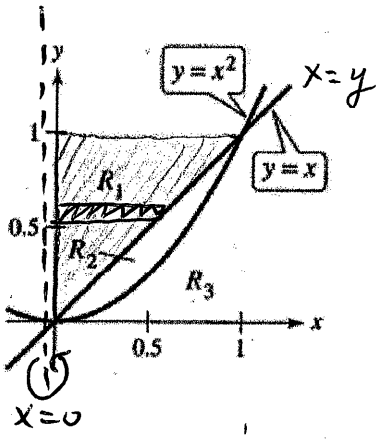
7.2

7.2 Additional Practice HW Problems (Disc & Washer Method) 2017 p. 454 # 41-48

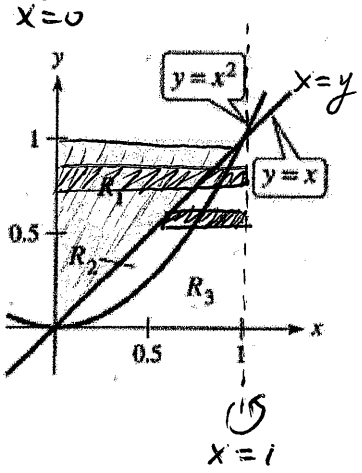
Finding the Volume of a Solid **In Exercises 41–48, find the volume generated by rotating the given region about the specified line.**



41. R_1 about $x = 0$ Disc Method, right-left

$R(y) = y - 0$

$$V = \pi \int_0^1 (y)^2 dy = \frac{y^3}{3}\Big]_0^1 = \left(\frac{1}{3} - 0\right)\pi = \boxed{\frac{\pi}{3}}$$

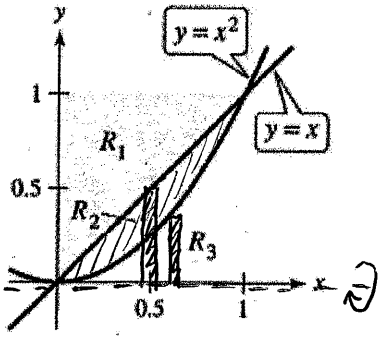


42. R_1 about $x = 1$ (washer Method, Right-Left)

$R(y) = 1 - 0 = 1 \quad r(y) = 1 - y$

$$V = \pi \int_0^1 (1)^2 - (1-y)^2 dy = \pi \int 1 - (1 - 2y + y^2) dy = \pi \int 2y - y^2 dy$$

$$\frac{2y^2}{2} - \frac{y^3}{3} = y^2 - \frac{y^3}{3}\Big]_0^1 = 1 - \frac{1}{3} = \boxed{\frac{2}{3}\pi}$$

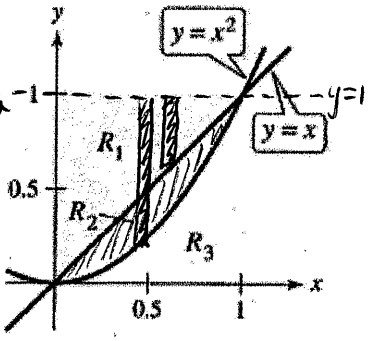


43. R_2 about $y = 0$ (washer Method, Top-Bottom)

$R(x) = x - 0 \quad r(x) = x^2 - 0$

$$V = \pi \int_0^1 [x]^2 - [x^2]^2 dx = \pi \int_0^1 x^2 - x^4 dx$$

$$= \frac{x^3}{3} - \frac{x^5}{5}\Big]_0^1 = \frac{1}{3} - \frac{1}{5} - (0 - 0) = \frac{5}{15} - \frac{3}{15} = \boxed{\frac{2\pi}{15}}$$



44. R_2 about $y = 1$ (washer Method, Top-Bottom)

$R(x) = 1 - x^2 \quad r(x) = 1 - x$

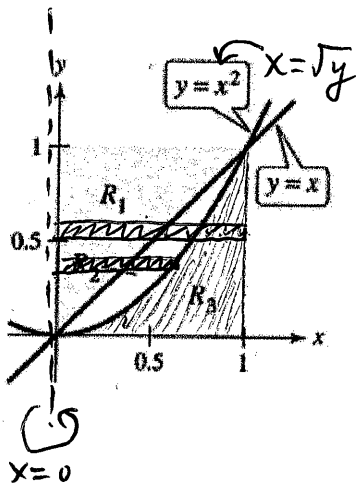
$$V = \pi \int_0^1 [1 - x^2]^2 - [1 - x]^2 dx$$

$$\pi \int 1 - 2x^2 + x^4 - (1 - 2x + x^2)$$

$$\int 2x - 3x^2 + x^4 dx$$

$$\frac{2x^2}{2} - \frac{3x^3}{3} + \frac{x^5}{5}\Big]_0^1$$

$$1 - 1 + \frac{1}{5} - (0 - 0 + 0) = \frac{1}{5}\pi = \boxed{\frac{\pi}{5}}$$

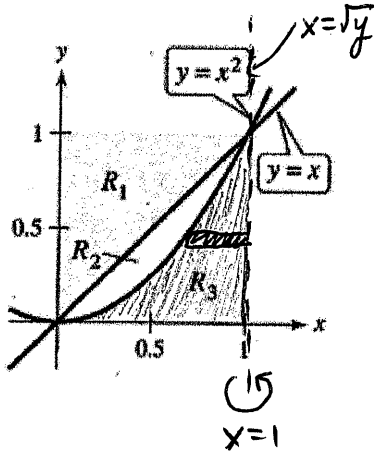


45. R_3 about $x = 0$ (Washer Method, Right-Left)

$R(y) = 1 - 0 \quad r(y) = \sqrt{y} - 0$

$$V = \pi \int_0^1 (1)^2 - (\sqrt{y})^2 \, dy = \pi \int 1 - y \, dy$$

$$y - \frac{y^2}{2}\Big]_0^1 = 1 - \frac{1}{2} - (0-0) = \frac{1}{2}\pi = \boxed{\frac{\pi}{2}}$$



46. R_3 about $x = 1$ (Disc Method, Right-Left)

$R(y) = 1 - \sqrt{y}$

$$V = \pi \int_0^1 [1 - \sqrt{y}]^2 \, dy$$

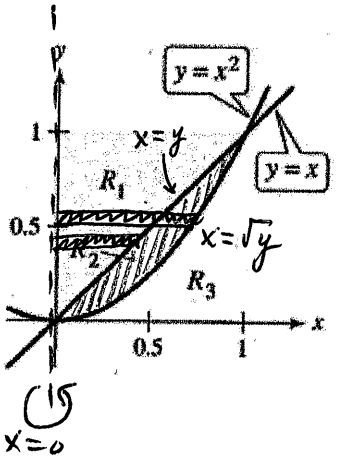
$$V = \pi \int 1 - 2\sqrt{y} + y \, dy$$

$$= \int 1 - 2y^{1/2} + y \, dy$$

$$y - \frac{2y^{3/2}}{3/2} + \frac{y^2}{2}$$

$$y - \frac{4}{3}y^{3/2} + \frac{1}{2}y^2\Big]_0^1 = 1 - \frac{4}{3}(1) + \frac{1}{2} - (0-0+0)$$

$$\frac{6}{6} - \frac{8}{3} + \frac{3}{6} = \frac{1}{6}\pi = \boxed{\frac{\pi}{6}}$$



47. R_2 about $x = 0$ (Washer Method, Right-Left)

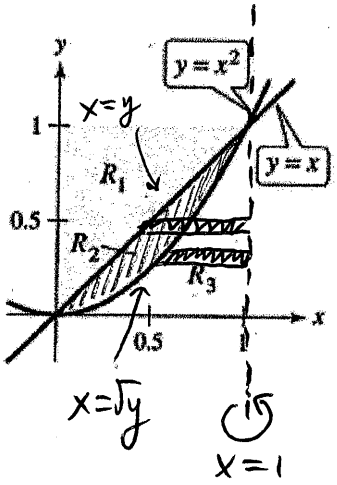
$R(y) = \sqrt{y} - 0 \quad r(y) = y - 0$

$$V = \pi \int_0^1 [\sqrt{y}]^2 - [y]^2 \, dy$$

$$V = \pi \int y - y^2 \, dy$$

$$V = \pi \cdot \frac{y^2}{2} - \frac{y^3}{3}\Big]_0^1 = \frac{1}{2} - \frac{1}{3} - (0-0)$$

$$\frac{3}{6} - \frac{2}{6} = \frac{1}{6}\pi = \boxed{\frac{\pi}{6}}$$



48. R_2 about $x = 1$ (Washer Method, Right-Left)

$R(y) = 1 - y \quad r(y) = 1 - \sqrt{y}$

$$V = \pi \int_0^1 [1-y]^2 - [1-\sqrt{y}]^2 \, dy$$

$$V = \pi \int 1 - 2y + y^2 - [1 - 2\sqrt{y} + y] \, dy$$

$$V = \pi \int 1 - 2y + y^2 - 1 + 2\sqrt{y} - y$$

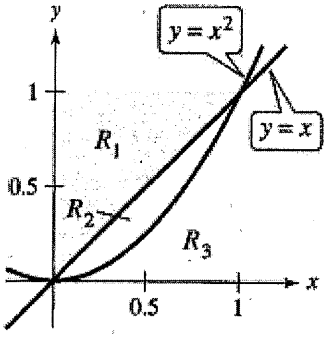
$$V = \pi \int 2\sqrt{y} - 3y + y^2 \, dy$$

$$\frac{2y^{3/2}}{3/2} - \frac{3y^2}{2} + \frac{y^3}{3}\Big]_0^1$$

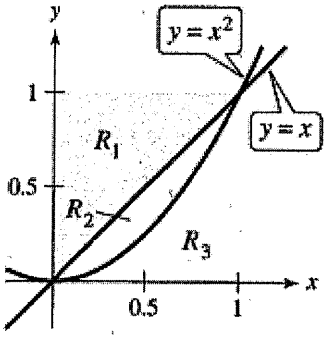
$$\frac{4}{3}(1) - \frac{3}{2}(1) + \frac{1}{3}(1) - (0-0-0)$$

$$\frac{8}{6} - \frac{9}{6} + \frac{2}{6} = \frac{1}{6}\pi = \boxed{\frac{\pi}{6}}$$

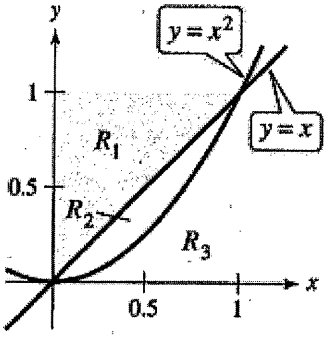
Finding the Volume of a Solid **In Exercises 41–48, find the volume generated by rotating the given region about the specified line.**



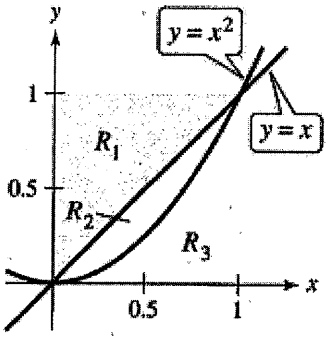
41. R_1 about $x = 0$



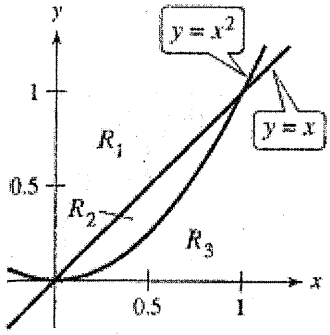
42. R_1 about $x = 1$



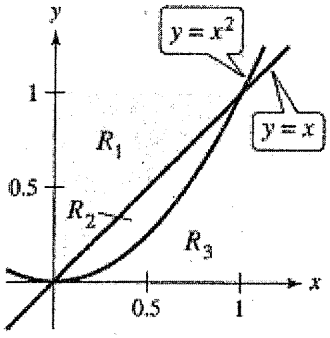
43. R_2 about $y = 0$



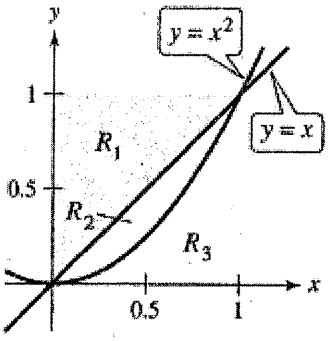
44. R_2 about $y = 1$



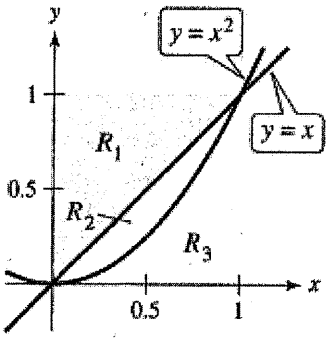
45. R_3 about $x = 0$



46. R_3 about $x = 1$



47. R_2 about $x = 0$



48. R_2 about $x = 1$