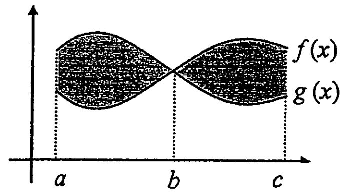
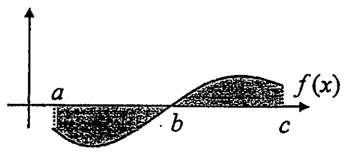
A.P. Calculus AB **7.1 Worksheet** **No Calculators!**

Write an integral that can be used to find the area of the shaded regions.

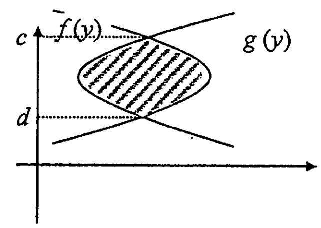
1.



2.



3.



Find the area bounded by the regions listed below:

4. the x-axis and $y = 2x - x^2$

5. the y-axis and $x = y^2 - y^3$

6. $y^2 = x$ and $x = 4$

7. $x = 3y - y^2$ and $x + y = 3$

8. $y = x^4 - 2x^2$ and $y = 2x^2$

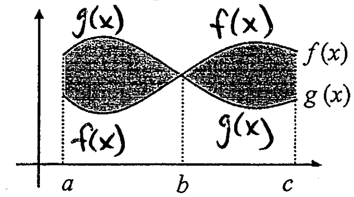
9. $y = x, \ y = \frac{1}{x^2}, x = 2$

10. $4x = y^2 - 4$ and $4x = y + 16$

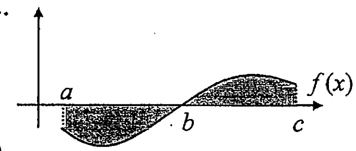
11. $y = -sinx \ and \ y = 2sinx \ , -\pi \le x \le 0$

A.P. Calculus AB 7.1 Worksheet No Calculators! Key

Write an integral that can be used to find the area of the shaded regions.

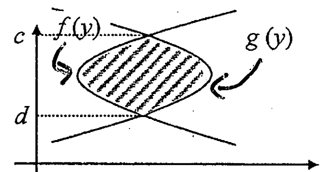
1.


$$A=\int_a^b g(x)-f(x)\,dx+\int_b^c f(x)-g(x)\,dx$$

2.


$$A=\int_a^b 0-f(x)\,dx+\int_b^c f(x)-0\,dx$$

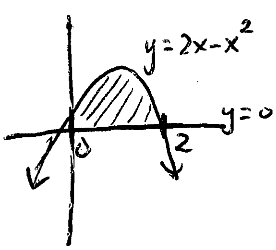
$$A=-\int_a^b f(x)\,dx+\int_b^c f(x)\,dx$$

3.


$$A=\int_d^c g(y)-f(y)\,dy$$

Find the area bounded by the regions listed below:

4. the x-axis and $y=2x-x^2$ Top-bottom



$$A=\int_0^2 \underbrace{2x-x^2}_{top}-\underbrace{0}_{bottom}\,dx$$

$$\left.\frac{2x^2}{2}-\frac{x^3}{3}\right]_0^2=4-8/3$$

$$=\boxed{4/3 \text{ units}^2}$$

$2x-x^2=0$

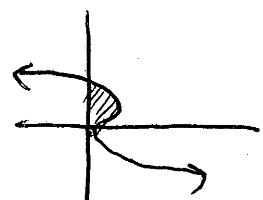
$x(2-x)=0$

$x=0,2$

5. the y-axis and $x=y^2-y^3$

$y^2(1-y)=0$

$y=0,1$



Right-Left Form

$$A=\int_0^1 \underbrace{y^2-y^3}_{Right}-\underbrace{0}_{Left}\,dy$$

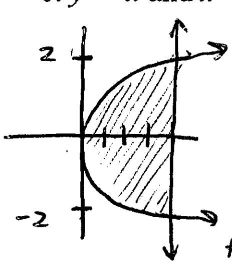
$$\left.\frac{y^3}{3}-\frac{y^4}{4}\right]_0^1$$

$$\frac{1}{3}-\frac{1}{4}=\boxed{\frac{1}{12}\text{ units}^2}$$

6. $y^2=x$ and $x=4$ Right-Left

set $y^2=4$ $y=\pm 2$

*find intersections



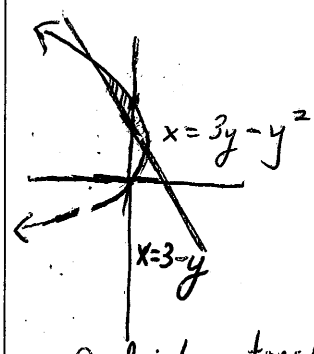
$$A=\int_{-2}^{2} \underbrace{4}_{Right}-\underbrace{y^2}_{Left}\,dy=\left.4y-\frac{y^3}{3}\right]_{-2}^{2}$$

$$=8-\frac{8}{3}-(-8+8/3)$$

$$=16-16/3=\boxed{\frac{32}{3}\text{ units}^2}$$

7. $x=3y-y^2$ and $x+y=3$

$x=3-y$



*find intersections (bounds)

$3-y=3y-y^2$

$y^2-4y+3=0$

$(y-3)(y-1)=0$

$y=1,3$

$$\int_1^3 \underbrace{3y-y^2}_{Right}-\underbrace{(3-y)}_{Left}\,dy$$

$$\int_1^3 3y-y^2-3+y\,dy$$

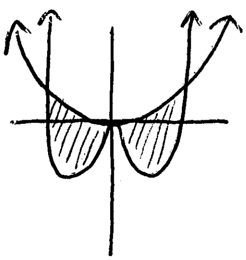
$$\int_1^3 -y^2+4y-3\,dy$$

$$\left.\frac{-y^3}{3}+\frac{4y^2}{2}-3y\right]_1^3$$

$$-\frac{27}{3}+18-9-\left(\frac{-1}{3}+2-3\right)$$

$$=\boxed{4/3\text{ units}^2}$$

8. $y = x^4 - 2x^2$ and $y = 2x^2$



*find intersection:

$x^4 - 2x^2 = 2x^2$

$x^4 - 4x^2 = 0$

$x^2(x^2 - 4) = 0$

$x = 0, 2, -2$

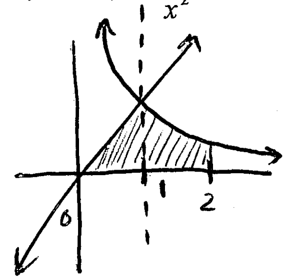
$$A = \int_{-2}^{2} \overbrace{2x^2}^{\text{Top}} - \overbrace{(x^4 - 2x^2)}^{\text{Bottom}}\, dx$$

$$\int_{-2}^{2} 4x^2 - x^4\, dx = \left.\frac{4x^3}{3} - \frac{x^5}{5}\right]_{-2}^{2}$$

$$\frac{32}{3} - \frac{32}{5} - \left(-\frac{32}{3} + \frac{32}{5}\right)$$

$$\frac{64}{3} - \frac{64}{5} = \boxed{\frac{128}{15}\text{ units}^2}$$

9. $y = x$, $y = \frac{1}{x^2}$, $x = 2$ $y = 0$



*find intersection:

$x = \frac{1}{x^2}$

$x^3 = 1$, $x = 1$

*split into 2 integrals:

$$A = \int_{0}^{1} \overbrace{x}^{\text{top}} - \overbrace{0}^{\text{bottom}}\, dx + \int_{1}^{2} \overbrace{\frac{1}{x^2}}^{\text{top}} - \overbrace{0}^{\text{bottom}}\, dx = \int x^{-2}\, dx$$

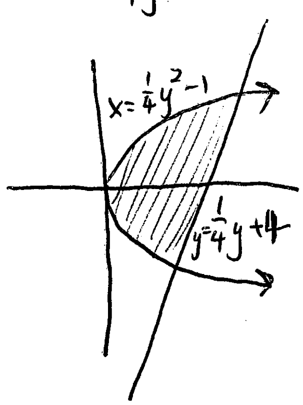
$$\left.\frac{x^2}{2}\right]_0^1 + \left.\frac{x^{-1}}{-1}\right]_1^2 = \left.-\frac{1}{x}\right]_1^2$$

$$\frac{1}{2} - 0 + -\frac{1}{2} - \left(-\frac{1}{1}\right)$$

$$\frac{1}{2} - \frac{1}{2} + 1 = \boxed{1}$$

10. $4x = y^2 - 4$ and $4x = y + 16$ $y = 4x - 16$

$x = \frac{1}{4}y^2 - 1$ $x = \frac{1}{4}y + 4$



*find intersection:

$\left[\frac{1}{4}y^2 - 1 = \frac{1}{4}y + 4\right] \cdot 4$

$y^2 - 4 = y + 16$

$y^2 - y - 20 = 0$

$(y - 5)(y + 4) = 0$

$y = -4, 5$

$$A = \int_{-4}^{5} \overbrace{\frac{1}{4}y + 4}^{\text{Right}} - \overbrace{\left(\frac{1}{4}y^2 - 1\right)}^{\text{Left}}\, dy$$

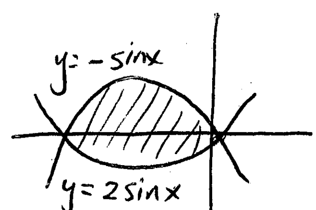
$$\int_{-4}^{5} -\frac{1}{4}y^2 + \frac{1}{4}y + 5\, dy$$

$$\left.-\frac{y^3}{12} + \frac{y^2}{8} + 5y\right]_{-4}^{5}$$

$$\frac{-125}{12} + \frac{25}{8} + 25 - \left(-\frac{64}{12} + \frac{16}{8} - 20\right)$$

$$= \boxed{\frac{243}{3}\text{ units}^2}$$

11. $y = -\sin x$ and $y = 2\sin x$, $-\pi \le x \le 0$



*find intersection:

$2\sin x = -\sin x$

$3\sin x = 0$

$x = 0, -\pi$

$$A = \int_{-\pi}^{0} \overbrace{-\sin x}^{\text{Top}} - \overbrace{2\sin x}^{\text{Bottom}}\, dx = \int_{-\pi}^{0} -3\sin x\, dx$$

$$= \int_{-\pi}^{0} -3\sin x\, dx = \left.3\cos x\right]_{-\pi}^{0} = 3\cos(0) - (3\cos(-\pi))$$

$$= 3 - (-3)$$

$$= \boxed{6\text{ units}^2}$$