Key (25)

7.1 AP Practice Problems

1. Identify the differential equation for which $y = \pi x + \sin^3 x$ is a solution.

(A) $\frac{dy}{dx} = \pi x + 3\sin^2 x \cos x$

$\boxed{\text{(B)}}$ $\frac{dy}{dx} = \pi + 3\sin^2 x \cos x$

(C) $\frac{dy}{dx} = -3\sin^2 x \cos x$

(D) $\frac{dy}{dx} = \pi + 3(\sin x)^2$

$y = \pi x + (\sin x)^3$

$y' = \pi + 3(\sin x)^2 \cos x$

$\boxed{y' = \pi + 3\sin^2 x \cos x}$

chain Rule

out: $(\ )^3$

in: $\sin x$

2. Identify the differential equation for which $y = e^{3x-4}$ is a solution.

$\boxed{\text{(A)}}$ $y' = 3e^{3x-4}$ (B) $y' = 3xe^{3x-4}$

(C) $y' = \frac{1}{3}e^{3x-4}$ (D) $y' = \frac{1}{3x-4}e^{3x-4}$

$y' = e^{3x-4} \cdot 3$

$\boxed{y' = 3e^{3x-4}}$

3. The general solution to the differential equation $y' = (x-3)^2(2x+1)$ is

(A) $y = \left(\frac{x-3}{3}\right)^3 (2x+4) + (x-3)\left(\frac{2x+1}{2}\right)^2 + C$

(B) $y = 6x^2 - 22x + 12 + C$

(C) $y = 2x^4 - 11x^3 + 12x^2 + C$

$\boxed{\text{(D)}}$ $y = \frac{1}{2}x^4 - \frac{11}{3}x^3 + 6x^2 + 9x + C$

$\int 1\,dy = \int (x-3)^2(2x+1)\,dx$

$= \int (2x+1)(x^2-6x+9)\,dx$

$\int 2x^3 - 12x^2 + 18x + x^2 - 6x + 9\,dx$

$\int 2x^3 - 11x^2 + 12x + 9\,dx$

$\frac{2x^4}{4} - \frac{11x^3}{3} + \frac{12x^2}{2} + 9x + C$

$\boxed{y = \frac{x^4}{2} - \frac{11}{3}x^3 + 6x^2 + 9x + C}$

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PAGE 542 4. The particular solution of the differential equation $\frac{dy}{dx} = x\sqrt[3]{x^2-1}$ with the initial condition, if $x = 3$, then $y = 2$ is

(A) $y = \frac{3}{4}(x^2-1)^{4/3} - 10$ $\boxed{\text{(B)}}$ $y = \frac{3}{8}(x^2-1)^{4/3} - 4$

(C) $y = \frac{3}{8}(x^2-1)^{4/3} + 6$ (D) $y = \frac{3}{4}(x^2-1)^{4/3} + 14$

$y = \int x(x^2-1)^{1/3}\,dx$

$u = x^2-1$ | $dx = \frac{du}{2x}$

$\frac{du}{dx} = 2x$

$\int \cancel{x} \cdot u^{1/3} \cdot \frac{du}{2\cancel{x}}$

$\frac{1}{2}\int u^{1/3}\,du$

$\frac{1}{2} \cdot \frac{u^{4/3}}{4/3} + C$

$y = \frac{1}{2}\cdot\frac{3}{4}(x^2-1)^{4/3} + C$

$y = \frac{3}{8}(x^2-1)^{4/3} + C$

* plug in $y(3)=2$ to solve for C

$2 = \frac{3}{8}(3^2-1)^{4/3} + C$

$2 = \frac{3}{8}(8)^{4/3} + C$

$2 = \frac{3}{8}(16) + C$

$2 = 6 + C$

$-4 = C$

$\boxed{y = \frac{3}{8}(x^2-1)^{4/3} + 4}$

7.2 AP Practice Problems

1. Find the solution of the differential equation $\frac{dy}{dx} = \frac{\cos x}{3y^2}$, with the boundary condition $y\left(\frac{\pi}{6}\right) = 1$.

(A) $y^3 = \sin x - \frac{1}{2}$ (B) $y = \sin x + \frac{1}{2}$

$\boxed{\text{(C)}}$ $y^3 = \sin x + \frac{1}{2}$ (D) $y^3 = \sin x + \frac{\sqrt{3}}{2}$

$3y^2\,dy = \cos x\,dx$

$y^2\,dy = \frac{1}{3}\cos x\,dx$

$\int y^2\,dy = \frac{1}{3}\int \cos x\,dx$

$\frac{y^3}{3} + C = \frac{1}{3}\sin x + C$

$\left[\frac{y^3}{3} = \frac{1}{3}\sin x + C\right](3)$

$y^3 = \sin x + C \leftarrow y\left(\frac{\pi}{6}\right) = 1$

$1 = \sin(\pi/6) + C$

$1 = \frac{1}{2} + C$

$\frac{1}{2} = C$

$\boxed{y^3 = \sin x + \frac{1}{2}}$

2. Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{x}{y}$, with the initial condition $y(0) = 1$?

$\boxed{\text{(A)}}$ $y = \sqrt{x^2+1}$ (B) $y = x^2 + 1$

(C) $y = \pm\sqrt{x^2+1}$ (D) $y = -\sqrt{x^2+1}$

$y\,dy = x\,dx$

$(2)\left[\frac{y^2}{2} = \frac{x^2}{2} + C\right]$

$y^2 = x^2 + C \leftarrow$ plug in (0,1)

$1^2 = 0^2 + C$

$1 = C$

$y^2 = x^2 + 1$

$y = \pm\sqrt{x^2+1}$

y-value is positive

$\boxed{y = \sqrt{x^2+1}}$