Non-AP Calculus **7.1-7.2 Area/Volume Review Worksheet #3**

$$\textbf{Area} = \int_{x_1}^{x_2} (Top\ graph - Bottom\ graph)\ dx$$

Radius $[R(x)] =$ *distance from the AOR (Axis of Revolution) to the graph curve*

$$\textbf{Disc Method}:\ Volume = \pi \int_{x_1}^{x_2} [R(x)]^2 dx$$

Radius $[R(x)] =$ *distance from the AOR (Axis of Revolution) to the* ***further*** *graph curve*

radius $[r(x)] =$ *distance from the AOR (Axis of Revolution) to the* ***closer*** *graph curve*

$$\textbf{Washer Method}:\ Volume = \pi \int_{x_1}^{x_2} [R(x)]^2 - [r(x)]^2 dx$$

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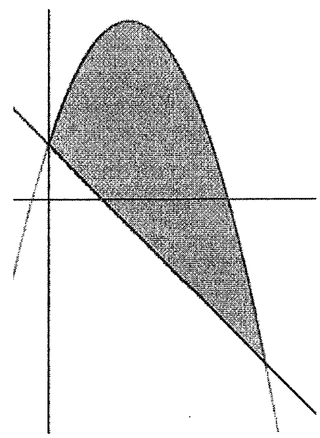
1)

The diagram shows the curve $y = -x^2 + 3x + 1$ and the line $y = -x + 1$. Calculate the shaded area. (Show work!)



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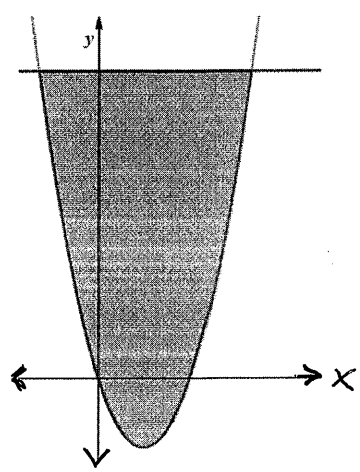
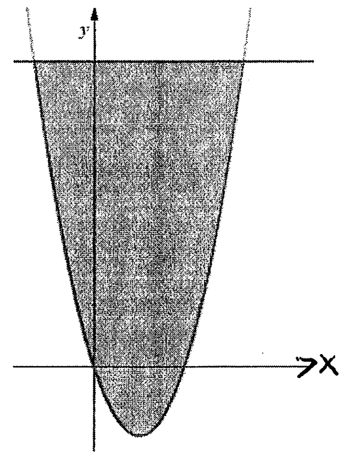
2)

Find the volume of the solid created by revolving the curve $y = x^2 - 3x$ and the line $y = 10$ about the AOR line $y = 10$



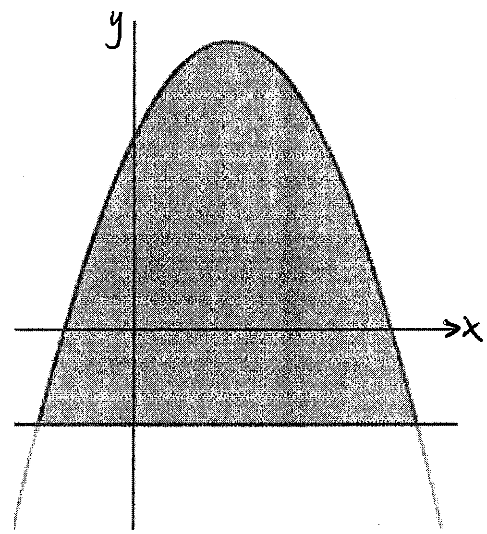
3)

Find the volume of the solid created by revolving the curve $y = x^2 - 3x$ and the line $y = 10$ about the AOR line $y = -3$



4)

Find the volume of the solid created by revolving the curve $y = -x^2 + 2x + 2$ and the line $y = -1$ about the AOR line $y = 3$



5) Find the volume of the solid created by revolving region created by $y = \sqrt{x}, \; x = 0, \; and \; y = 4$ about the AOR y = 4 (Show Work!)

Non-AP Calculus **7.1-7.2 Area/Volume Review Worksheet #3**

Key

$$Area = \int_{x_1}^{x_2} (Top\ graph - Bottom\ graph)\, dx$$

Radius $[R(x)]$ = *distance from the AOR (Axis of Revolution) to the graph curve*

$$\textbf{Disc Method}: Volume = \pi \int_{x_1}^{x_2} [R(x)]^2 dx$$

Radius $[R(x)]$ = *distance from the AOR (Axis of Revolution) to the* ***further*** *graph curve*

radius $[r(x)]$ = *distance from the AOR (Axis of Revolution) to the* ***closer*** *graph curve*

$$\textbf{Washer Method}: Volume = \pi \int_{x_1}^{x_2} [R(x)]^2 - [r(x)]^2 dx$$

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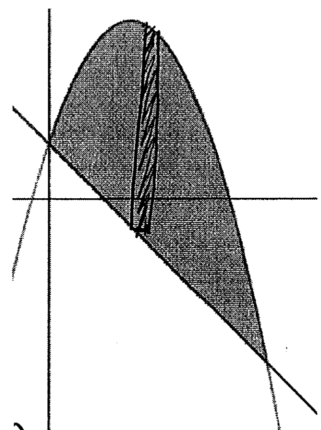
1)

The diagram shows the curve $y = -x^2 + 3x + 1$ and the line $y = -x+1$. Calculate the shaded area. (Show work!)



*intersections:

$-x+1 = -x^2+3x+1$

$x^2-4x=0$

$x(x-4)=0$

$x=0,4$

$$A = \int_0^4 -x^2+3x+1-(-x+1)\,dx$$

$$\int -x^2+3x+1+x-1\,dx \rightarrow \int_0^4 -x^2+4x\,dx$$

$$\left.\frac{-x^3}{3}+\frac{4x^2}{2}\right]_0^4 = -\frac{4^3}{3}+2(4)^2-\left(-\frac{0}{3}+\frac{0}{2}\right)$$

$$= \boxed{\frac{32}{3}\text{ units}^2}$$

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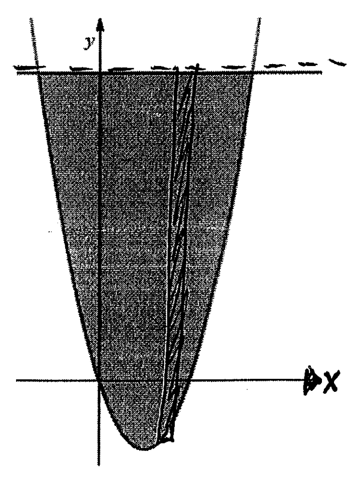
2)

Find the volume of the solid created by revolving the curve $y = x^2 - 3x$ and the line $y = 10$ about the AOR line $y = 10$

AOR $y=10$



*intersection:

$x^2-3x=10$

$x^2-3x-10=0$

$(x-5)(x+2)=0$

$x=5,-2$

*Disc Method

$R(x)=10-(x^2-3x)$

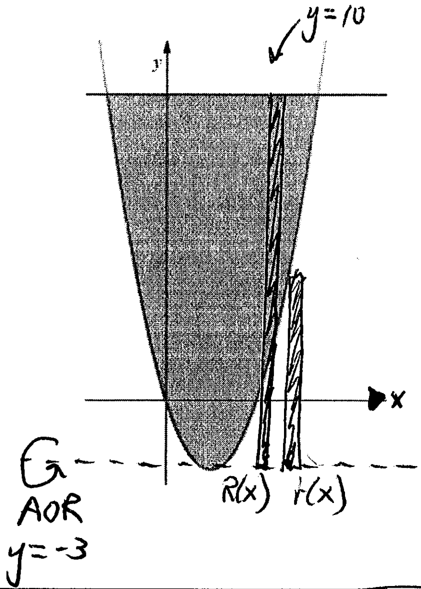
$R(x)=10-x^2+3x$

$$V = \pi\int_{-2}^{5} (-x^2+3x+10)^2\,dx$$

$$\boxed{V = 560.233\pi \text{ or } \frac{16807}{30}\pi \text{ units}^3}$$

3)

Find the volume of the solid created by revolving the curve $y = x^2 - 3x$ and the line $y = 10$ about the AOR line $y = -3$



*intersection:

$x^2-3x=10$

$x^2-3x-10=0$

$(x-5)(x+2)=0$

$x=5, -2$

washer method:

$R(x) = 10-(-3) = 13$

$r(x) = x^2-3x-(-3) = x^2-3x+3$

$$V = \pi \int_{-2}^{5} [13]^2 - [x^2-3x+3]^2 \, dx$$

$$\boxed{V = 926.1\pi \text{ or } \frac{9261}{10}\pi \text{ units}^3}$$

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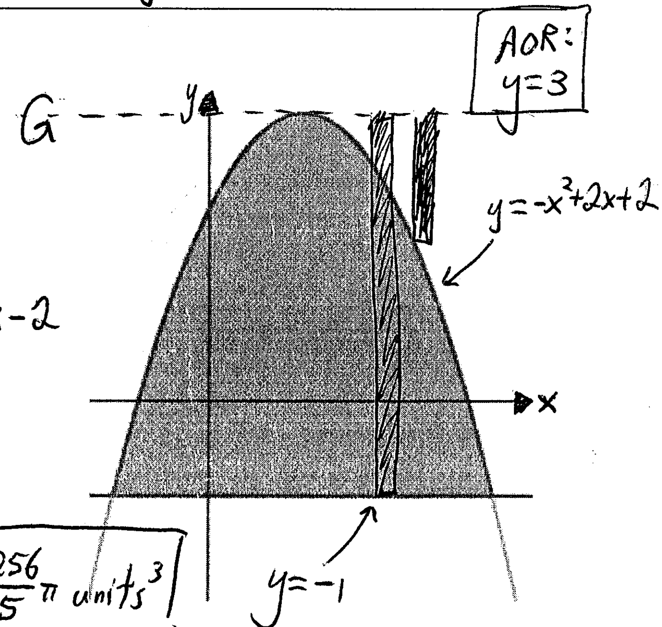
4)

Find the volume of the solid created by revolving the curve $y = -x^2 + 2x + 2$ and the line $y = -1$ about the AOR line $y = 3$



*Washer Method

*intersections:

$-1 = -x^2+2x+2$

$x^2-2x-3=0$

$(x-3)(x+1)=0$

$x=3, -1$

$R(x) = 3-(-1) = 4$

$r(x) = 3-(-x^2+2x+2) = 3+x^2-2x-2$

$= x^2-2x+1$

$$V = \pi \int_{-1}^{3} [4]^2 - [x^2-2x+1]^2 \, dx$$

$$\boxed{V = 51.2\pi \text{ or } \frac{256}{5}\pi \text{ units}^3}$$

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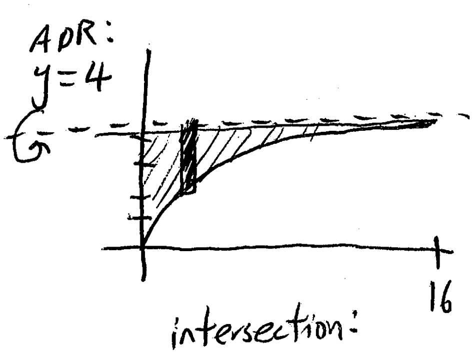
5) Find the volume of the solid created by revolving region created by $y = \sqrt{x}$, $x = 0$, and $y = 4$ about the AOR $y = 4$ (Show Work!)



intersection:

$\sqrt{x} = 4$

$(\sqrt{x})^2 = (4)^2$

$x = 16$

*Disc Method:

$R(x) = 4 - \sqrt{x}$

$$V = \pi \int_0^{16} [4-\sqrt{x}]^2 \, dx$$

$$V = \pi \int (4-\sqrt{x})(4-\sqrt{x}) \, dx$$

$$V = \pi \int 16 - 8\sqrt{x} + x \, dx$$

$$V = \pi \int_0^{16} 16 - 8x^{1/2} + x \, dx$$

$$= 16x - \frac{8x^{3/2}}{3/2} + \frac{x^2}{2} \Big]_0^{16}$$

$$= 16(16) - \tfrac{2}{3} \cdot 8(16)^{3/2} + \frac{16^2}{2} - (0 - 0 + 0)$$

$$= \boxed{42.667\pi \text{ units}^3}$$