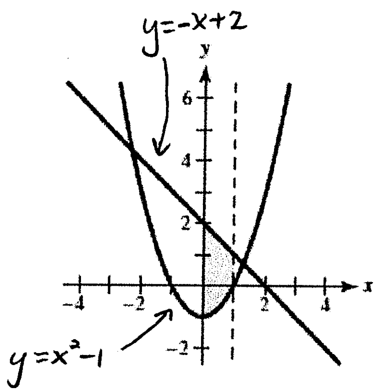
Ch. 7.1a Area Between Curves   p. 442–443   2017

\# 17–35 odd

Finding the Area of a Region In Exercises 17–30, sketch the region bounded by the graphs of the equations and find the area of the region.

17. $y = x^2 - 1, \quad y = -x + 2, \quad x = 0, \quad x = 1$



Top − bottom

$$A = \int_0^1 \left[(-x + 2) - (x^2 - 1)\right] dx$$

$$= \int_0^1 \left(-x^2 - x + 3\right) dx$$

$$= \left[\frac{-x^3}{3} - \frac{x^2}{2} + 3x\right]_0^1$$

$$= \left(-\frac{1}{3} - \frac{1}{2} + 3\right) - 0 = \frac{13}{6}$$

19. $f(x) = x^2 + 2x, \quad g(x) = x + 2$

The points of intersection are given by:

$$x^2 + 2x = x + 2$$

$$x^2 + x - 2 = 0$$

$$(x + 2)(x - 1) = 0 \text{ when } x = -2, 1$$

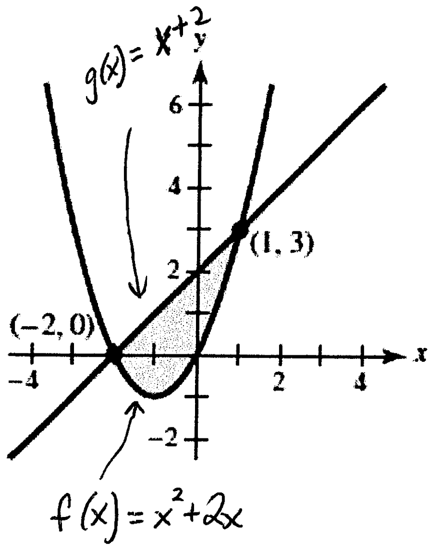
Top − Bottom

$$A = \int_{-2}^1 \left[g(x) - f(x)\right] dx$$

$$= \int_{-2}^1 \left[(x + 2) - (x^2 + 2x)\right] dx$$

$$= \left[\frac{-x^3}{3} - \frac{x^2}{2} + 2x\right]_{-2}^1$$

$$= \left(-\frac{1}{3} - \frac{1}{2} + 2\right) - \left(\frac{8}{3} - 2 - 4\right) = \frac{9}{2}$$



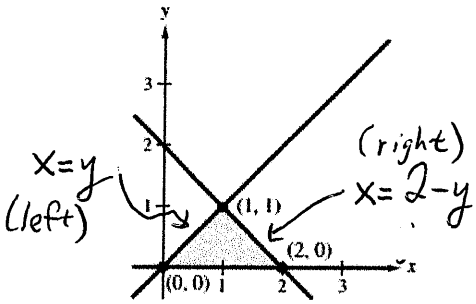
21. $y = x, \quad y = 2 - x, \quad y = 0$

The points of intersection are given by:

$x = 2 - x$ and $x = 0$ and $2 - x = 0$

$x = 1 \qquad x = 0 \qquad x = 2$

Right-Left Method

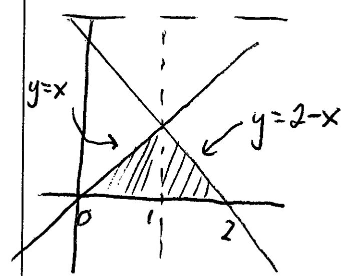


$$A = \int_0^1 [(2 - y) - (y)]\, dy = \left[2y - y^2\right]_0^1 = 1$$

right − left

Note that if you integrate with respect to x, you need two integrals. Also, note that the region is a triangle.

Top-Bottom Method:



$$A = \int_0^1 x - 0\, dx + \int_1^2 2 - x - 0\, dx$$

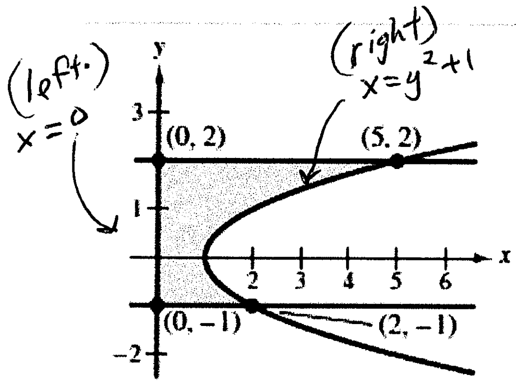
$\left.\frac{x^2}{2}\right]_0^1 = \frac{1}{2} - 0$; $\left.2x - \frac{x^2}{2}\right]_1^2 = 4 - 2 - (2 - \frac{1}{2})$

$= 2 - \frac{3}{2}$

$= \frac{1}{2}$

$= \frac{1}{2} + \frac{1}{2} = \boxed{1}$

27. $f(y) = y^2 + 1, \quad g(y) = 0, \quad y = -1, \quad y = 2$



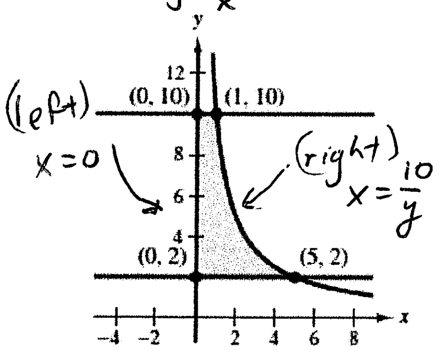
$$\text{Area} = \int_{-1}^{2} y^2 + 1 - 0\, dy$$

$$\left.\frac{y^3}{3} + y\right]_{-1}^{2} = \frac{8}{3} + 2 - \left(\frac{-1}{3} - 1\right)$$

$$= \frac{8}{3} + 2 + \frac{1}{3} + 1 = \frac{9}{3} + 3 = \boxed{6}$$

29. $f(x) = \dfrac{10}{x}, \quad x = 0, \quad y = 2, \quad y = 10$

$y = \frac{10}{x}$



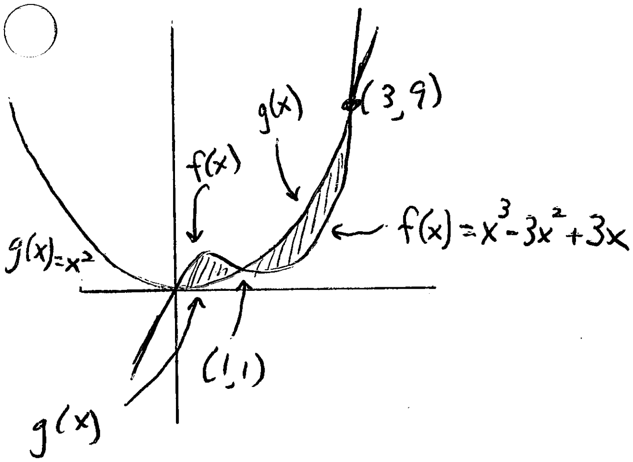
$$\text{Area} = \int_2^{10} \frac{10}{y} - 0\, dy$$

$$\left.10 \ln|y|\right]_2^{10} = 10\ln 10 - 10\ln 2 = 10\ln\left(\frac{10}{2}\right)$$

$$\boxed{= 10\ln 5 \approx 16.0944}$$

2017

31. $f(x) = x(x^2 - 3x + 3)$, $g(x) = x^2$



*Find intesections: $x^3-3x^2+3x = x^2$

$x^3-4x^2+3x=0$

$x(x^2-4x+3)=0$

$x(x-3)(x-1)=0$

$x=0,1,3$

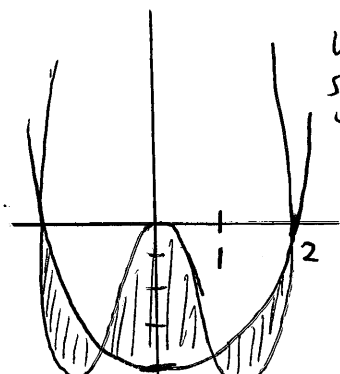
$$\text{Area} = \int_0^1 f(x)-g(x)\,dx + \int_1^3 g(x)-f(x)\,dx$$

$$\int_0^1 x^3-3x^2+3x-x^2\,dx + \int_1^3 x^2-(x^3-3x^2+3x)\,dx$$

$$\int_0^1 x^3-4x^2+3x + \int_1^3 -x^3+4x^2-3x\,dx$$

$$\frac{5}{12} + \frac{8}{3} = \boxed{\frac{37}{12}}$$

33. $f(x) = x^4 - 4x^2$, $g(x) = x^2 - 4$



use symmetry

*Find intesections

$x^4-4x^2 = x^2-4 \rightarrow x^4-5x^2+4=0$

$(x^2-4)(x^2-1)=0$

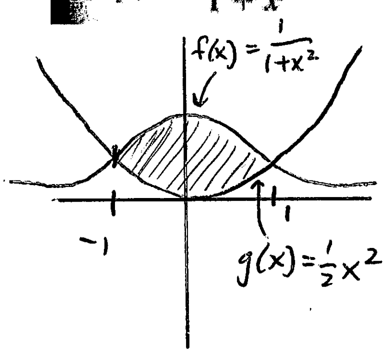
$(x-2)(x+2)(x+1)(x-1)=0$

$x=\pm 2, \pm 1$

$$\text{Area} = 2\int_0^1 x^4-4x^2-(x^2-4)\,dx + 2\int_1^2 x^2-4-(x^4-4x^2)\,dx$$

$\boxed{\text{Area} = 8}$

35. $f(x) = \dfrac{1}{1+x^2}$ $\quad g(x) = \dfrac{1}{2}x^2$



$$\text{Area} = \int_{-1}^{1} \frac{1}{1+x^2} - \frac{x^2}{2}\,dx = \arctan x - \frac{x^3}{6}\Big]_{-1}^{1}$$

$$= \arctan 1 - \frac{1}{6} - \left(\arctan(-1) + \frac{1}{6}\right)$$

$$\frac{\pi}{4} - \frac{1}{6} - \left(\frac{-\pi}{4}\right) - \frac{1}{6} = \boxed{\frac{\pi}{2} - \frac{1}{3} = \approx 1.237}$$

*find intesections

$\dfrac{1}{x^2+1} = \dfrac{x^2}{2}$

$2 = x^4+x^2$

$0 = x^4+x^2-2$

$0 = (x^2+2)(x^2-1)$

$(x^2+2)(x+1)(x-1)$

$\underline{x=-1,1}$

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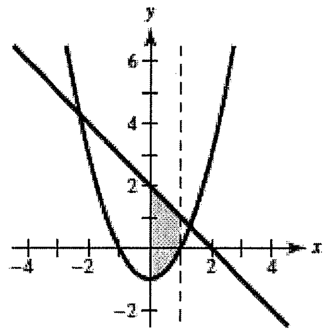

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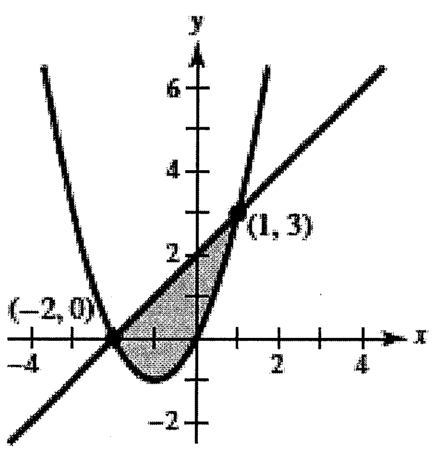

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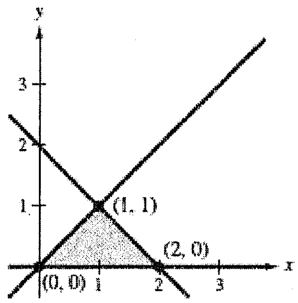

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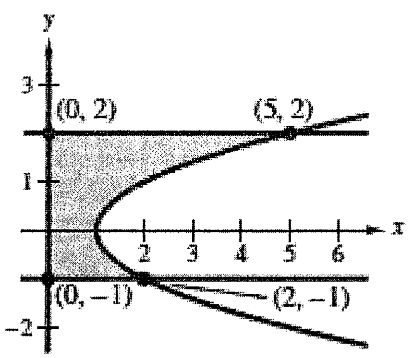

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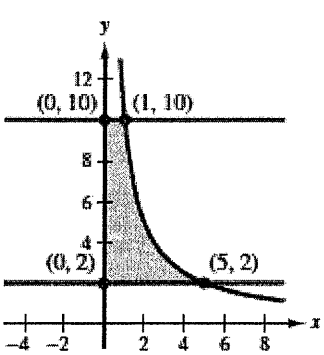

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