7.2 Morning Quiz Review

1. Given a bounded region by $y = 6 - x$, $y = 0$, $y = 4$, and the y-axis. Determine the volume of the solid generated by revolving the region about the given line:
   a. $x = 0$
   b. $x = -3$

2. Given a bounded region by $y = 4 - x^2$ and $y = 1$. Determine the volume of the solid generated by revolving the region about the given line:
   a. $y = 5$
   b. $x = -4$

3. The base of a solid is enclosed by $x^2 + y^2 = 16$. Find the volume of the solid:

a. Rectangles whose height is 3 times the length of the base (parallel to the x-axis)

b. Equilateral triangles (parallel to the x-axis)

c. right isosceles triangles whose hypotenuse lie on the base of the solid. (perpendicular to the x-axis)

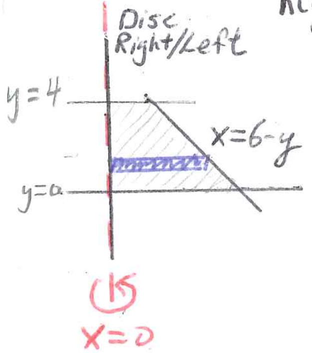
d. Semicircles (perpendicular to the x-axis)

7.2 Morning Quiz Review $\boxed{\text{KEY}}$

1. Determine the volume of the solid generated by revolving the region bounded by $y = 6 - x$, $y = 0$, $y = 4$, and the y-axis about

a. $x = 0$

Disc Right/Left



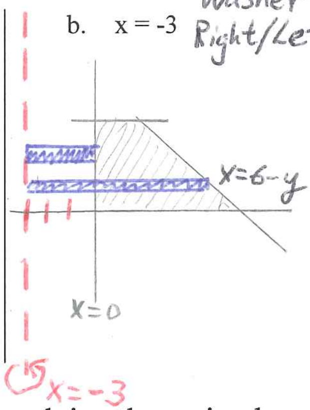
$R(y) = 6 - y - 0$

$V = \pi \int_0^4 [6-y]^2 \, dy$

$= \boxed{\frac{208}{3}\pi \text{ units}^3}$

b. $x = -3$

Washer Right/Left



$R(y) = 6 - y - (-3)$

$= 9 - y$

$r(y) = 0 - (-3) = 3$

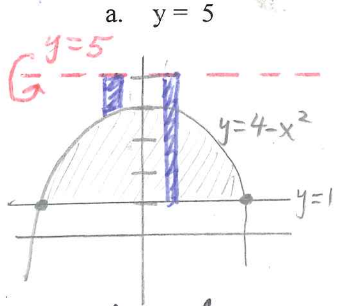
$V = \pi \int_0^4 (9-y)^2 - (3)^2 \, dy$

$\boxed{V = \frac{496}{3}\pi \text{ units}^3}$

2. Determine the volume of the solid generated by revolving the region bounded by $y = 4 - x^2$ and $y = 1$

a. $y = 5$

Washer, Top/bottom



$R(x) = 5 - 1 = 4$

$r(x) = 5 - (4 - x^2)$

$= 1 + x^2$

$V = \pi \int_{-\sqrt{3}}^{\sqrt{3}} 4^2 - [1+x^2]^2 \, dx$

*Find bounds:

$4 - x^2 = 1$

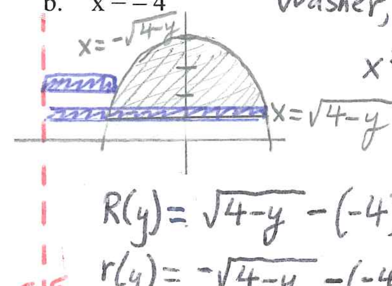
$x^2 = 3$

$x = \pm\sqrt{3}$

$\boxed{V = 38.798\pi \text{ units}^3}$

b. $x = -4$

Washer, Right/Left



$x^2 = 4 - y \qquad x = \pm\sqrt{4-y}$

$\boxed{V = 55.426\pi \text{ units}^3}$

$R(y) = \sqrt{4-y} - (-4)$

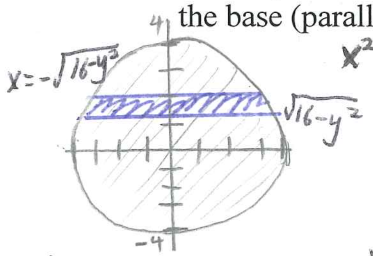
$r(y) = -\sqrt{4-y} - (-4)$

$x = -4$

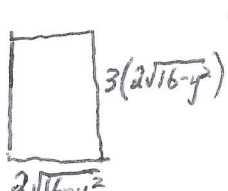
$V = \pi \int_1^4 [\sqrt{4-y} + 4]^2 - [-\sqrt{4-y} + 4]^2 \, dy$

3. The base of a solid is enclosed by $x^2 + y^2 = 16$. Find the volume of the solid:

a. Rectangles whose height is 3 times the length of the base (parallel to the x-axis)



$x^2 = 16 - y^2 \quad x = \pm\sqrt{16-y^2}$



$V = \int_{-4}^{4} 12(16 - y^2) \, dy$

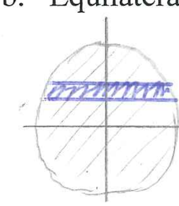
$\boxed{V = 1024 \text{ units}^3}$

$base = \sqrt{16-y^2} - (-\sqrt{16-y^2})$

$= 2\sqrt{16-y^2}$

$Area = 2\sqrt{16-y^2} \cdot 6\sqrt{16-y^2}$

b. Equilateral triangles (parallel to the x-axis)



$A = \frac{\sqrt{3}}{4}(base)^2$

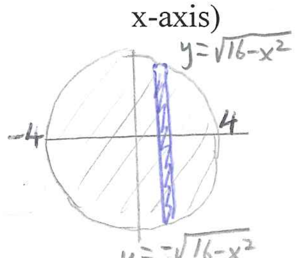
$V = \frac{\sqrt{3}}{4} \int_{-4}^{4} [2\sqrt{16-y^2}]^2 \, dy$

$base = \sqrt{16-y^2} - (-\sqrt{16-y^2})$

$= 2\sqrt{16-y^2}$

$V = \frac{\sqrt{3}}{4}\left(\frac{1024}{3}\right) = \boxed{\frac{256\sqrt{3}}{3} \text{ units}^3}$

c. right isosceles triangles whose hypotenuse lie on the base of the solid. (perpendicular to the x-axis)



$A = \frac{1}{4}(hypotenuse)^2$

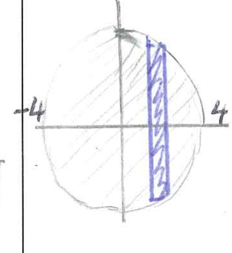
$V = \frac{1}{4} \int_{-4}^{4} (2\sqrt{16-x^2})^2 \, dx$

$V = \frac{1}{4}\left(\frac{1024}{3}\right) = \boxed{\frac{256}{3} \text{ units}^3}$

$base = \sqrt{16-x^2} - (-\sqrt{16-x^2}) = 2\sqrt{16-x^2}$

d. Semicircles (perpendicular to the x-axis)

$A = \frac{\pi}{8}(diameter)^2$



$V = \frac{\pi}{8} \int_{-4}^{4} (2\sqrt{16-x^2})^2 \, dx$

$V = \frac{\pi}{8}\left(\frac{1024}{3}\right)$

$base = \sqrt{16-x^2} - (-\sqrt{16-x^2})$

$= 2\sqrt{16-x^2}$

$\boxed{V = \frac{128}{3}\pi \text{ units}^3}$