Non-AP Calculus **7.1-7.2 Area/Volume Review Worksheet #2**

$$\textbf{Area} = \int_{x_1}^{x_2} (Top\ graph - Bottom\ graph)\, dx$$

$Radius\ [R(x)] = distance\ from\ the\ AOR\ (Axis\ of\ Revolution)\ to\ the\ graph\ curve$

$$\textbf{Disc Method}: Volume = \pi \int_{x_1}^{x_2} [R(x)]^2 dx$$

$Radius\ [R(x)] = distance\ from\ the\ AOR\ (Axis\ of\ Revolution)\ to\ the\ \textbf{further}\ graph\ curve$

$radius\ [r(x)] = distance\ from\ the\ AOR\ (Axis\ of\ Revolution)\ to\ the\ \textbf{closer}\ graph\ curve$

$$\textbf{Washer Method}: Volume = \pi \int_{x_1}^{x_2} [R(x)]^2 - [r(x)]^2 dx$$

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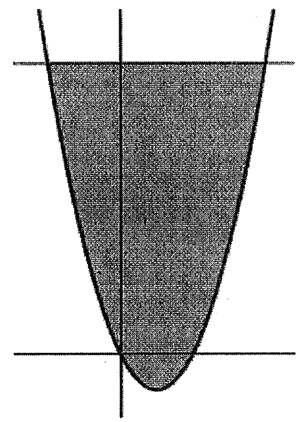
1)

The diagram shows the curve $y = x^2 - 2x$ and the line $y = 8$.

a) Calculate the intersections

b) Calculate the shaded area. (Show work!)



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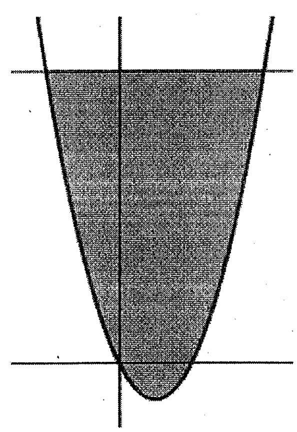
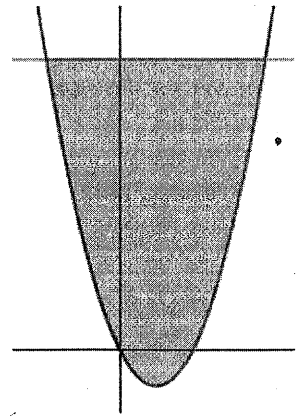
2)

Find the volume of the solid created by revolving the curve $y = x^2 - 2x$ and the line $y = 8$ about the AOR line $y = 8$



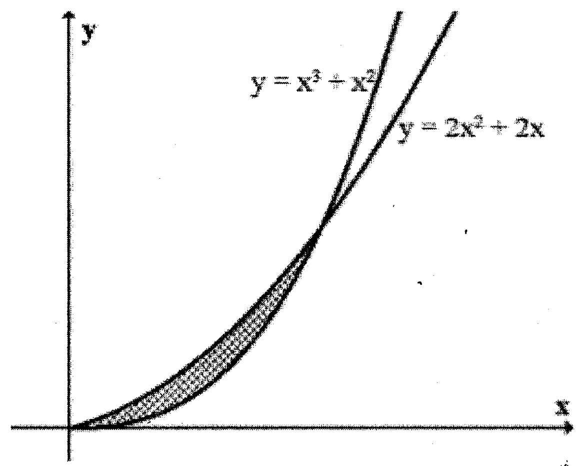
3)

Find the volume of the solid created by revolving the curve $y = x^2 - 2x$ and the line $y = 8$ about the AOR line $y = -1$



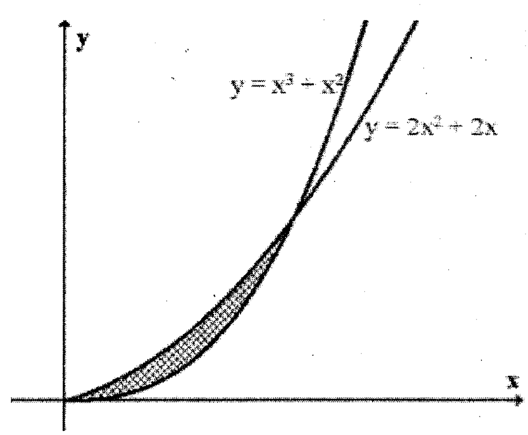
4)

Calculate the area of the shaded region. (Show work!)



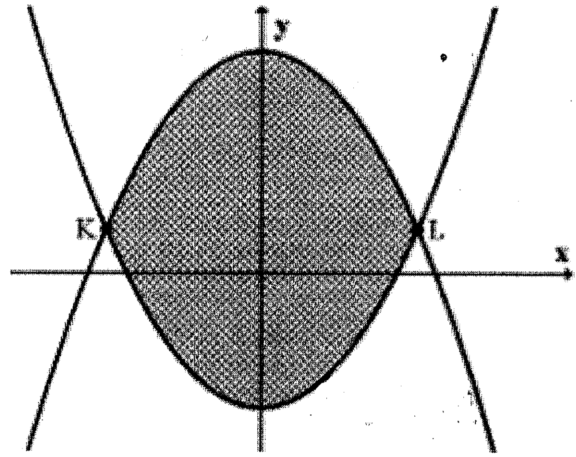
5)

Find the volume of the solid created by revolving the shaded region about the AOR $y = -3$



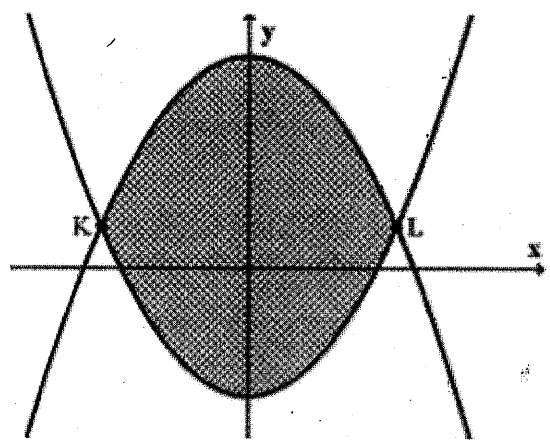
6)

Calculate the area of the shaded region (bounded by $y = 2x^2 - 6$ and $y = 10 - 2x^2$) . (Show work!)



7)

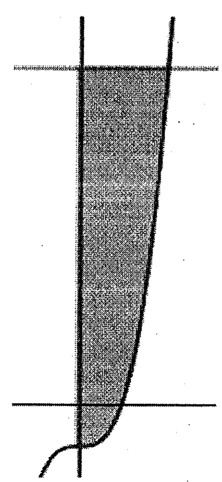
Find the volume of the solid created by revolving the shaded region (bounded by $y = 2x^2 - 6$ and $y = 10 - 2x^2$) about the AOR $y = 10$



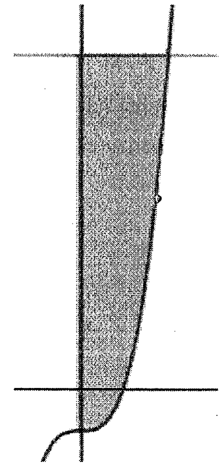
8)

The diagram shows the shaded region bounded by $y = x^3 - 1$, $y = 7$, and $x = 0$.

a) Calculate the intersections
b) Calculate the shaded area. (Show work!)



9) The diagram shows the shaded region bounded by $y = x^3 - 1$, $y = 7$, and $x = 0$. Find the volume of the solid created by revolving the shaded region about the AOR of $y = 8$.



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10) Find the area of the region bounded by the following graphs.
$y = \sqrt{x}$, $x = 0$, $and\ y = 3$ (Show Work!)

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11) Find the volume of the solid created by revolving region created by
$y = \sqrt{x}$, $x = 0$, $and\ y = 3$ about the AOR $y = 3$ (Show Work!)

Non-AP Calculus **7.1-7.2 Area/Volume Review Worksheet #2**

Key

$$\textit{Area} = \int_{x_1}^{x_2} (\textit{Top graph} - \textit{Bottom graph})\, dx$$

Radius $[R(x)]$ = *distance from the AOR (Axis of Revolution) to the graph curve*

$$\textbf{Disc Method}\text{: } \textit{Volume} = \pi \int_{x_1}^{x_2} [R(x)]^2 dx$$

Radius $[R(x)]$ = *distance from the AOR (Axis of Revolution) to the* ***further*** *graph curve*

radius $[r(x)]$ = *distance from the AOR (Axis of Revolution) to the* ***closer*** *graph curve*

$$\textbf{Washer Method}\text{: } \textit{Volume} = \pi \int_{x_1}^{x_2} [R(x)]^2 - [r(x)]^2 dx$$

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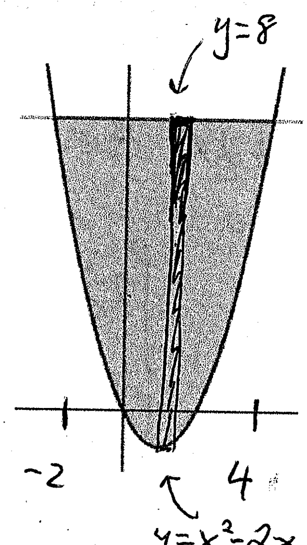
1)

The diagram shows the curve $y = x^2 - 2x$ and the line $y = 8$.

a) Calculate the intersections

b) Calculate the shaded area. (Show work!)



*intersection

a) $x^2 - 2x = 8$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4, x = -2$

b) $\text{Area} = \int_{-2}^{4} 8 - (x^2 - 2x)\, dx$

$\int 8 - x^2 + 2x\, dx = 8x - \frac{x^3}{3} + \frac{2x^2}{2}\Big]_{-2}^{4}$

$= 32 - \frac{4^3}{3} + 4^2 - \left(-16 + \frac{8}{3} + 4\right)$

$= \boxed{36}$ units^2

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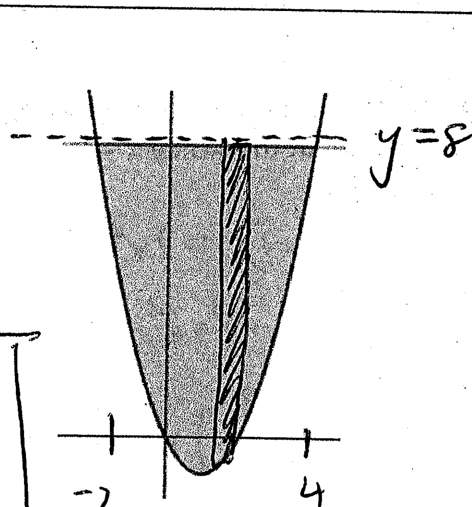
2) *Disc Method

Find the volume of the solid created by revolving the curve $y = x^2 - 2x$ and the line $y = 8$ about the AOR line $y = 8$



$R(x) = 8 - (x^2 - 2x) = 8 - x^2 + 2x$

$V = \pi \int_{x_1}^{x_2} [R(x)]^2 dx$

$V = \pi \int_{-2}^{4} [8 - x^2 + 2x]^2 dx$

$V = 259.2\pi$

OR

$V = \frac{1296}{5}\pi$ units^3

3)

Find the volume of the solid created by revolving the curve $y = x^2 - 2x$ and the line $y = 8$ about the AOR line $y = -1$

*intersection

$x^2-2x=8$

$x^2-2x-8=0$

$(x-4)(x+2)=0$

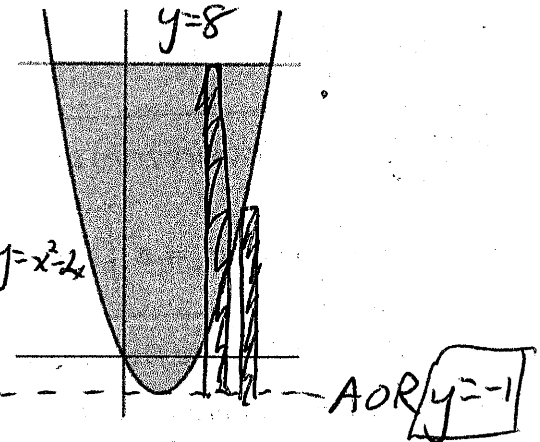
$x=4,-2$

washer method:

$R(x)=8-(-1)=9$

$r(x)=x^2-2x-(-1)=x^2-2x+1$

$$V=\pi\int_{-2}^{4}[9]^2-[x^2-2x+1]^2\,dx=\boxed{\frac{1944\pi}{5} \text{ or } 388.8\pi \text{ units}^3}$$



4)

Calculate the area of the shaded region. (Show work!)

*intersections:

$x^3+x^2=2x^2+2x$

$x^3-x^2-2x=0$

$x(x^2-x-2)=0$

$x(x-2)(x+1)=0$

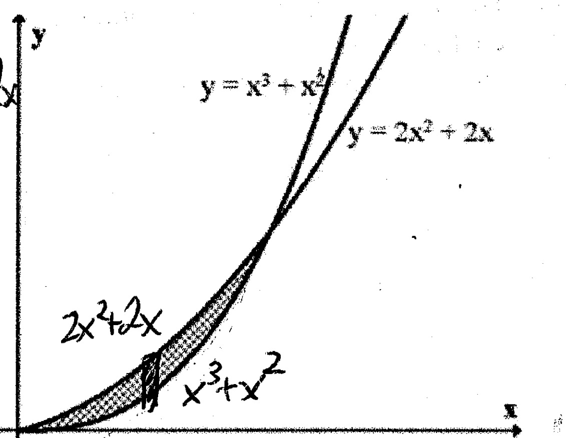
$x=0,2,\cancel{-1}$

$$\text{Area}=\int_0^2 2x^2+2x-(x^3+x^2)\,dx$$

$$\int 2x^2+2x-x^3-x^2\,dx$$

$$\int x^2-x^3+2x\,dx$$

$$\left.\frac{x^3}{3}-\frac{x^4}{4}+\frac{2x^2}{2}\right]_0^2=\frac{2^3}{3}-\frac{2^4}{4}+2^2-(0-0+0)=\boxed{\frac{8}{3}\text{ units}^2}$$



5)

Find the volume of the solid created by revolving the shaded region about the AOR $y = -3$

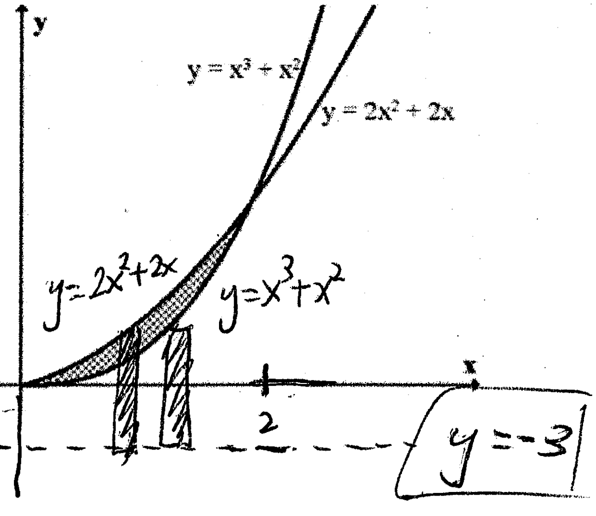
*washer method

$R(x)=2x^2+2x-(-3)=2x^2+2x+3$

$r(x)=x^3+x^2-(-3)=x^3+x^2+3$

$$V=\pi\int_0^2[2x^2+2x+3]^2-[x^3+x^2+3]^2\,dx$$

$$\boxed{V=38.248\pi \text{ or } \frac{4016}{105}\pi \text{ units}^3}$$



6)

Calculate the area of the shaded region (bounded by $y = 2x^2 - 6$ and $y = 10 - 2x^2$). (Show work!)

*find intersections:

$2x^2 - 6 = 10 - 2x^2$

$4x^2 - 16 = 0$

$4(x^2 - 4) = 0$

$4(x-2)(x+2) = 0$

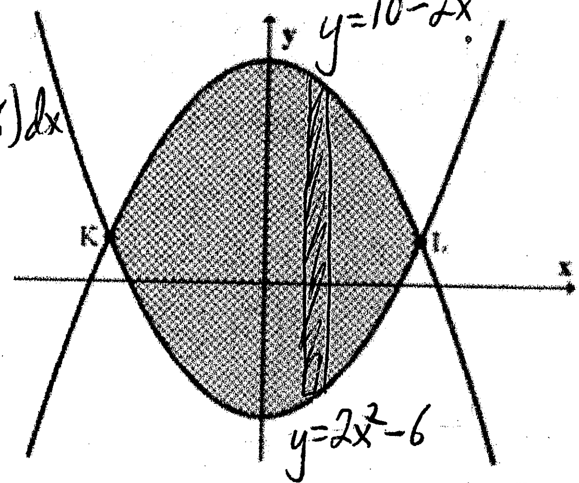
$x = 2, -2$

$$\text{Area} = \int_{-2}^{2} (10 - 2x^2) - (2x^2 - 6)\,dx$$

$$\int_{-2}^{2} 10 - 2x^2 - 2x^2 + 6\,dx$$

$$\int 16 - 4x^2\,dx = 16x - \frac{4x^3}{3}\Bigg]_{-2}^{2}$$

$$32 - \frac{4(2)^3}{3} - \left(-32 + \frac{32}{3}\right) = 42.667 \text{ or } \boxed{\frac{128}{3} \text{ units}^2}$$



7)

*washer method

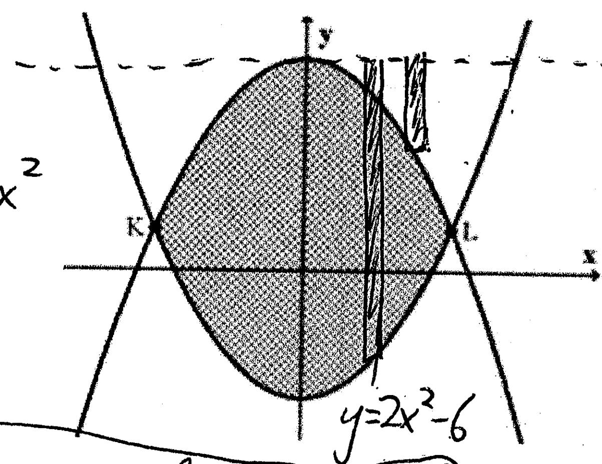
Find the volume of the solid created by revolving the shaded region (bounded by $y = 2x^2 - 6$ and $y = 10 - 2x^2$) about the AOR y = 10

$R(x) = 10 - (2x^2 - 6) = 10 - 2x^2 + 6 = 16 - 2x^2$

$r(x) = 10 - (10 - 2x^2) = 2x^2$

$$V = \pi \int_{-2}^{2} [16 - 2x^2]^2 - [2x^2]^2\,dx$$

$$= \boxed{682.667\pi \text{ or } \frac{2048}{3}\pi \text{ units}^3}$$



8)

The diagram shows the shaded region bounded by $y = x^3 - 1$, $y = \cancel{7}$, and $x = 0$.

a) Calculate the intersections

b) Calculate the shaded area. (Show work!)

bound: $x^3 - 1 = \cancel{7}$

$x^3 = 8$

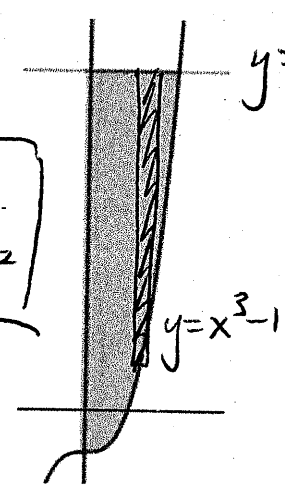
$\sqrt[3]{x^3} = \sqrt[3]{8}$

$x = \sqrt[3]{8} = 2$

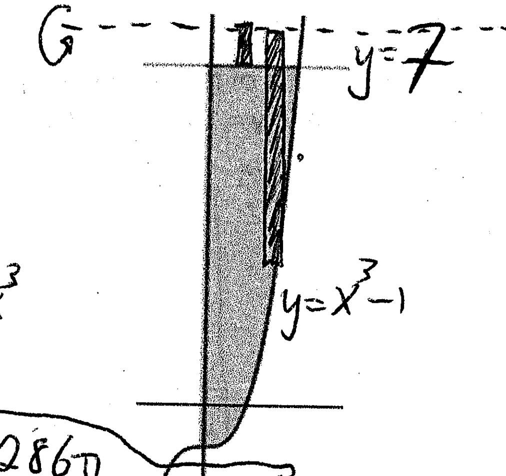
$$A = \int_0^2 7 - (x^3 - 1)\,dx$$

$$\int_0^2 7 - x^3 + 1\,dx$$

$$\boxed{A = 12 \text{ units}^2}$$



9) The diagram shows the shaded region bounded by $y = x^3 - 1$, $y = 7$ and $x = 0$. Find the volume of the solid created by revolving the shaded region about the AOR of $y = 8$. (Show Work!)



*intersection:

$x^3 - 1 = 7$

$x^3 = 8$

$x = 2$

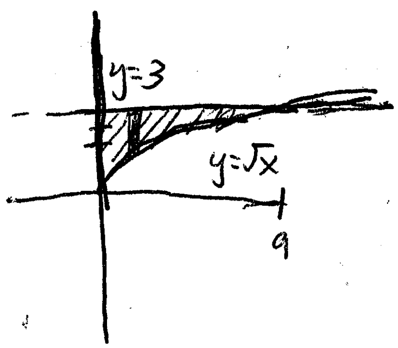
* Washer method

$R(x) = 8 - (x^3 - 1) = 8 - x^3 + 1 = 9 - x^3$

$r(x) = 8 - 7 = 1$

$V = \pi \int_0^2 [9 - x^3]^2 - [1]^2 \, dx \boxed{= 106.286\pi \text{ or } \frac{744}{7}\pi \text{ units}^3}$

10) Find the area of the region bounded by the following graphs. $y = \sqrt{x}$, $x = 0$, and $y = 3$ (Show Work!)



*intersection

$\sqrt{x} = 3$

$(\sqrt{x})^2 = (3)^2$

$x = 9$

$A = \int_0^9 3 - \sqrt{x} \, dx$

$\int 3 - x^{1/2} \, dx$

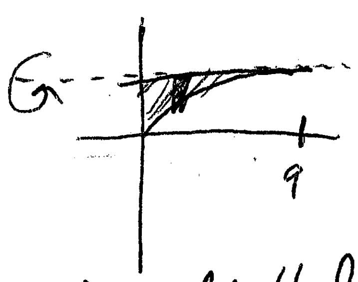
$3x - \frac{x^{3/2}}{3/2} = 3x - \frac{2}{3}x^{3/2} \Big]_0^9$

$= 27 - \frac{2}{3}(9)^{3/2} - (0 - 0)$

$= 27 - \frac{2}{3}(27)$

$= 27 - 18 = \boxed{9 \text{ units}^2}$

11) Find the volume of the solid created by revolving region created by $y = \sqrt{x}$, $x = 0$, and $y = 3$ about the AOR $y = 3$ (Show Work!)



Disc Method

$R(x) = 3 - \sqrt{x}$

$V = \pi \int_0^9 [3 - \sqrt{x}]^2 \, dx$

$\rightarrow (3 - \sqrt{x})(3 - \sqrt{x})$

$V = \pi \int_0^9 9 - 6\sqrt{x} + x \, dx$

$\pi \int_0^9 9 - 6x^{1/2} + x \, dx$

$9x - \frac{6x^{3/2}}{3/2} + \frac{x^2}{2}$

$9x - 4x^{3/2} + \frac{x^2}{2} \Big]_0^9$

$81 - 4(9)^{3/2} + \frac{81}{2} - (0 - 0 + 0)$

$\boxed{= 13.5\pi \text{ or } \frac{27}{2}\pi \text{ units}^3}$