Ch. 7.1b Area between Curves **Area FRQ Graphing Calculator Practice Problems**

Key

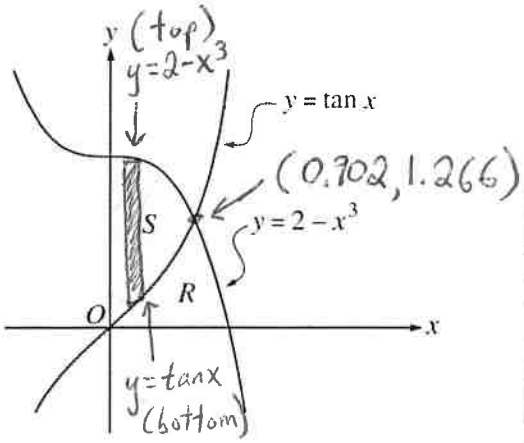
1. Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.

(a) Find the area of S

$$\textbf{Area} = \int_{x_1}^{x_2} (Top\ graph - Bottom\ graph)\,dx \qquad \int_{y_1}^{y_2} (Right\ graph - Left\ graph)\,dy$$

(in the forms of "y = __") (in the form of "x = ___")

i) (Top – Bottom Method)

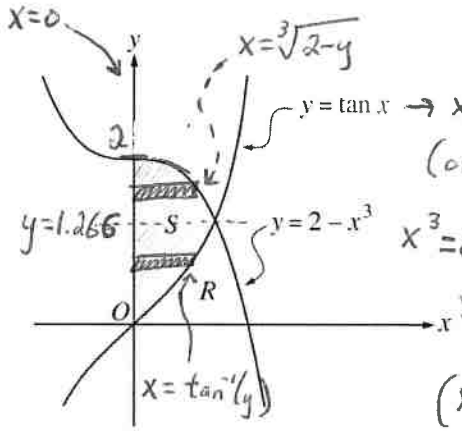


$$\text{Area} = \int_0^{0.902} 2 - x^3 - (\tan x)\,dx$$

Top - Bottom

$$\boxed{\text{Area} = 1.161}$$

ii) (Right – Left Method)



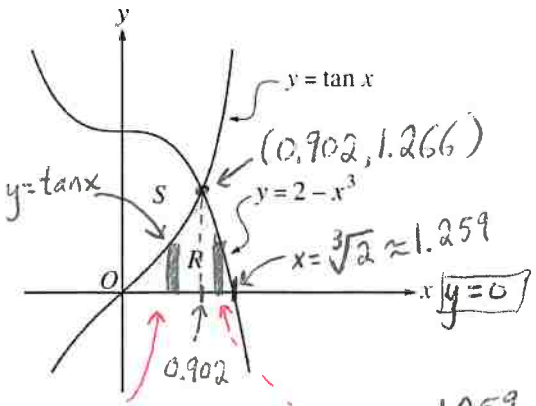
$$\int_0^{1.266} \tan^{-1}(y) - 0\,dy + \int_{1.266}^{2} \sqrt[3]{2-y} - 0\,dy$$

(Right) - (Left) (Right) - (Left)

$$\text{Area} = 0.664 + 0.4965 = \boxed{1.161}$$

(b) Find the area of R

i) (Top – Bottom Method)

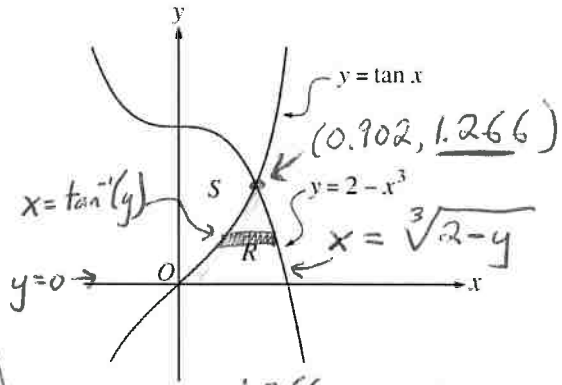


$$\text{Area} = \int_0^{0.902} \tan x - 0\,dx + \int_{0.902}^{1.259} 2 - x^3 - 0\,dx$$

(top) - (bottom) (top) - (bottom)

$$\text{Area} = 0.478 + 0.251 = \boxed{0.729}$$

ii) (Right – Left Method)



$$\text{Area} = \int_0^{1.266} \sqrt[3]{2-y} - \tan^{-1}(y)\,dy$$

(Right) - (Left)

$$\boxed{\text{Area} = 0.729}$$

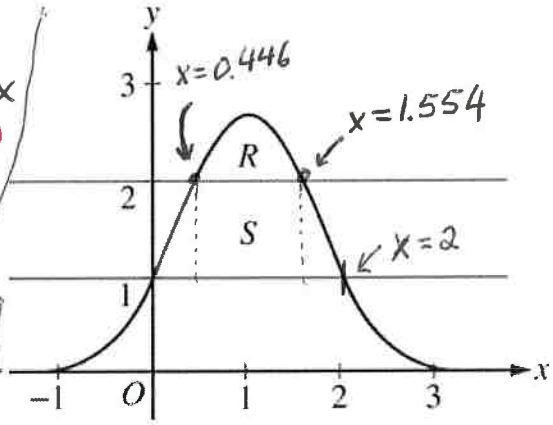
2) Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.

(a) Find the area of R.

(b) Find the area of S.

a) $\text{Area}_{(R)} = \int_{0.446}^{1.554} e^{2x-x^2} - 2\,dx$ (top) − (bottom)

$\text{Area}_{(R)} = 0.514$



b) option 1:

$$\int_0^{0.446} e^{2x-x^2} - 1\,dx + \int_{0.446}^{1.554} 2 - 1\,dx + \int_{1.554}^{2} e^{2x-x^2} - 1\,dx = 1.546$$

option 2: $\int_{(R+S)} - \int_R$ (top) − (bottom)

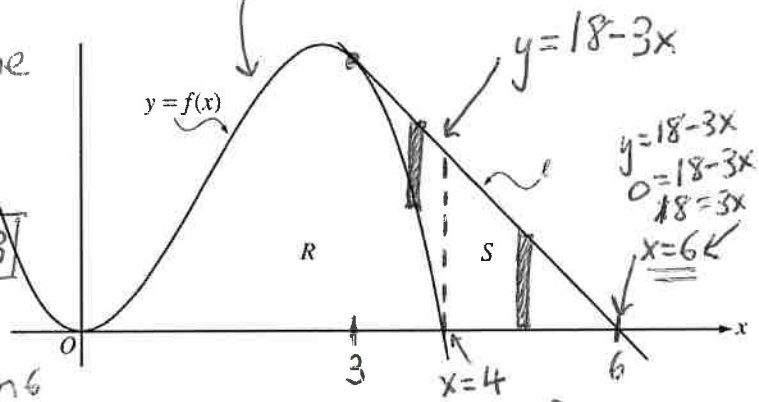
$$\int_0^2 e^{2x-x^2} - 1\,dx - \int_{0.446}^{1.554} e^{2x-x^2} - 2\,dx = 1.546$$

3) Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $y = 18 - 3x$, where ℓ is tangent to the graph of f. Let R be the region bounded by the graph of f and the x-axis, and let S be the region bounded by the graph of f, the line ℓ, and the x-axis, as shown above.

(a) Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$.

(b) Find the area of S.

$f(x) = 4x^2 - x^3$



a) * Show that the graph $f(x)$ has same slope as line $y = 18 - 3x$ at $x = 3$

* slope of graph: $f'(x) = 8x - 3x^2$

$f'(3) = 8(3) - 3(3)^2 = -3$

slope of line: $y = -3x + 18 \rightarrow m = -3$ ← same slope

$y = 18 - 3x$

$0 = 18 - 3x$

$18 = 3x$

$x = 6$

$0 = 4x^2 - x^3$

$0 = x^2(4 - x)$

$x = 4$

b) $\text{Area}_{(S)} = \int_3^4 18 - 3x - (4x^2 - x^3)\,dx + \int_4^6 18 - 3x - 0\,dx$

(Top) − (Bottom) (Top) − (Bottom)

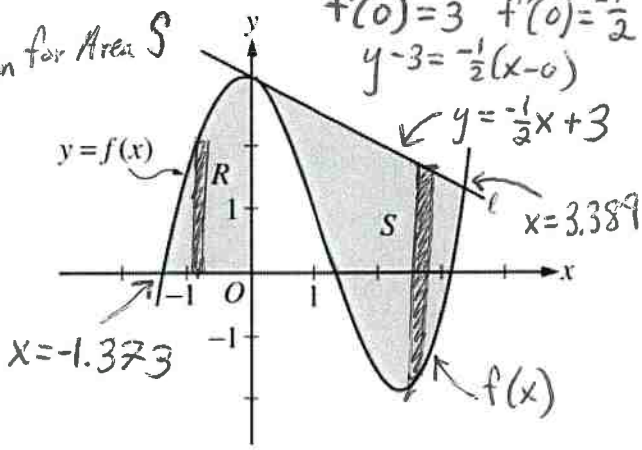
$\text{Area}_{(S)} = 1.917 + 6 = 7.917$

4) Let f be the function given by $f(x) = \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x$. Let R be the shaded region in the second quadrant bounded by the graph of f, and let S be the shaded region bounded by the graph of f and line ℓ, the line tangent to the graph of f at $x = 0$, as shown above.

(a) Find the area of R.

(b) Write integral expression for Area S

a) $\int_{-1.373}^{0} f(x) - 0\,dx = 2.903$



$f(0) = 3 \quad f'(0) = -\frac{1}{2}$

$y - 3 = -\frac{1}{2}(x - 0)$

$y = -\frac{1}{2}x + 3$

b) Area of S: $\int_0^{3.389} -\frac{1}{2}x + 3 - f(x)\,dx$

(top) − (bottom)

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a) Find the area of **S**

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(in the forms of " y = ___ ") (in the form of " x = ___ ")

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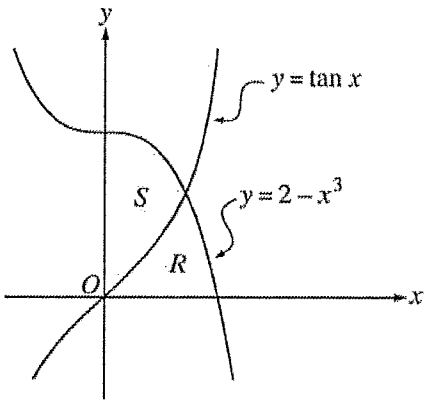
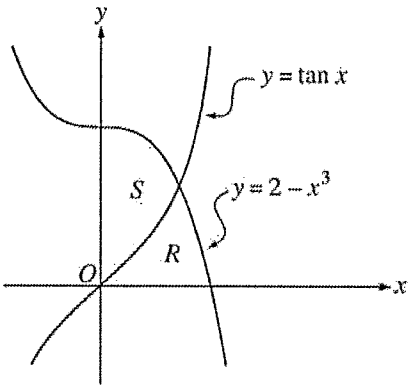
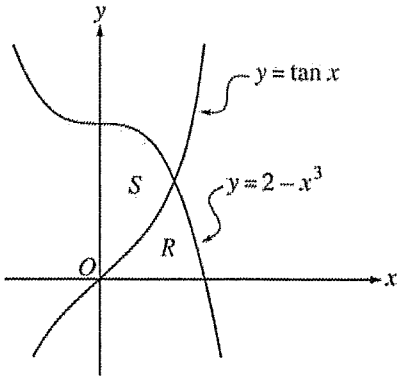


ii) (Right – Left Method)



b) Find the area of **R**

i) (Top – Bottom Method)



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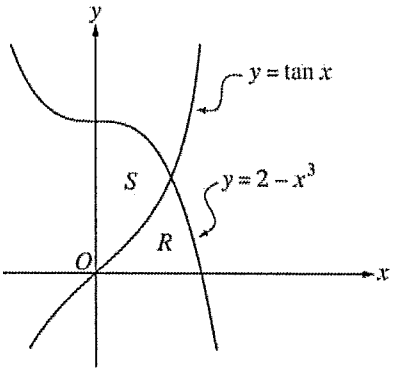

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(a) Find the area of R.

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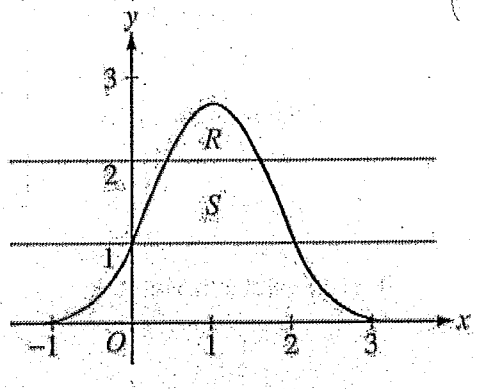

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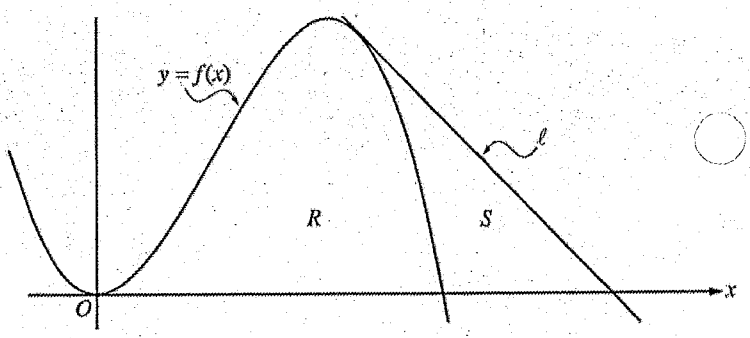
(a) Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$.

(b) Find the area of S.



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(a) Find the area of R. **(b)** Write an integral expression for Area of **S**