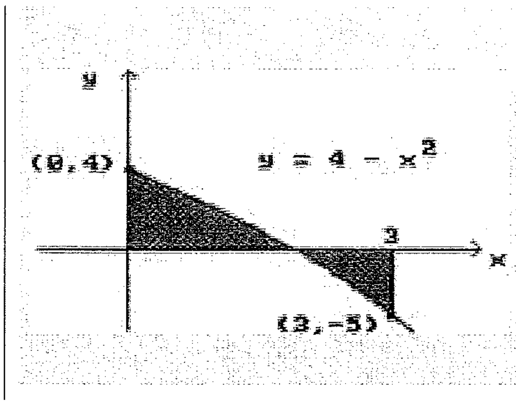
AP Calculus AB **7.1 Quiz Morning Review** No calculators!

1. Write the integral(s) that would give the area of the shaded region to the right. (Do not solve)



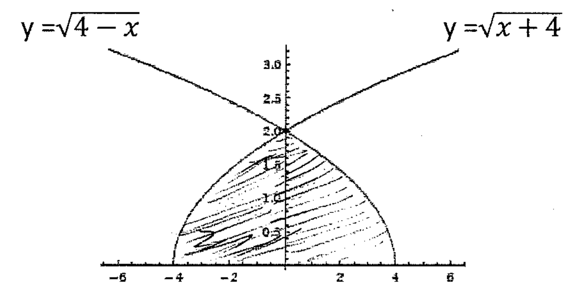
Find the area bounded by the following regions:

2. $y = x^2$ and $x + y = 6$

3. $x - y^2 = 0$ and $x - y = 2$

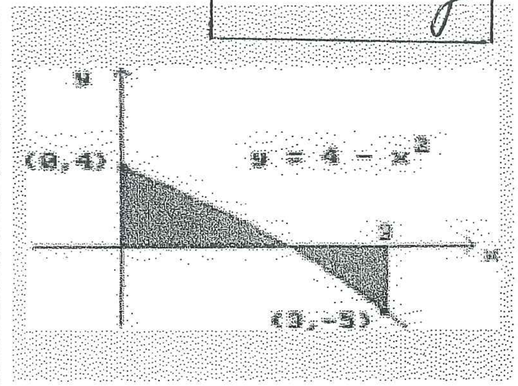
4. $y = x^3 + x^2$ and $y = 6x$

5. Find area of shaded region



AP Calculus AB_ **7.1 Quiz Morning Review** No calculators! Solution Key

1. Write the integral(s) that would give the area of the shaded region to the right. (Do not solve)



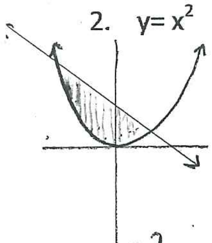
*Find intersection: $4-x^2=0 \quad x=2, \cancel{-2}$

$$A=\int_0^2 4-x^2-0\,dx+\int_2^3 0-(4-x^2)\,dx$$

$$\boxed{A=\int_0^2 4-x^2\,dx+\int_2^3 -4+x^2\,dx}$$

Find the area bounded by the following regions:

2. $y=x^2$ and $x+y=6$ $\quad y=6-x$



*Find intersection:

$x^2=6-x \qquad (x+3)(x-2)=0$

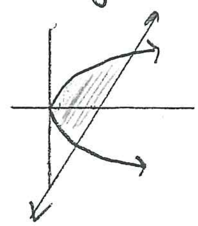
$x^2+x-6=0 \qquad x=-3, 2$

$$A=\int_{-3}^{2} \underbrace{6-x}_{top}-\underbrace{x^2}_{bottom}\,dx = 6x-\frac{x^2}{2}-\frac{x^3}{3}\Big]_{-3}^{2}$$

$$=12-2-\frac{8}{3}-\left(-18-\frac{9}{2}+\frac{27}{3}\right) = \boxed{\frac{125}{6}}$$

3. $x-y^2=0$ and $x-y=2$

$x=y^2 \quad x=y+2$



*find intersection

$y^2=y+2$

$y^2-y-2=0$

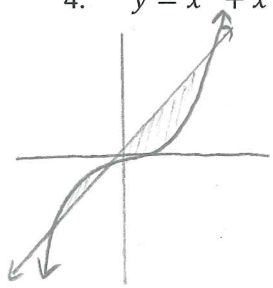
$(y-2)(y+1)=0$

$y=2, -1$

$$\int_{-1}^{2} \underbrace{y+2}_{right}-\underbrace{y^2}_{left}\,dy$$

$$\frac{y^2}{2}+2y-\frac{y^3}{3}\Big]_{-1}^{2} = \frac{4}{2}+4-\frac{8}{3}-\left(\frac{1}{2}-2+\frac{1}{3}\right)=\boxed{\frac{9}{2}}$$

4. $y=x^3+x^2$ *and* $y=6x$



Find intersection: $x(x^2+x-6)=0$

$x^3+x^2=6x \qquad x(x+3)(x-2)=0$

$x^3+x^2-6x=0 \qquad x=0, -3, 2$

$$A=\int_{-3}^{0} \underbrace{x^3+x^2}_{top}-\underbrace{6x}_{bottom}\,dx+\int_0^2 \underbrace{6x}_{top}-\underbrace{(x^3+x^2)}_{bottom}\,dx = \int 6x-x^3-x^2$$

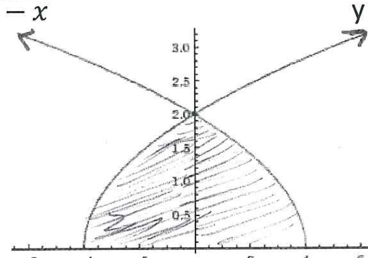
$$\frac{x^4}{4}+\frac{x^3}{3}-\frac{6x^2}{2}\Big]_{-3}^{0} \qquad \frac{6x^2}{2}-\frac{x^4}{4}-\frac{x^3}{3}\Big]_0^2$$

$$=(0+0-0)-\left(\frac{81}{4}-\frac{27}{3}-27\right) \qquad 3(4)-\frac{16}{4}-\frac{8}{3}-(0-0-0)$$

$$\frac{63}{4}+\frac{16}{3}=\boxed{\frac{253}{12}\text{ units}^2}$$

5. Find area of shaded region

$y=\sqrt{4-x}$ $\qquad y=\sqrt{x+4}$



$y^2=4-x \qquad y^2=x+4$

$x=4-y^2 \qquad x=y^2-4$

Top/bottom

$$A=\int_{-4}^{0}\sqrt{x+4}-0\,dx+\int_0^4\sqrt{4-x}-0\,dx=\frac{16}{3}+\frac{16}{3}=\boxed{\frac{32}{3}}$$

$u=x+4 \quad \int u^{1/2}du = \frac{u^{3/2}}{3/2}\Big]_0^4=\frac{16}{3}$

$du/dx=1$

Right/Left

$$\int_0^2 4-y^2-(y^2-4)\,dy$$

$$\int -2y^2+8\,dy$$

$$\frac{-2y^3}{3}+8y\Big]_0^2=-\frac{16}{3}+16-0$$

$$=\boxed{\frac{32}{3}}$$