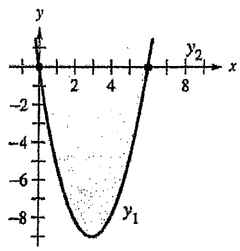
$$Area = \int_{x_1}^{x_2} (Top\ graph - Bottom\ graph)\ dx$$

Writing a Definite Integral **In Exercises 1–6, set up the definite integral that gives the area of the region.**

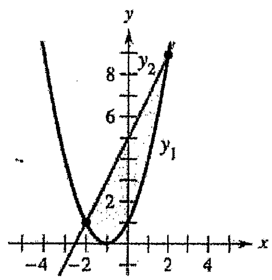
1. $y_1 = x^2 - 6x$

   $y_2 = 0$



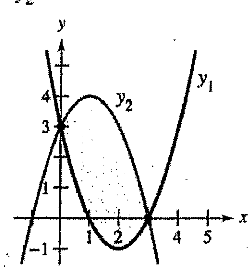
2. $y_1 = x^2 + 2x + 1$

   $y_2 = 2x + 5$



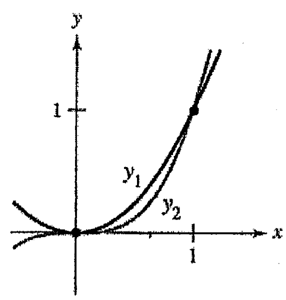
3. $y_1 = x^2 - 4x + 3$

   $y_2 = -x^2 + 2x + 3$



4. $y_1 = x^2$

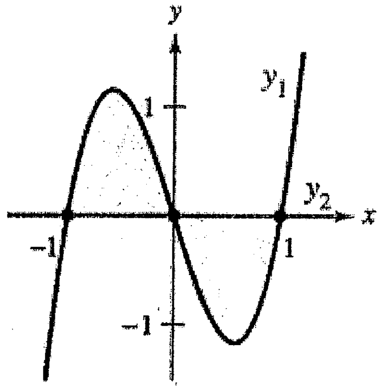
   $y_2 = x^3$



$$Area = \int_{x_1}^{x_2} (Top\ graph - Bottom\ graph)\, dx$$

5. $y_1 = 3(x^3 - x)$

$y_2 = 0$



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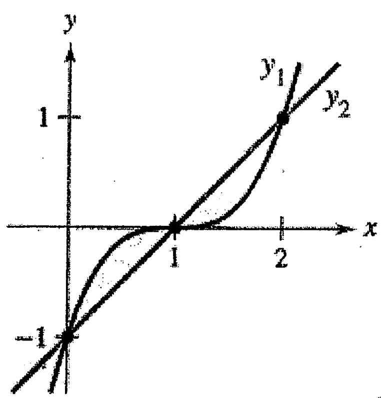
6. $y_1 = (x - 1)^3$

$y_2 = x - 1$



Finding the Area of a Region In Exercises 17–30, sketch the region bounded by the graphs of the equations and find the area of the region.

$$Area = \int_{x_1}^{x_2} (Top\ graph - Bottom\ graph)\, dx$$

17. $y = x^2 - 1, \quad y = -x + 2,$

$x = 0, \quad x = 1$

18. $y = -x^3 + 2, \quad y = x - 3,$

$x = -1, \quad x = 1$

19. $f(x) = x^2 + 2x, \quad g(x) = x + 2$

20. $y = -x^2 + 3x + 1, \quad y = -x + 1$

$$Area = \int_{x_1}^{x_2} (Top\ graph - Bottom\ graph)\, dx$$

Finding the Area of a Region In Exercises 17–30, sketch the region bounded by the graphs of the equations and find the area of the region.

21. $y = x, \quad y = 2 - x, \quad y = 0$

22. $y = \dfrac{4}{x^3}, \quad y = 0, \quad x = 1, \quad x = 4$

23. $f(x) = \sqrt{x} + 3, \quad g(x) = \frac{1}{2}x + 3$

24. $f(x) = \sqrt[3]{x - 1}, \quad g(x) = x - 1$

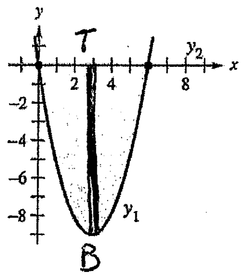
Non-AP Calculus Chapter 7.1: Area between Curves Classwork Worksheet

Key

$$Area = \int_{x_1}^{x_2} (Top\ graph - Bottom\ graph)\, dx$$

Writing a Definite Integral **In Exercises 1–6, set up the definite integral that gives the area of the region.**

1. $y_1 = x^2 - 6x$
   $y_2 = 0$



$x(x-6)=0$

$x=0,\ x=6$

$Area = \int_0^6 0-(x^2-6x)\,dx$

$\int -x^2+6x\,dx$

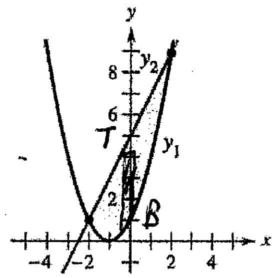
$-\frac{x^3}{3}+\frac{6x^2}{2}$

$\left.-\frac{x^3}{3}+3x^2\right]_0^6$

$-\frac{6^3}{3}+3(6)^2-(0-0)$

$= \boxed{36}$

2. $y_1 = x^2 + 2x + 1$
   $y_2 = 2x + 5$



find bounds:

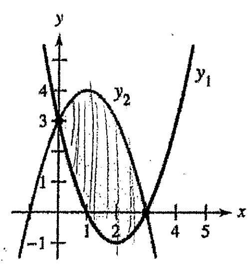
$x^2+2x+1=2x+5$

$x^2-4=0$

$(x-2)(x+2)=0 \quad x=2,-2$

$Area = \int_{-2}^{2} 2x+5-(x^2+2x+1)\,dx$

3. $y_1 = x^2 - 4x + 3$
   $y_2 = -x^2 + 2x + 3$



$-x^2+2x+3 = x^2-4x+3$

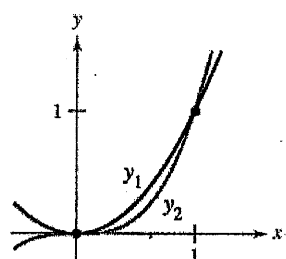
$0=2x^2-6x$

$0=2x(x-3)$

$x=0,3$

$A=\int_0^3 -x^2+2x+3-(x^2-4x+3)\,dx$

4. $y_1 = x^2$
   $y_2 = x^3$



bounds: $x^2=x^3$

$x^2-x^3=0$

$x^2(1-x)=0$

$x=0,1$

$A=\int_0^1 x^2-x^3\,dx$

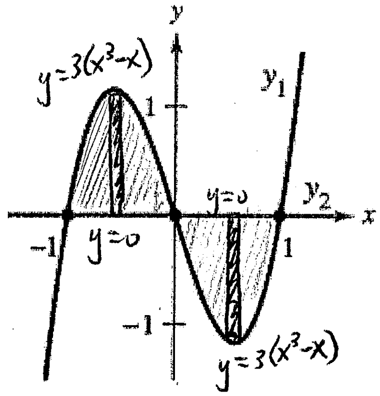
$=\left.\frac{x^3}{3}-\frac{x^4}{4}\right]_0^1 = \frac{1^3}{3}-\frac{1}{4}-(0-0)$

$=\frac{1}{3}-\frac{1}{4}=\boxed{\frac{1}{12}}$

$$Area = \int_{x_1}^{x_2} (Top\ graph - Bottom\ graph)\ dx$$

5. $y_1 = 3(x^3 - x)$

$y_2 = 0$



$$\boxed{A = \int_{-1}^{0} 3(x^3-x) - 0\, dx + \int_{0}^{1} 0 - 3(x^3-x)\, dx}$$

*bounds:

$3(x^3-x) = 0$

$x^3 - x = 0$

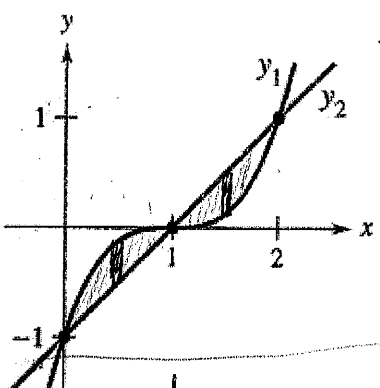
$x(x^2-1) = 0$

$x(x+1)(x-1) = 0$

$x = 0, 1, -1$

6. $y_1 = (x-1)^3$

$y_2 = x - 1$



*bounds:

$x - 1 = (x-1)^3$

$x - 1 - (x-1)^3 = 0$

$(x-1)[1-(x-1)^2] = 0$

$(x-1)(1-x^2+2x-1) = 0$

$(x-1)(x)(2-x) = 0$

$x = 1, 0, 2$

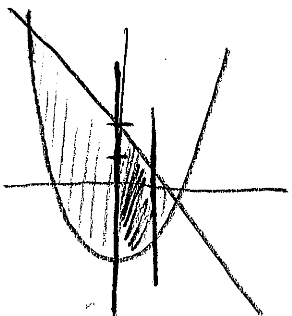
$$\boxed{Area = \int_{0}^{1} (x-1)^3 - (x-1)\, dx + \int_{1}^{2} x - 1 - (x-1)^3\, dx}$$

Finding the Area of a Region In Exercises 17–30, sketch the region bounded by the graphs of the equations and find the area of the region.

$Area = \int_{x_1}^{x_2} (Top\ graph - Bottom\ graph)\, dx$

17. $y = x^2 - 1, \quad y = -x + 2,$

$x = 0, \quad x = 1$



$$Area = \int_0^1 -x+2-(x^2-1)\,dx$$

$$\int -x+2-x^2+1\,dx$$

$$\int_0^1 -x^2-x+3\,dx$$

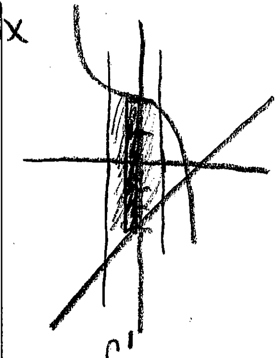
$$= \frac{-x^3}{3} - \frac{x^2}{2} + 3x \Big]_0^1$$

$$= \left(\frac{-1}{3} - \frac{1}{2} + 3\right) - (0-0+0)$$

$$= \boxed{\frac{13}{6}}$$

18. $y = -x^3 + 2, \quad y = x - 3,$

$x = -1, \quad x = 1$



$$A = \int_{-1}^{1} -x^3+2-(x-3)\,dx$$

$$\int -x^3+2-x+3\,dx$$

$$\int_{-1}^{1} -x^3-x+5\,dx$$

$$\frac{-x^4}{4} - \frac{x^2}{2} + 5x \Big]_{-1}^{1}$$

$$\left[\frac{-1}{4} - \frac{1}{2} + 5\right] - \left[\frac{-1}{4} - \frac{1}{2} - 5\right]$$

$$= \boxed{10}$$

19. $f(x) = x^2 + 2x, \quad g(x) = x + 2$

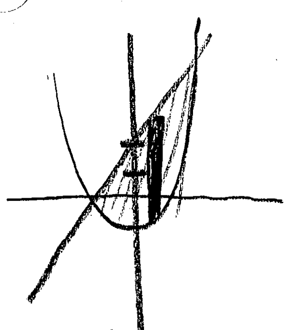
find intersection:

$$x^2+2x = x+2$$

$$x^2+x-2=0$$

$$(x+2)(x-1)=0$$

$$x=-2, 1$$



$$A = \int_{-2}^{1} x+2-(x^2+2x)\,dx$$

$$\int x+2-x^2-2x\,dx$$

$$\int_{-2}^{1} -x^2-x+2\,dx$$

$$\frac{-x^3}{3} - \frac{x^2}{2} + 2x \Big]_{-2}^{1}$$

$$\left(\frac{-1}{3} - \frac{1}{2} + 2\right) - \left(\frac{2^3}{3} - \frac{4}{2} - 4\right)$$

$$= \boxed{\frac{9}{2}}$$

20. $y = -x^2 + 3x + 1, \quad y = -x + 1$

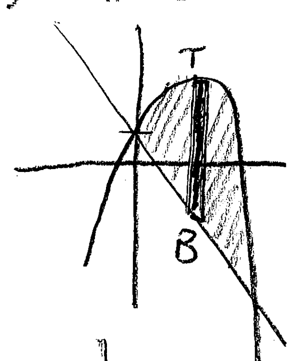
*find intersection:

$$-x+1 = -x^2+3x+1$$

$$x^2-4x=0$$

$$x(x-4)=0$$

$$x=0, 4$$



$$A = \int_0^4 -x^2+3x+1-(-x+1)\,dx$$

$$\int -x^2+3x+1+x-1\,dx$$

$$\int_0^4 -x^2+4x\,dx$$

$$\frac{-x^3}{3} + \frac{4x^2}{2}$$

$$-\frac{x^3}{3} + 2x^2 \Big]_0^4$$

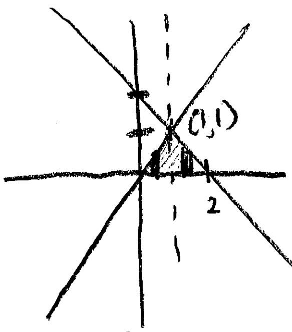
$$-\frac{4^3}{3} + 2(4)^2 - \left(\frac{0}{3} + 2(0)^2\right)$$

$$\frac{-64}{3} + 32 = \boxed{\frac{32}{3}}$$

$$Area = \int_{x_1}^{x_2} (Top\ graph - Bottom\ graph)\,dx$$

Finding the Area of a Region In Exercises 17–30, sketch the region bounded by the graphs of the equations and find the area of the region.

21. $y = x, \quad y = 2 - x, \quad y = 0$



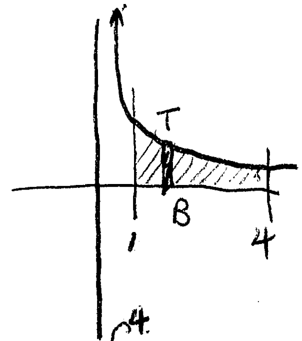
$$A = \int_0^1 x - 0\,dx + \int_1^2 2 - x - 0\,dx$$

$$\left.\frac{x^2}{2}\right]_0^1 \qquad \left. 2x - \frac{x^2}{2}\right]_1^2$$

$$\frac{1}{2} - 0 \qquad 4 - \frac{4}{2} - \left(2 - \frac{1}{2}\right)$$

$$\frac{1}{2} + \frac{1}{2} = \boxed{1}$$

22. $y = \dfrac{4}{x^3}, \quad y = 0, \quad x = 1, \quad x = 4$



$$A = \int_1^4 \frac{4}{x^3} - 0\,dx$$

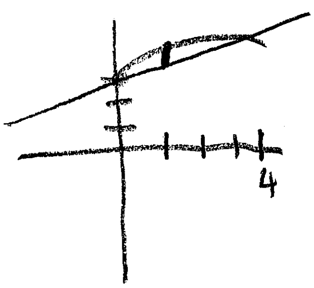
$$\int 4x^{-3}\,dx$$

$$\left.\frac{4x^{-2}}{-2} = \frac{-2}{x^2}\right]_1^4$$

$$\frac{-2}{4^2} - \frac{-2}{1^2}$$

$$\frac{-2}{16} + 2 = \boxed{\frac{15}{8}}$$

23. $f(x) = \sqrt{x} + 3, \quad g(x) = \frac{1}{2}x + 3$



*find intersection:

$$\sqrt{x} + 3 = \frac{1}{2}x + 3$$

$$\left(\sqrt{x}\right)^2 = \left(\frac{x}{2}\right)^2 \qquad 4x - x^2 = 0$$

$$x = \frac{x^2}{4} \qquad x(4-x) = 0$$

$$4x = x^2 \qquad x = 0, 4$$

$$\int_0^4 \sqrt{x} + 3 - \left(\frac{1}{2}x + 3\right) dx$$

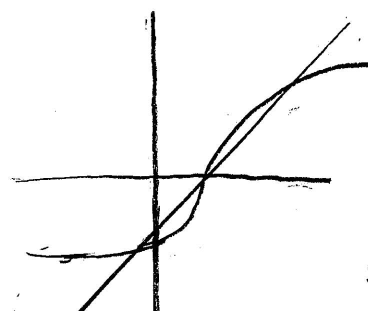
$$\int \sqrt{x} + 3 - \frac{1}{2}x - 3\,dx \qquad \frac{x^{3/2}}{3/2} - \frac{1}{2}\left(\frac{x^2}{2}\right)$$

$$\int x^{1/2} - \frac{1}{2}x\,dx \qquad \left.\frac{2}{3}x^{3/2} - \frac{x^2}{4}\right]_0^4$$

$$\frac{2}{3}(4)^{3/2} - \frac{4^2}{4} = \frac{2}{3}(8) - 4$$

$$= \frac{16}{3} - \frac{12}{3} = \boxed{\frac{4}{3}}$$

24. $f(x) = \sqrt[3]{x - 1}, \quad g(x) = x - 1$



*bounds:

$$\sqrt[3]{x-1} = x - 1$$

$$x - 1 = (x-1)^3$$

$$x - 1 - (x-1)^3 = 0$$

$$x - 1\left[1 - (x-1)^2\right] = 0$$

$$(x-1)(1 - x^2 + 2x - 1) = 0$$

$$(x-1)(x)(2-x) = 0$$

$$x = 0, 1, 2$$

$$A = \int_0^1 x - 1 - (x-1)^{1/3} + \int_1^2 (x-1)^{1/3} - (x-1)\,dx = \boxed{\frac{1}{2}}$$