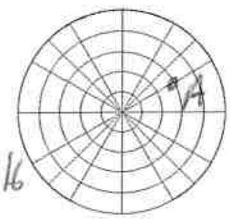
Updated Key

7.12b: Test Review **WS #2** ***No calculator***

First plot each point, given as polar coordinates. Then, determine 3 other coordinates for the same point. Use -360° ≤ θ ≤ 360° if in degrees, or use -2π ≤ θ ≤ 2π if in radians.

1. A = $(-3, \frac{7\pi}{6})$

$7\pi/6$



$(3, \pi/6)$

$(3, -11\pi/6)$

$(-3, -5\pi/6)$

$7\pi/6 - \frac{12\pi}{6} = \frac{-5\pi}{6}$

Convert the given rectangular coordinates into polar coordinates, where 0 ≤ θ ≤ 2π.

2. $(-2, 2\sqrt{3})$

x y

$r = \sqrt{2^2 + (2\sqrt{3})^2} = \sqrt{4+12} = \sqrt{16} = 4$

$\theta = \tan^{-1}\left(\frac{2\sqrt{3}}{-2}\right)$

$\theta = \tan^{-1}(-\sqrt{3})$

$\theta = \tan^{-1}\left(-\frac{\sqrt{3}}{1}\right) \rightarrow \frac{\pi}{3}$

in Q2 is $\frac{2\pi}{3}$

$\boxed{\left(4, \frac{2\pi}{3}\right)}$

Convert the given polar coordinates into rectangular coordinates.

3. $\left(2\sqrt{3}, \frac{11\pi}{6}\right)$ r θ

$*(r\cos\theta, r\sin\theta)$

$x = 2\sqrt{3}\cos\left(\frac{11\pi}{6}\right) \rightarrow 2\sqrt{3} \cdot \frac{\sqrt{3}}{2} \rightarrow 3$

$y = 2\sqrt{3}\sin\left(\frac{11\pi}{6}\right) \rightarrow 2\sqrt{3} \cdot -\frac{1}{2} \rightarrow -\sqrt{3}$

$\boxed{(3, -\sqrt{3})}$

Find the distance between the complex numbers. ***No calculator*** $*|z_1 - z_2|$

4. $(-4 + 6)$ and $(1 + 7i)$

$-4-1 \quad 6-7$

$|-5-1i| = \sqrt{5^2+1^2} = \boxed{\sqrt{26}}$

Find the midpoint between the complex numbers. ***No calculator*** $* \frac{z_1+z_2}{2}$

5. $(14 - 3i)$ and $(3 - 5i)$

$\frac{14+3}{2} \quad \frac{-3-5}{2}$

$\boxed{\frac{17}{2} - 4i}$

Graph each complex number, find its modulus (absolute value) and argument (direction), and then write in polar form, where 0 ≤ θ ≤ 2π. ***No calculator***

6. $z = -\sqrt{2} - \sqrt{2}i$

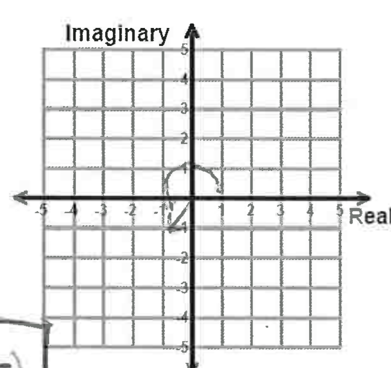
Q3

180

Modulus: $\sqrt{\sqrt{2}^2 + \sqrt{2}^2} = \sqrt{2+2} = \sqrt{4} = 2$

Argument: $\theta = \tan^{-1}\left(\frac{-\sqrt{2}}{-\sqrt{2}}\right) \rightarrow \tan^{-1}(1) \rightarrow {}^{+}45°$

$\theta = 225$ or $\frac{5\pi}{4}$



Polar:

$2\left[\cos\left(\frac{5\pi}{4}\right) + i\sin\left(\frac{5\pi}{4}\right)\right]$ or $\boxed{2\,\text{cis}\left(\frac{5\pi}{4}\right)}$

polar form

7. Convert z = -2 + $2\sqrt{3}i$ to polar form, where $0 \le \theta \le 2\pi$.

$r=\sqrt{2^2+(2\sqrt{3})^2}=\sqrt{4+12}=\sqrt{16}=4$

$\theta=\tan^{-1}\left(\frac{-2\sqrt{3}}{2}\right)\to\tan^{-1}\left(\frac{-\sqrt{3}}{1}\right)\to 60° \to Q2 \to 180-60=120 \to \frac{2\pi}{3}$

$\boxed{\left(4,\frac{2\pi}{3}\right)} \to \boxed{4\text{cis}\frac{2\pi}{3}}$

8. Convert $z = 4\sqrt{3}(\cos 240° + i\sin 240°)$ to rectangular form. ***No calculator**

$x=4\sqrt{3}\cos 240 \to 4\sqrt{3}\cdot\left(-\frac{1}{2}\right)\to -2\sqrt{3}$

$y=4\sqrt{3}\sin 240 \to 4\sqrt{3}\left(-\frac{\sqrt{3}}{2}\right)\to -6$

$\boxed{-2\sqrt{3}-6i}$

Simplify each expression using polar methods. Answer in polar form, where $0 \le \theta \le 2\pi$.
***No calculator**

Given: $z_1 = 2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$, $z_2 = 3\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$, $z_3 = 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

subtract 2π

9. $z_1 \cdot z_2$ $\quad 6\text{cis}\left(\frac{5\pi}{3}+\frac{7\pi}{6}\right)\to\frac{10\pi}{6}+\frac{7\pi}{6}\to\frac{17\pi}{6}-\frac{12\pi}{6}\to\frac{5\pi}{6}$

$\boxed{6\text{cis}\left(\frac{5\pi}{6}\right)}$

10. $\frac{z_1}{z_3}$ $\quad \frac{2}{2}\text{cis}\left(\frac{5\pi}{3}-\frac{\pi}{4}\right)\to\frac{20\pi}{12}-\frac{3\pi}{12}\to\frac{17\pi}{12}$

$\boxed{1\text{cis}\left(\frac{17\pi}{12}\right)}$

11. $(z_3)^3$ $\quad 2^3\left[\cos\left(3\cdot\frac{\pi}{4}\right)+i\sin\left(3\cdot\frac{\pi}{4}\right)\right]$

$\boxed{8\text{cis}\left(\frac{3\pi}{4}\right)}$

12. Find the cube roots of $\sqrt{3}-i$.

$*r=\sqrt{\sqrt{3}^2+1^2}=\sqrt{3+1}=\sqrt{4}=2$

$\theta=\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)\to 30° \to 360-30=330° \to {}^{11\pi}/_{6}$

$z=2\text{cis}\left(\frac{11\pi}{6}\right)$

$z^{1/3}=\sqrt[3]{2}\text{ cis}\left[\frac{1}{3}\cdot\frac{11\pi}{6}\right]\to$ 1 $\boxed{\sqrt[3]{2}\text{cis}\frac{11\pi}{18}}$

$*$interval is $\frac{2\pi}{3}\to$ Add $\frac{12\pi}{18}$

2 $\sqrt[3]{2}\text{ cis}\frac{23\pi}{18}$

3 $\sqrt[3]{2}\text{ cis}\frac{35\pi}{18}$

13. Find the fourth roots of z_3

$z^{1/4}=\sqrt[4]{2}\left[\cos\left(\frac{1}{4}\cdot\frac{\pi}{4}\right)+i\sin\left(\frac{1}{4}\cdot\frac{\pi}{4}\right)\right]$

$*$interval is $\frac{2\pi}{4}\to\frac{\pi}{2}\to$ Add $\frac{8\pi}{16}$

1 $\sqrt[4]{2}\text{ cis}\left(\frac{\pi}{16}\right)$

2 $\sqrt[4]{2}\text{ cis}\left(\frac{9\pi}{16}\right)$

3 $\sqrt[4]{2}\text{ cis}\left(\frac{17\pi}{16}\right)$

4 $\sqrt[4]{2}\text{cis}\left(\frac{25\pi}{16}\right)$