5^3}{3} + (25)(-5)\right)$$

$$\boxed{\text{Area} = \frac{500}{3} \text{ units}^2}$$

6)

Find the volume of the solid created by revolving the function $y = 2x^2 - 25$ bounded by the curve $y = x^2$ about the line $y = -30$.

* washer method

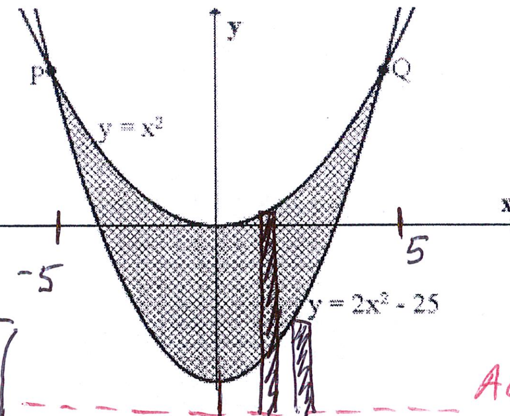
*intersections: $x = -5, 5$

$R(x) = x^2 - (-30) = x^2 + 30$

$r(x) = 2x^2 - 25 - (-30) = 2x^2 - 25 + 30 = 2x^2 + 5$

$$V = \pi \int_{-5}^{5} [x^2+30]^2 - [2x^2+5]^2 \, dx$$

$$\boxed{V = \frac{25000}{3}\pi \text{ units}^3}$$



AOR $y = -30$

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7)

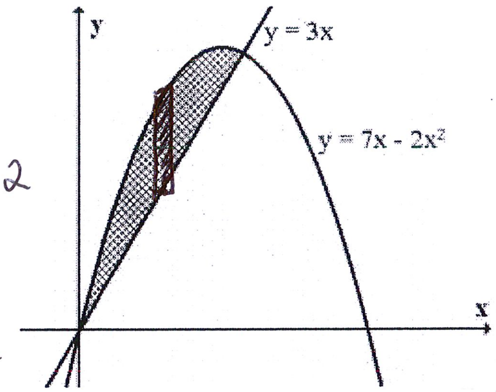
The diagram shows the curve $y = 3x$ and the curve $y = 7x - 2x^2$

a) Calculate the coordinates of A and B
b) Calculate the shaded area. (Show work!)

a) intersections: $3x = 7x - 2x^2$ | $x = 0, x = 2$

$2x^2 - 4x = 0$

$2x(x-2) = 0$



$$\text{Area} = \int_0^2 7x - 2x^2 - (3x)\, dx \quad \left. \frac{4x^2}{2} - \frac{2x^3}{3} \right]_0^2$$

$$\int_0^2 4x - 2x^2 \, dx \quad \left. 2x^2 - \frac{2}{3}x^3 \right]_0^2 = 2(2)^2 - \frac{2}{3}(2)^3 - (0-0) = \boxed{\frac{8}{3} \text{ units}^2}$$

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8)

Find the volume of the solid created by revolving the curve $y = 3x$ and the curve $y = 7x - 2x^2$ about the line $y = -3$

* washer method

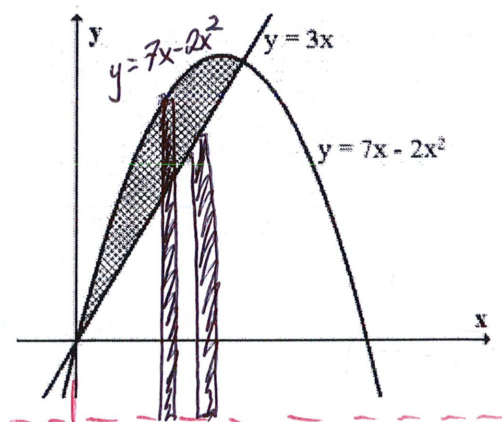
*intersections: $x = 0, 2$

$R(x) = 7x - 2x^2 - (-3) = 7x - 2x^2 + 3$

$r(x) = 3x - (-3) = 3x + 3$

$$V = \pi \int_0^2 [-2x^2 + 7x + 3]^2 - [3x+3]^2 \, dx$$

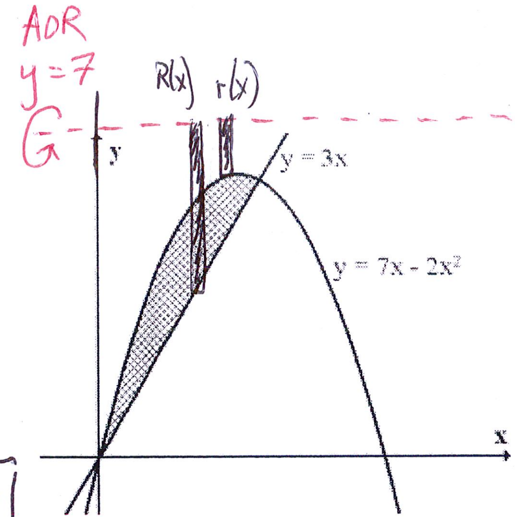
$$\boxed{V = \frac{544}{15}\pi \text{ units}^3}$$



AOR $y = -3$

9)

Find the volume of the solid created by revolving the curve y = 3x and the curve y = 7x – 2x² about the line y = 7



*washer method

*intersections: $x=0,\ x=2$

$R(x) = 7 - 3x$

$r(x) = 7-(7x-2x^2) = 7-7x+2x^2$

$$V = \pi \int_0^2 [7-3x]^2 - [2x^2-7x+7]^2 \, dx$$

$$\boxed{V = \frac{256}{15}\pi \text{ units}^3}$$

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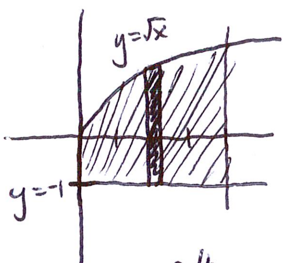
10) Find the area of the region bounded by the following graphs.
$y = \sqrt{x}$, $x = 0$, $y = -1$, *and* $x = 4$ (Show Work!)



$$\text{Area} = \int_0^4 \sqrt{x} - (-1)\, dx$$

$$\int_0^4 x^{1/2} + 1\, dx$$

$$\frac{x^{3/2}}{3/2} + x$$

$$\left.\frac{2}{3}x^{3/2} + x\right]_0^4 = \frac{2}{3}(4)^{3/2} + 4 - (0+0)$$

$$= \frac{16}{3} + 4 = \boxed{\frac{28}{3}}$$

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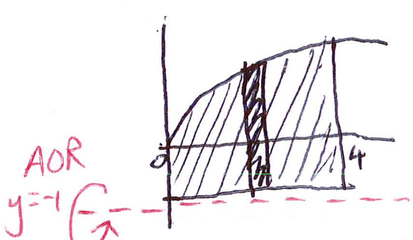
11) Find the volume of the solid created by revolving region created by
$y = \sqrt{x}$, $x = 0$, $y = -1$, *and* $x = 4$ about the line y = -1 (Show Work!)



*Disc Method

$R(x) = \sqrt{x} - (-1)$

$R(x) = \sqrt{x} + 1$

$$V = \pi \int_0^4 \left[\sqrt{x}+1\right]^2 dx$$

$$\pi \int (\sqrt{x}+1)(\sqrt{x}+1)\, dx$$

$$\pi \int x + 2\sqrt{x} + 1\, dx$$

$$\pi \int x + 2x^{1/2} + 1\, dx \rightarrow \frac{x^2}{2} + \frac{2x^{3/2}}{3/2} + x$$

$$\left.\frac{x^2}{2} + \frac{4}{3}x^{3/2} + x\right]_0^4$$

$$\frac{16}{2} + \frac{4}{3}(4)^{3/2} + 4 - (0+0+0)$$

$$8 + \frac{32}{3} + 4 = 12 + \frac{32}{3} = \frac{68}{3}\pi \text{ units}^3$$

$$\boxed{V = \frac{68}{3}\pi \text{ units}^3}$$