## 7.2abc Volumes with Disc, Washer, Cross Section Problems Worksheet 2

Disc Method:

$$V = \pi \int_{x_1}^{x_2} [R(x)]^2 \, dx \quad \text{or} \quad V = \pi \int_{y_1}^{y_2} [R(y)]^2 \, dy$$

Washer Method:

$$V = \pi \int_{x_1}^{x_2} [R(x)^2 - r(x)^2] dx \quad \text{or} \quad V = \pi \int_{y_1}^{y_2} [R(y)^2 - r(y)^2] dy$$

1. Find the volume of the solid created by rotating the region bounded by $y = 2x - 4$, $y = 0$, and $x = 3$ about the x-axis.

2. Find the volume of the solid created by rotating the region bounded by $y = 2x^2 - 3$, $y = -3$, and $x = 2$ about the line $x = -1$.

3. Find the volume of the solid created by rotating the region bounded by $y = 2x^2 - 3$, $y = -3$, and $x = 2$ about the line $y = 7$.

4. Find the volume of the solid whose base is bounded by $y = x + 1$ and $y = x^2 - 1$, the cross sections parallel to the y-axis are:

a) Rectangles of height 5

b) Equilateral triangles

5. Find the volume of the solid whose base is bounded by $x = 0$, $y = 0$, and $y = \frac{4}{3}x - 3$ the cross sections perpendicular to the y-axis are:

a) Semicircles

b) Isosceles right triangles with hypotenuse on the base

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Key

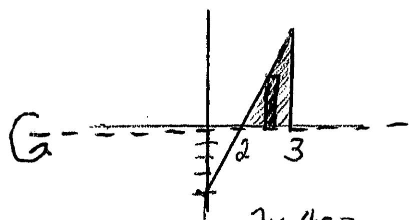
**Disc Method:**

$$V = \pi \int_{x_1}^{x_2} [R(x)]^2\,dx \quad \text{or} \quad V = \pi \int_{y_1}^{y_2} [R(y)]^2\,dy$$

**Washer Method:**

$$V = \pi \int_{x_1}^{x_2} [R(x)^2 - r(x)^2]\,dx \quad \text{or} \quad V = \pi \int_{y_1}^{y_2} [R(y)^2 - r(y)^2]\,dy$$

1. Find the volume of the solid created by rotating the region bounded by y = 2x – 4, y = 0, and x = 3 about the x-axis. *Disc Method, Top/Bottom*




$R(x) = 2x - 4 - 0 = 2x - 4$

$V = \pi \int R(x)^2\,dx$

$2x - 4 = 0$

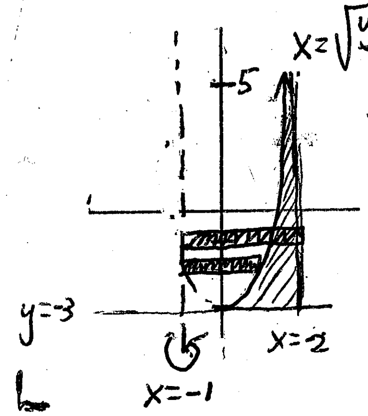
$2x = 4$

$x = 2$

$V = \pi \int_2^3 (2x-4)^2\,dx = \boxed{\frac{4\pi}{3} \text{ units}^3}$

$$\pi \int_2^3 (2x-4)^2\,dx = \frac{4\pi}{3}$$

2. Find the volume of the solid created by rotating the region bounded by y = 2x$^2$ - 3, y = -3, and x = 2 about the line x = -1. *Washer Method, Right/Left*




$x = \sqrt{\frac{y+3}{2}}$

$R(y) = 2 - (-1) = 3$

$r(y) = \sqrt{\frac{y+3}{2}} - (-1) = \sqrt{\frac{y+3}{2}} + 1$

$2x^2 = 3 + y$

$x^2 = \frac{3+y}{2}$

$x = \sqrt{\frac{3+y}{2}}$

* right bound

$\sqrt{\frac{3+y}{2}} = 2$

$\frac{3+y}{2} = 4$

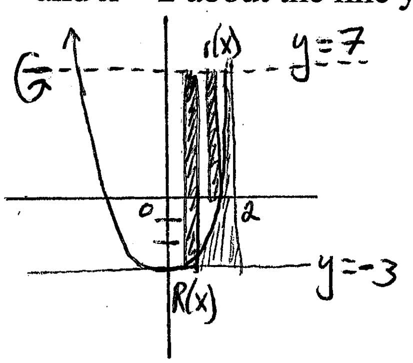
$3 + y = 8$

$y = 5$

$V = \pi \int_{-3}^{5} 3^2 - \left[\sqrt{\frac{y+3}{2}} + 1\right]^2 dy$

$\boxed{V = \frac{80\pi}{3} \text{ units}^3}$

3. Find the volume of the solid created by rotating the region bounded by y = 2x$^2$ – 3, y = -3, and x = 2 about the line y = 7. *Washer Method, Top/Bottom*




$R(x) = 7 - (-3) = 10$

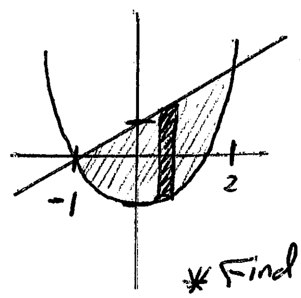
$r(x) = 7 - (2x^2 - 3) = 10 - 2x^2$

$V = \pi \int_0^2 10^2 - [10 - 2x^2]^2\,dx$

$\boxed{V = \frac{1216}{15}\pi \text{ units}^3}$

$$\pi \int_0^2 \left[10^2 - (7 - (2x^2 - 3))^2\right] dx = \frac{1216\pi}{15}$$

4. Find the volume of the solid whose base is bounded by $y = x + 1$ and $y = x^2 - 1$, the cross sections parallel to the y-axis are: Top/Bottom




$$Base = x+1-(x^2-1) = x+1-x^2+1 = -x^2+x+2$$

*Find intersections/bounds: $x^2-1 = x+1$ | $(x-2)(x+1)=0$
$x^2-x-2=0$ | $x = 2, -1$

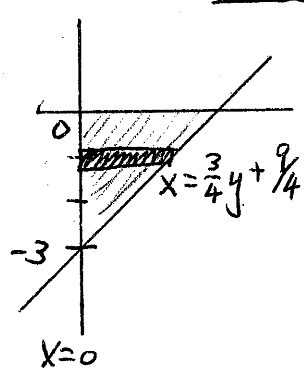
a) Rectangles of height 5

$$Area = (base)(height)$$
$$Area = (-x^2+x+2)(5)$$
$$V = \int_{-1}^{2} 5(-x^2+x+2)\,dx$$

b) Equilateral triangles

$$A = \frac{\sqrt{3}}{4}(base)^2$$
$$V = \frac{\sqrt{3}}{4}\int_{-1}^{2} (-x^2+x+2)\,dx$$

5. Find the volume of the solid whose base is bounded by $x = 0$, $y = 0$, and $y = \frac{4}{3}x - 3$ the cross sections perpendicular to the y-axis are:




$$Base = \frac{3}{4}y + \frac{9}{4} - 0$$
$$= \frac{3}{4}y + 9/4$$

$$\frac{4}{3}x = y+3$$
$$x = \frac{3}{4}(y+3)$$
$$x = \frac{3}{4}y + \frac{9}{4}$$

a) Semicircles

$$A = \frac{\pi}{8}[base]^2 = \frac{\pi}{8}\left[\frac{3}{4}y + 9/4\right]^2$$
$$V = \frac{\pi}{8}\int_{-3}^{0} \left[\frac{3}{4}y + \frac{9}{4}\right]^2 dy$$

b) Isosceles right triangles with hypotenuse on the base

$$V = \frac{1}{4}[base]^2 = \frac{1}{4}\left[\frac{3}{4}y + 9/4\right]^2$$
$$V = \frac{1}{4}\int_{-3}^{0} \left[\frac{3}{4}y + 9/4\right]^2 dy$$