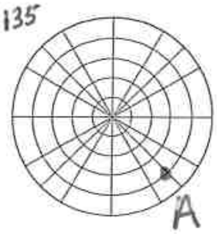
7.11: Test Review Date Key

First plot each point, given as polar coordinates. Then, determine 3 other coordinates for the same point. Use -360° ≤ θ ≤ 360° if in degrees, or use -2π ≤ θ ≤ 2π if in radians. ***No calculator**

1. A = (-4, 135°)

135



A

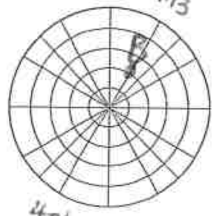
$(4, 315°)$

$(4, -45°)$

$(-4, -225°)$

2. B = $\left(2, \frac{\pi}{3}\right)$

π/3



4π/3

$\left(2, -\frac{5\pi}{3}\right)$

$\left(-2, \frac{4\pi}{3}\right)$

$\left(-2, -\frac{2\pi}{3}\right)$

Find the distance between the polar points. Use the polar method: $\sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$

r_1 θ_1 r_2 θ_2

3. (-6, 210°) and (4, -45°)

$\sqrt{6^2+4^2-(2\cdot 6\cdot 4\cos(-45-210))}$

$\sqrt{36+16+48\cos(-255)} = \sqrt{39.57668}$

$\boxed{\text{distance} = 6.291}$

4. $\left(1, \frac{2\pi}{3}\right)$ and $\left(-5, -\frac{7\pi}{6}\right)$

$\sqrt{1^2+5^2-\left(2\cdot 1\cdot -5\cos\left(-\frac{7\pi}{6}-\frac{2\pi}{3}\right)\right)}$

$\sqrt{1+25+10\cos\left(-\frac{11\pi}{6}\right)} = \sqrt{34.66025}$

$\boxed{\text{distance} = 5.887}$

Convert the given rectangular coordinates into polar coordinates, where 0 ≤ θ ≤ 2π. (r, θ)

5. (-3, 3) ***No calculator**

$r = \sqrt{3^2+3^2} = \sqrt{18} = 3\sqrt{2}$

$\theta = \tan^{-1}\left(\frac{3}{-3}\right) = -45° + 180 = 135°$

Q2 r, θ

$\theta = 135 \cdot \frac{\pi}{180} = \frac{3\pi}{4}$ $\left(3\sqrt{2}, \frac{3\pi}{4}\right)$

6. (-4√5, -2)

$r = \sqrt{(4\sqrt{5})^2+2^2} = \sqrt{84} = 2\sqrt{21}$

$\theta = \tan^{-1}\left(\frac{-2}{-4\sqrt{5}}\right) \rightarrow 0.219 + \pi = 3.362$

Q3 $\boxed{(2\sqrt{21}, 3.362 \text{ rad})}$

Convert the given polar coordinates into rectangular coordinates. $*(r\cos\theta, r\sin\theta)$

Q3 7. (14, 210°) ***No calculator**

$x = 14\cos 210 \rightarrow 14\left(-\frac{\sqrt{3}}{2}\right)$

$y = 14\sin 210 \rightarrow 14\left(-\frac{1}{2}\right)$

$\boxed{(-7\sqrt{3}, -7)}$

8. $\left(2\sqrt{3}, \frac{11\pi}{7}\right)$

$x = 2\sqrt{3}\cos\left(\frac{11\pi}{7}\right)$

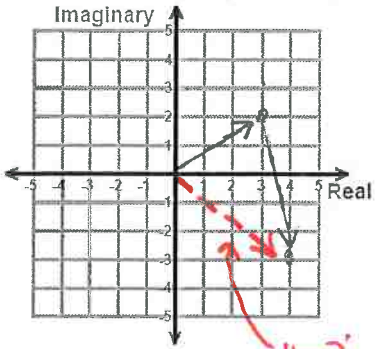
$y = 2\sqrt{3}\sin\left(\frac{11\pi}{7}\right)$

Radian Mode

$\boxed{(0.771, -3.377)}$

Simply each expression using geometric methods. ***No calculator**

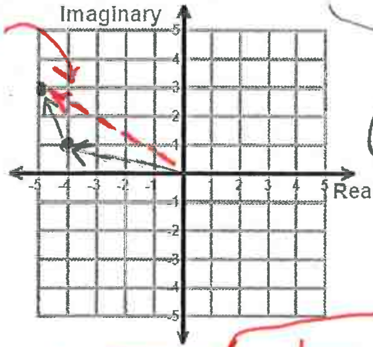
9. $(3+2i)+(1-5i)$ $\rightarrow 4-3i$



4-3i

10. $(-4+i)-(1-2i)$ $= -5+3i$

-5+3i



$(-4+i)+(-1+2i)$

$\boxed{* \, |z_1 - z_2|}$

Find the distance between the complex numbers. ***No calculator**

11. $(13+2i)$ and $(9-5i)$

$|4+7i| = \sqrt{4^2+7^2}$
$= \sqrt{16+49} = \boxed{\sqrt{65}}$

12. $(-8+5i)$ and $(-2-i)$

$|-6+6i| = \sqrt{6^2+6^2} = \sqrt{72} = \boxed{6\sqrt{2}}$

$\boxed{\frac{z_1+z_2}{2}}$

Find the midpoint between the complex numbers. ***No calculator**

13. $(13+2i)$ and $(9-5i)$

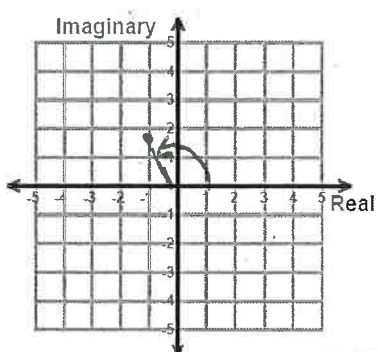
$\frac{13+9}{2} + \frac{2-5}{2}i \rightarrow \boxed{11-\frac{3}{2}i}$

14. $(-8+5i)$ and $(-2-i)$

$\frac{-8-2}{2} + \frac{5-1}{2}i \rightarrow \boxed{-5+2i}$

Graph each complex number, find its modulus (absolute value) and argument (direction), and then write in polar form, where $\boxed{0 \le \theta \le 2\pi.}$ ***No calculator**

15. $z = -1+\sqrt{3}i$



Angle(θ)

Modulus: $\sqrt{1^2+\sqrt{3}^2} = \boxed{2}$

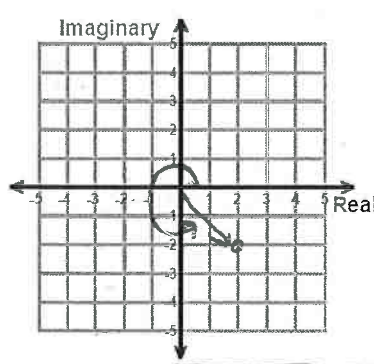
Argument: $\theta = \tan^{-1}\left(\frac{\sqrt{3}}{-1}\right)$

$\theta = -60 + 180 = 120°$

Q2 $\rightarrow$ $\boxed{\theta = \frac{2\pi}{3}}$

$\boxed{\text{Polar: } 2\,\text{cis}\left(\frac{2\pi}{3}\right)}$

16. $z = 2-2i$



Modulus: $\sqrt{2^2+2^2} = \boxed{2\sqrt{2}}$

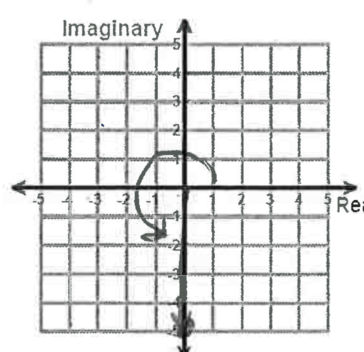
Argument: $\theta = \tan^{-1}\left(\frac{-2}{2}\right)$

$\theta = -45 + 360 = 315°$

$\boxed{\theta = 7\pi/4}$

$\boxed{\text{Polar: } 2\sqrt{2}\,\text{cis}\left(\frac{7\pi}{4}\right)}$

17. $z = -5i$ $\rightarrow 0-5i$



Modulus: $\sqrt{0^2+5^2} = \boxed{5}$

Argument: $\boxed{\frac{3\pi}{2}}$

$\boxed{\text{Polar: } 5\,\text{cis}\left(\frac{3\pi}{2}\right)}$

#15) Polar: $2\,cis\left(\frac{2\pi}{3}\right)$

#16) Polar: $2\sqrt{2}\,cis\left(\frac{7\pi}{4}\right)$

#17) Polar: $5\,cis\left(\frac{3\pi}{2}\right)$

18. Convert $z = -5 + 12i$ to polar form, where $0 \le \Theta \le 2\pi$.

$r = \sqrt{5^2 + 12^2} = 13$

Q2 → $\Theta = \tan^{-1}\left(\frac{12}{-5}\right) = -1.176 + \pi = 1.967$

$\boxed{13\,cis\,1.967\text{ rad}}$

19. Convert $z = 4\sqrt{3}(\cos 30° + i \sin 30°)$ to rectangular form. ***No calculator**

* distribute $4\sqrt{3}$ through parentheses

$4\sqrt{3}\cos 30 \to 4\sqrt{3}\left(\frac{\sqrt{3}}{2}\right) \to 2\sqrt{9} \to 2(3) = 6$

$4\sqrt{3}\sin 30 \to 4\sqrt{3}\left(\frac{1}{2}\right) \to 2\sqrt{3}$

$\boxed{6 + 2\sqrt{3}i}$

Simplify each expression using polar methods. Answer in polar form, where $0 \le \Theta \le 2\pi$.
***No calculator**

Given: $z_1 = 3\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$, $z_2 = 4\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$, $z_3 = 2\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

*multiply r, add Θ

20. $z_1 \cdot z_2$ $\quad 3 \cdot 4\,cis\left(\frac{4\pi}{3} + \frac{5\pi}{6}\right)$

$12\left[\cos\frac{13\pi}{6} + i\sin\frac{13\pi}{6}\right]$

$\boxed{12\,cis\,\pi/6}$ subtract 2π

21. $z_2 \cdot z_3$ $\quad 4 \cdot 2\,cis\left[\frac{5\pi}{6} + \frac{3\pi}{4}\right]$

$\boxed{8\,cis\left(\frac{19\pi}{12}\right)}$

$\frac{9\pi}{12} - \frac{10\pi}{12} = \frac{-\pi}{12}$

*divide r, subtract Θ

22. $\frac{z_1}{z_2}$ $\quad \frac{3}{4}\,cis\left(\frac{4\pi}{3} - \frac{5\pi}{6}\right)$

$\frac{3}{4}\,cis\left(\frac{3\pi}{6}\right) \to \boxed{\frac{3}{4}\,cis\,\frac{\pi}{2}}$

23. $\frac{z_3}{z_2}$ $\quad \frac{2}{4}\,cis\left(\frac{3\pi}{4} - \frac{5\pi}{6}\right)$

$\frac{1}{2}\,cis\left(\frac{-\pi}{12}\right) \to \boxed{\frac{1}{2}\,cis\left(\frac{23\pi}{12}\right)}$

Add 2π

*raise r to exponent multiply Θ

24. $(z_2)^4$

$4^4\left(\cos\left(4 \cdot \frac{5\pi}{6}\right) + i\sin\left(4 \cdot \frac{5\pi}{6}\right)\right)$

$4^4\,cis\left(\frac{10\pi}{3}\right) \to \boxed{4^4\,cis\left(\frac{4\pi}{3}\right)}$

subtract 2π

25. $(z_3)^3$

$2^3\left[\cos\left(3 \cdot \frac{3\pi}{4}\right) + i\sin\left(3 \cdot \frac{3\pi}{4}\right)\right]$

$8\,cis\left(\frac{9\pi}{4}\right)$ $\quad \boxed{8\,cis\left(\frac{\pi}{4}\right)}$

Subtract 2π

26. Find the cube roots of z_2.

$z_2 = 4\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

$z^{1/3} = 4^{1/3}\left[\cos\left(\frac{1}{3} \cdot \frac{5\pi}{6}\right) + i\sin\left(\frac{1}{3} \cdot \frac{5\pi}{6}\right)\right]$

① $\sqrt[3]{4}\left[\cos\frac{5\pi}{18} + i\sin\frac{5\pi}{18}\right]$

② $\sqrt[3]{4}\,cis\,\frac{17\pi}{18}$

③ $\sqrt[3]{4}\,cis\,\frac{29\pi}{18}$

$+\frac{2\pi}{3}$

*Add $\frac{2\pi}{n} \to \frac{2\pi}{3}$

27. Find the fourth roots of z_1

$z_1 = 3\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$

$z^{1/4} = \sqrt[4]{3}\left[\cos\left(\frac{1}{4} \cdot \frac{4\pi}{3}\right) + i\sin\left(\frac{1}{4} \cdot \frac{4\pi}{3}\right)\right] = \boxed{\sqrt[4]{3}\,cis\left(\frac{\pi}{3}\right)}$ ①

② $\sqrt[4]{3}\,cis\,\frac{5\pi}{6}$

③ $\sqrt[4]{3}\,cis\,8\pi/6$

④ $\sqrt[4]{3}\,cis\,\frac{11\pi}{6}$

$+\frac{\pi}{2}$ or $\frac{3\pi}{6}$

*Add $\frac{2\pi}{n} \to \frac{2\pi}{4}$

$+\pi/2$