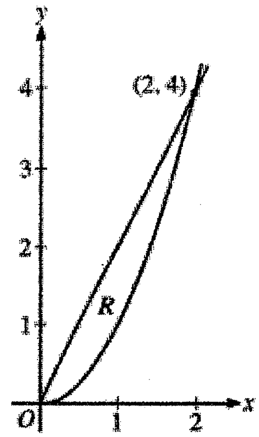
7.1-.74 BC Review WS 2

1)

(AP 2009-4) Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure.

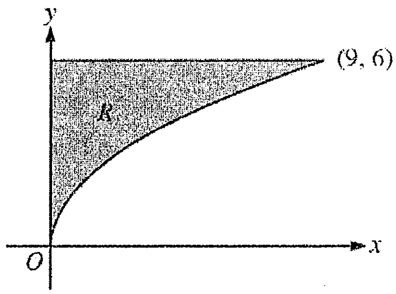


(a) Find the area of R.

b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line y = 7. (choose disc or washer)

c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the vertical line x = -3 . (set up using washer, then set up using shell)

2) (AP 2010-4) Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure below.



Region R is the base of a solid. For each y, where the cross section of the solid taken perpendicular to the y-axis is: (Write, but do not evaluate, and integral expression that gives the volume of this solid.)

a) a square	b) a semicircle
c) a right isosceles triangle with hypotenuse on the base	d) a rectangle with the height that is 1/3 times the length of the base

3. a) Find the length of the curve described by $y = \frac{2}{3}x^{3/2}$ from $x = 0$ to $x = 8$.

3b) Find the area of surface revolving curve about the given axis (set up integral expression)

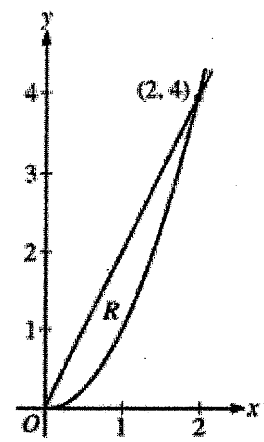
a) x-axis	b) y-axis
c) x = -3	d) y = 10

Key

7.1-.74 BC Review WS 2

1)

(AP 2009-4) Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure.

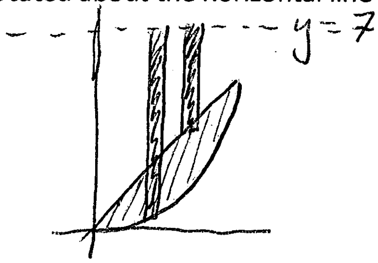


(a) Find the area of R.

$$\text{Area} = \int_0^2 2x - x^2\,dx = \frac{2x^2}{2} - \frac{x^3}{3} = x^2 - \frac{x^3}{3}\Big]_0^2 = 4 - \frac{8}{3} - 0 - 0$$

$$= \frac{12}{3} - \frac{8}{3} = \boxed{\frac{4}{3}}$$

b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line y = 7. (choose disc or washer)

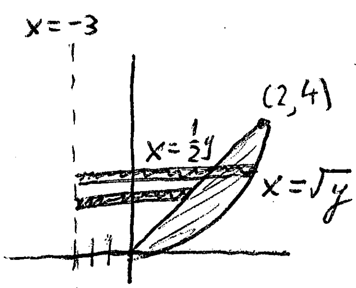


$R(x) = 7 - x^2$

$r(x) = 7 - 2x$

$$V = \pi \int_0^2 [7 - x^2]^2 - [7 - 2x]^2\,dx$$

c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the vertical line x = -3. (set up using washer, then set up using shell)



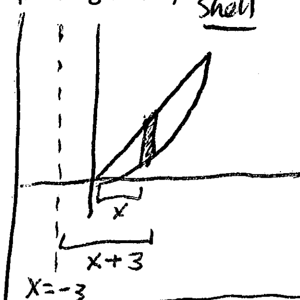
$$\boxed{V = \pi \int_0^4 \left[\sqrt{y} + 3\right]^2 - \left[\tfrac{1}{2}y + 3\right]^2 dy}$$

Washer

$R(y) = \sqrt{y} - (-3) = \sqrt{y} + 3$

$r(y) = \frac{1}{2}y - (-3) = \frac{1}{2}y + 3$

Shell



$$V = 2\pi \int_0^2 r \cdot h(x)\,dx$$

$h(x) = 2x - x^2$

$$\boxed{V = 2\pi \int_0^2 (x+3)\left[2x - x^2\right] dx}$$

(AP 2010-4) Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure below.

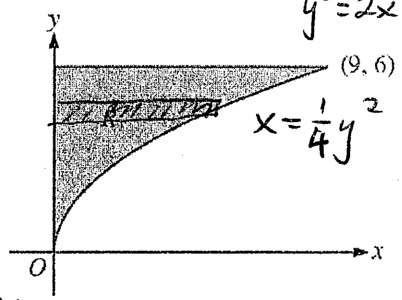
$y = 2\sqrt{x}$

$y^2 = (2\sqrt{x})^2$

$y^2 = 4x$

$\frac{1}{4}y^2 = x$

$y^2 = 2x$



$x = \frac{1}{4}y^2$

$y = 0$

base: $\frac{1}{4}y^2 - 0 = \frac{1}{4}y^2$

Region R is the base of a solid. For each y, where the cross section of the solid taken perpendicular to the y-axis is: (Write, but do not evaluate, and integral expression that gives the volume of this solid.)

a) a square

$$V = \int_0^6 \left[\frac{1}{4}y^2\right]^2 dy$$

b) a semicircle $\frac{\pi}{8}[\text{base}]^2$

$$V = \frac{\pi}{8}\int_0^6 \left[\frac{1}{4}y^2\right]^2 dy$$

c) a right isosceles triangle with hypotenuse on the base $A = \frac{1}{4}[\text{base}]^2$

$$V = \frac{1}{4}\int_0^6 \left[\frac{1}{4}y^2\right]^2 dy$$

d) a rectangle with the height that is 1/3 times the length of the base

$$\text{Area} = \left[\frac{1}{4}y^2\right] \cdot \frac{1}{3}\left[\frac{1}{4}y^2\right]$$

$$V = \frac{1}{48}\int_0^6 y^4\, dy$$

Find the length of the curve described by $y = \frac{2}{3}x^{3/2}$ from $x = 0$ to $x = 8$.

$$L = \int_0^8 \sqrt{1 + f'(x)^2}\, dx$$

3. a)

$$L = \int_0^8 \sqrt{1 + (\sqrt{x})^2}\, dx = \int_0^8 \sqrt{1+x}\, dx$$

$u = 1+x \quad \frac{du}{dx} = 1$

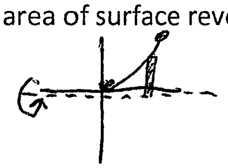
$y' = \frac{2}{3}\cdot\frac{3}{2}x^{1/2} = x^{1/2} = \sqrt{x}$

$$\int_0^8 u^{1/2} du = \frac{u^{3/2}}{3/2} = \frac{2}{3}(1+x)^{3/2}\Big]_0^8 = \frac{2}{3}(9)^{3/2} - \frac{2}{3}(1)^{3/2}$$

$$= \frac{2}{3}(27) - \frac{2}{3} = \boxed{\frac{52}{3}}$$

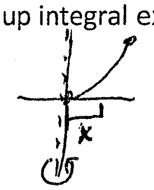
3b) Find the area of surface revolving curve about the given axis (set up integral expression)

a) x-axis



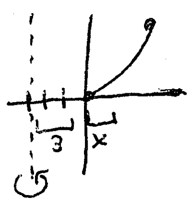
$$S = 2\pi\int_0^8 \frac{2}{3}x^{3/2}\sqrt{1+x}\, dx$$

b) y-axis



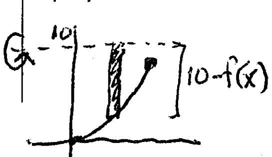
$$S = 2\pi\int_0^8 x\sqrt{1+x}\, dx$$

c) x = -3



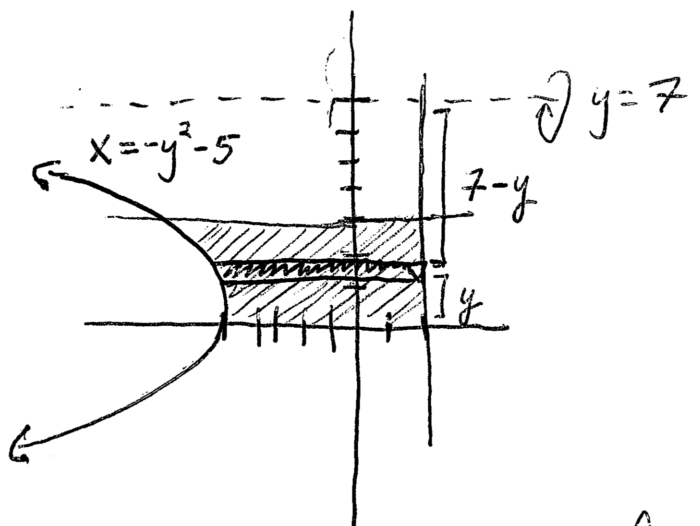
$$S = 2\pi\int_0^8 (x+3)\sqrt{1+x}\, dx$$

d) y = 10



$$S = 2\pi\int_0^8 \left(10 - \frac{2}{3}x^{3/2}\right)\sqrt{1+x}\, dx$$

4) Let R be the region bounded by $x+y^2+5=0$, $x=2$, $y=3$, and the x-axis rotated about the line $y=7$. (set up integral)



$h(y) = 2-(-y^2-5) = 2+y^2+5$

$h(y) = 7+y^2$

$r(y) = 7-y$

shell method: $V = 2\pi \int r(y) \cdot h(y)\, dy$

$$V = 2\pi \int_0^3 (7-y)(7+y^2)\, dy$$

$$V = 2\pi \int_0^3 49-7y+7y^2-y^3\, dy = \boxed{\frac{633}{2}\pi \text{ units}^3}$$