Non-AP Calculus 7.1-7.2 Area/Volume Review Worksheet #4

$$\textbf{Area} = \int_{x_1}^{x_2} (Top\ graph - Bottom\ graph)\, dx$$

$Radius\ [R(x)] = distance\ from\ the\ AOR\ (Axis\ of\ Revolution)\ to\ the\ graph\ curve$

$$\textbf{Disc Method}:\ Volume = \pi \int_{x_1}^{x_2} [R(x)]^2 dx$$

$Radius\ [R(x)] = distance\ from\ the\ AOR\ (Axis\ of\ Revolution)\ to\ the\ \textbf{further}\ graph\ curve$

$radius\ [r(x)] = distance\ from\ the\ AOR\ (Axis\ of\ Revolution)\ to\ the\ \textbf{closer}\ graph\ curve$

$$\textbf{Washer Method}:\ Volume = \pi \int_{x_1}^{x_2} [R(x)]^2 - [r(x)]^2 dx$$

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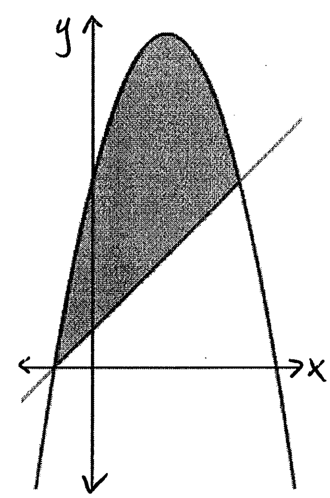
1)

The diagram shows the curve $y = -x^2 + 4x + 5$ and the line $y = x + 1$. Calculate the shaded area. (Show work!)



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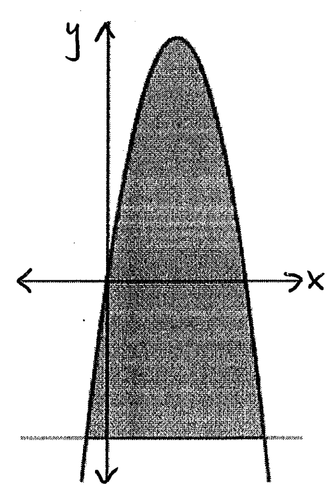
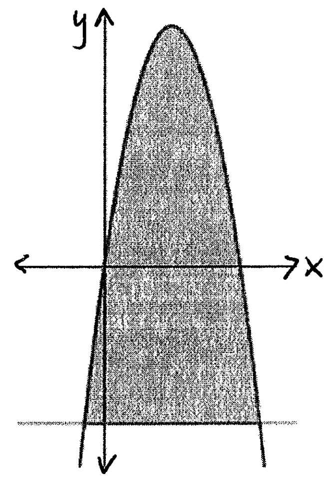
2)

Find the volume of the solid created by revolving the curve $y = -x^2 + 7x$ and the line $y = -8$ about the AOR line $y = -8$



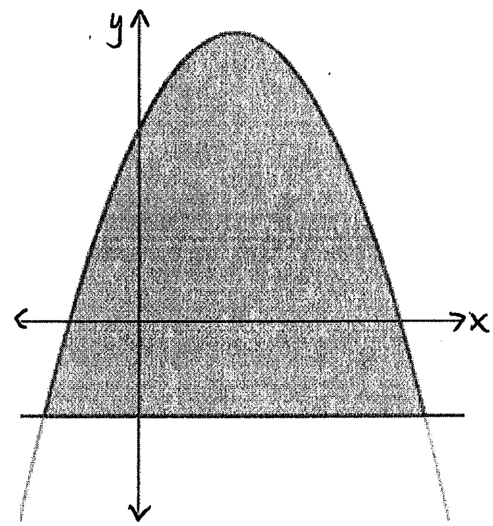
3)

Find the volume of the solid created by revolving the curve $y = -x^2 + 7x$ and the line $y = -8$ about the AOR line $y = -10$



4)

Find the volume of the solid created by revolving the curve $y = -x^2 + 2x + 2$ and the line $y = -1$ about the AOR line $y = 5$



5) Find the volume of the solid created by revolving region created by $y = \sqrt{x}$, $y = -2$, *and* $x = 5$ about the AOR y = -2 (Show Work!)

Non-AP Calculus 7.1-7.2 Area/Volume Review Worksheet #4

Key

$$\textit{Area} = \int_{x_1}^{x_2} (\textit{Top graph} - \textit{Bottom graph})\, dx$$

$\textit{Radius } [R(x)] = \textit{distance from the AOR (Axis of Revolution) to the graph curve}$

$$\textbf{Disc Method}:\ \textit{Volume} = \pi \int_{x_1}^{x_2} [R(x)]^2 dx$$

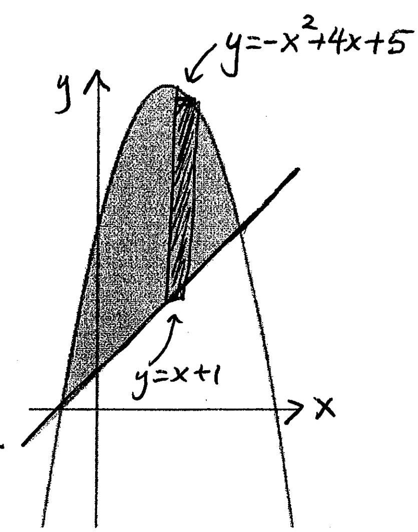
$\textit{Radius } [R(x)] = \textit{distance from the AOR (Axis of Revolution) to the } \textbf{further} \textit{ graph curve}$

$\textit{radius } [r(x)] = \textit{distance from the AOR (Axis of Revolution) to the } \textbf{closer} \textit{ graph curve}$

$$\textbf{Washer Method}: \textit{Volume} = \pi \int_{x_1}^{x_2} [R(x)]^2 - [r(x)]^2 dx$$

1)

The diagram shows the curve $y = -x^2 + 4x + 5$ and the line $y = x + 1$. Calculate the shaded area. (Show work!)



*intersections:

$x + 1 = -x^2 + 4x + 5$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4, -1$

$$\text{Area} = \int_{-1}^{4} -x^2 + 4x + 5 - (x+1)\, dx$$

$$= \int_{-1}^{4} -x^2 + 4x + 5 - x - 1\, dx$$

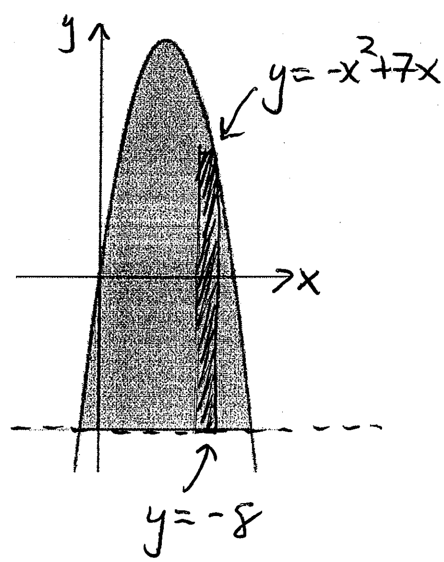
$$= \int_{-1}^{4} -x^2 + 3x + 4\, dx = \left. -\frac{x^3}{3} + \frac{3x^2}{2} + 4x \right]_{-1}^{4}$$

$$= -\frac{4^3}{3} + \frac{3(4)^2}{2} + 4(4) - \left(-\frac{(-1)^3}{3} + \frac{3}{2}(-1)^2 - 4\right) = \boxed{20.833 \text{ or } \frac{125}{6} \text{ units}^2}$$

2)

Find the volume of the solid created by revolving the curve $y = -x^2 + 7x$ and the line $y = -8$ about the AOR line $y = -8$

*Disc Method



*intersection:

$-8 = -x^2 + 7x$

$x^2 - 7x - 8 = 0$

$(x-8)(x+1) = 0$

$x = 8, -1$

$R(x) = -x^2 + 7x - (-8) = -x^2 + 7x + 8$

$$V = \pi \int_{-1}^{8} [-x^2 + 7x + 8]^2 dx$$

$$\boxed{V = 1968.3\pi \text{ units}^3}$$

3)

Find the volume of the solid created by revolving the curve y = – x² +7x and the line y = – 8 about the AOR line y = – 10

*washer method

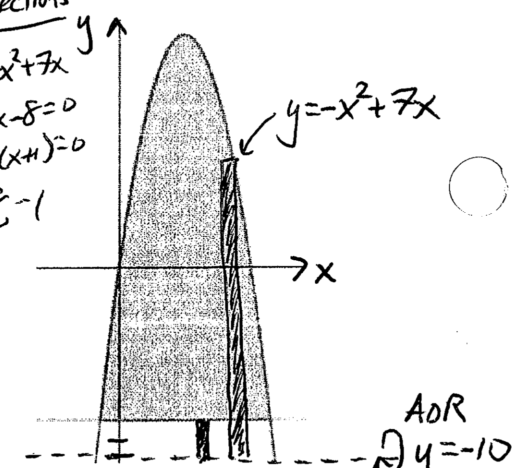
intersections:

$-8 = -x^2+7x$

$x^2-7x-8=0$

$(x-8)(x+1)=0$

$x=8,-1$



$$R(x) = -x^2+7x-(-10) = -x^2+7x+10$$

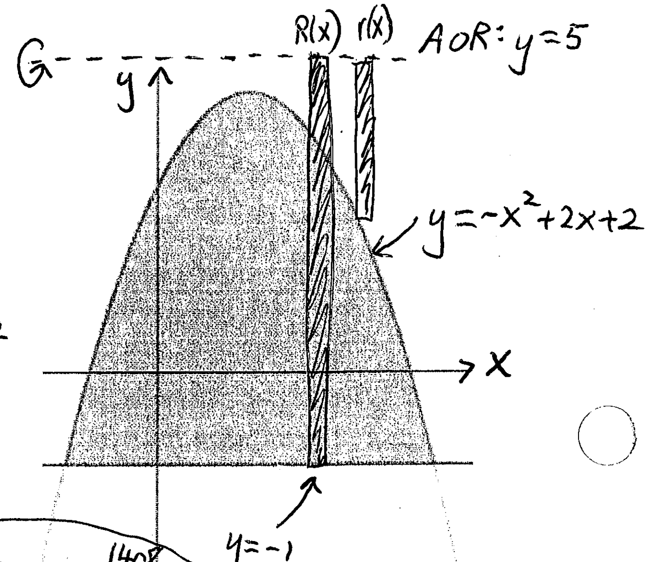
$$r(x) = -8-(-10) = 2$$

$$V = \pi\int_{-1}^{8} [-x^2+7x+10]^2 - [2]^2\,dx$$

$$\boxed{V = 2454.3\pi \text{ units}^3}$$

4)

Find the volume of the solid created by revolving the curve y = –x² +2x+2 and the line y = –1 about the AOR line y = 5



intersection:

$-1 = -x^2+2x+2$

$x^2-2x-3=0$

$(x-3)(x+1)=0$

$x=3,-1$

Washer method:

$R(x) = 5-(-1) = 6$

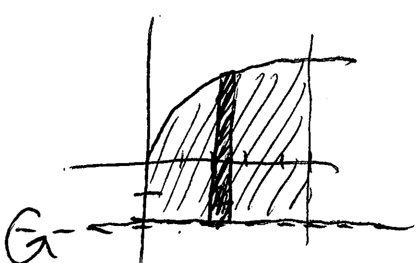
$r(x) = 5-(-x^2+2x+2) = 5+x^2-2x-2$

$= 3+x^2-2x$

$$V = \pi\int_{-1}^{3} [6]^2 - [x^2-2x+3]^2\,dx$$

$$\boxed{V = 93.867\pi \text{ or } \frac{1408}{15}\pi \text{ units}^3}$$

5) Find the volume of the solid created by revolving region created by $y = \sqrt{x}$, $y = -2$, *and* $x = 5$ about the AOR y = -2 (Show Work!)



bounds: $x=0, x=5$

*Disc Method

$R(x) = \sqrt{x}-(-2) = \sqrt{x}+2$

$$V = \pi\int_0^5 [\sqrt{x}+2]^2\,dx$$

$$\int (\sqrt{x}+2)(\sqrt{x}+2)\,dx$$

$$\int x + 2\sqrt{x} + 2\sqrt{x} + 4\,dx$$

$$\int x + 4\sqrt{x} + 4\,dx$$

$$\pi\int_0^5 x + 4x^{1/2} + 4\,dx$$

$$\frac{x^2}{2} + \frac{4x^{3/2}}{3/2} + 4x$$

$$\left.\frac{x^2}{2} + \frac{8}{3}x^{3/2} + 4x\right]_0^5$$

$$\frac{5^2}{2} + \frac{8}{3}(5)^{3/2} + 4(5) - (0+0+0)$$

$$\boxed{V = 62.314\pi \text{ units}^3 \text{ or } \frac{351390}{5639}\pi \text{ units}^3}$$