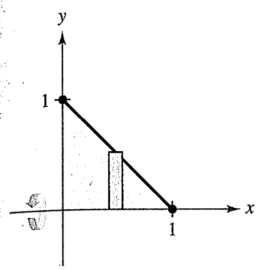
7.2 Exercises

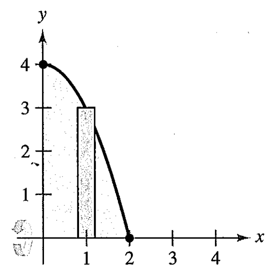
See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Finding the Volume of a Solid In Exercises 1–6, set up and evaluate the integral that gives the volume of the solid formed by revolving the region about the x-axis.

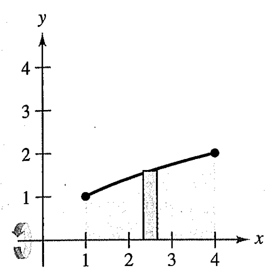
1. $y = -x + 1$



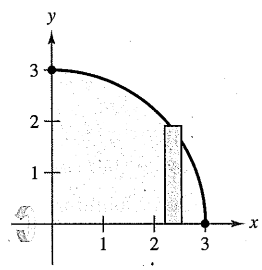
2. $y = 4 - x^2$



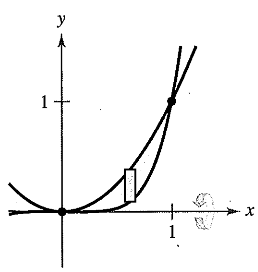
3. $y = \sqrt{x}$



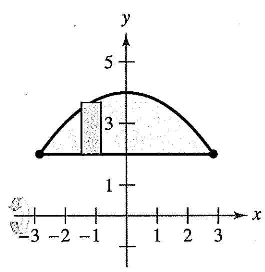
4. $y = \sqrt{9 - x^2}$



5. $y = x^2, \quad y = x^5$

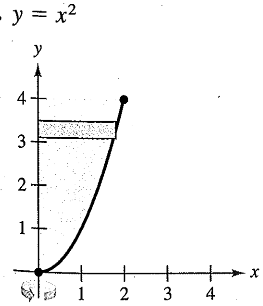


6. $y = 2, \quad y = 4 - \dfrac{x^2}{4}$

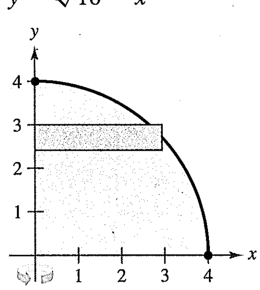


Finding the Volume of a Solid In Exercises 7–10, set up and evaluate the integral that gives the volume of the solid formed by revolving the region about the y-axis.

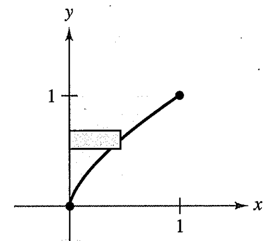
7. $y = x^2$



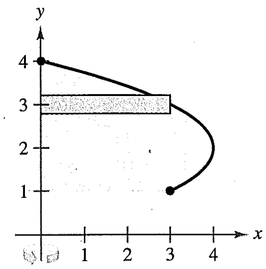
8. $y = \sqrt{16 - x^2}$



9. $y = x^{2/3}$



10. $x = -y^2 + 4y$



Finding the Volume of a Solid In Exercises 11–14, find the volumes of the solids generated by revolving the region bounded by the graphs of the equations about the given lines.

11. $y = \sqrt{x}, \quad y = 0, \quad x = 3$

(a) the x-axis (b) the y-axis

(c) the line $x = 3$ (d) the line $x = 6$

12. $y = 2x^2, \quad y = 0, \quad x = 2$

(a) the y-axis (b) the x-axis

(c) the line $y = 8$ (d) the line $x = 2$

13. $y = x^2, \quad y = 4x - x^2$

(a) the x-axis (b) the line $y = 6$

14. $y = 4 + 2x - x^2, \quad y = 4 - x$

(a) the x-axis (b) the line $y = 1$

Finding the Volume of a Solid In Exercises 15–18, find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the line $y = 4$.

15. $y = x, \quad y = 3, \quad x = 0$

16. $y = \frac{1}{2}x^3, \quad y = 4, \quad x = 0$

17. $y = \dfrac{3}{1 + x}, \quad y = 0, \quad x = 0, \quad x = 3$

18. $y = \sec x, \quad y = 0, \quad 0 \le x \le \dfrac{\pi}{3}$

Finding the Volume of a Solid In Exercises 19–22, find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the line $x = 5$.

19. $y = x, \quad y = 0, \quad y = 4, \quad x = 5$

20. $y = 3 - x, \quad y = 0, \quad y = 2, \quad x = 0$

21. $x = y^2, \quad x = 4$

22. $xy = 3, \quad y = 1, \quad y = 4, \quad x = 5$

Finding the Volume of a Solid In Exercises 23–30, find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the x-axis.

23. $y = \dfrac{1}{\sqrt{x + 1}}, \quad y = 0, \quad x = 0, \quad x = 4$

24. $y = x\sqrt{4 - x^2}, \quad y = 0$

25. $y = \frac{1}{x}, \quad y = 0, \quad x = 1, \quad x = 3$

26. $y = \frac{2}{x+1}, \quad y = 0, \quad x = 0, \quad x = 6$

27. $y = e^{-x}, \quad y = 0, \quad x = 0, \quad y = 1$

28. $y = e^{x/4}, \quad y = 0, \quad x = 0, \quad x = 6$

29. $y = x^2 + 1, \quad y = -x^2 + 2x + 5, \quad x = 0, \quad x = 3$

30. $y = \sqrt{x}, \quad y = -\frac{1}{2}x + 4, \quad x = 0, \quad x = 8$

Finding the Volume of a Solid **In Exercises 31 and 32, find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the y-axis.**

31. $y = 3(2 - x), \quad y = 0, \quad x = 0$

32. $y = 9 - x^2, \quad y = 0, \quad x = 2, \quad x = 3$

Finding the Volume of a Solid **In Exercises 33–36, find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the x-axis. Verify your results using the integration capabilities of a graphing utility.**

33. $y = \sin x, \quad y = 0, \quad x = 0, \quad x = \pi$

34. $y = \cos 2x, \quad y = 0, \quad x = 0, \quad x = \frac{\pi}{4}$

35. $y = e^{x-1}, \quad y = 0, \quad x = 1, \quad x = 2$

36. $y = e^{x/2} + e^{-x/2}, \quad y = 0, \quad x = -1, \quad x = 2$

Finding the Volume of a Solid **In Exercises 37–40, use the integration capabilities of a graphing utility to approximate the volume of the solid generated by revolving the region bounded by the graphs of the equations about the x-axis.**

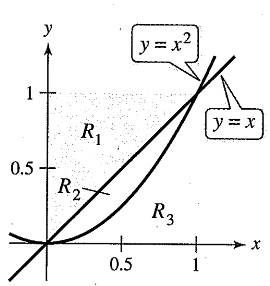
37. $y = e^{-x^2}, \quad y = 0, \quad x = 0, \quad x = 2$

38. $y = \ln x, \quad y = 0, \quad x = 1, \quad x = 3$

39. $y = 2\arctan(0.2x), \quad y = 0, \quad x = 0, \quad x = 5$

40. $y = \sqrt{2x}, \quad y = x^2$

Finding the Volume of a Solid **In Exercises 41–48, find the volume generated by rotating the given region about the specified line.**



41. R_1 about $x = 0$

42. R_1 about $x = 1$

43. R_2 about $y = 0$

44. R_2 about $y = 1$

45. R_3 about $x = 0$

46. R_3 about $x = 1$

47. R_2 about $x = 0$

48. R_2 about $x = 1$

WRITING ABOUT CONCEPTS

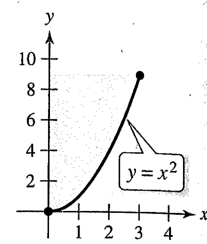
Describing a Solid **In Exercises 49 and 50, the integral represents the volume of a solid. Describe the solid.**

49. $\pi \int_0^{\pi/2} \sin^2 x \, dx$

50. $\pi \int_2^4 y^4 \, dy$

51. **Comparing Volumes** A region bounded by the parabola $y = 4x - x^2$ and the x-axis is revolved about the x-axis. A second region bounded by the parabola $y = 4 - x^2$ and the x-axis is revolved about the x-axis. Without integrating, how do the volumes of the two solids compare? Explain.

52. **Comparing Volumes** The region in the figure is revolved about the indicated axes and line. Order the volumes of the resulting solids from least to greatest. Explain your reasoning.



(a) x-axis

(b) y-axis

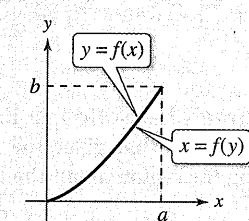
(c) $x = 3$

53. **Analyzing Statements** Discuss the validity of the following statements.

(a) For a solid formed by rotating the region under a graph about the x-axis, the cross sections perpendicular to the x-axis are circular disks.

(b) For a solid formed by rotating the region between two graphs about the x-axis, the cross sections perpendicular to the x-axis are circular disks.

54. **HOW DO YOU SEE IT?** Use the graph to match the integral for the volume with the axis of rotation.



(a) $V = \pi \int_0^b (a^2 - [f(y)]^2) \, dy$ (i) x-axis

(b) $V = \pi \int_0^a (b^2 - [b - f(x)]^2) \, dx$ (ii) y-axis

(c) $V = \pi \int_0^a [f(x)]^2 \, dx$ (iii) $x = a$

(d) $V = \pi \int_0^b [a - f(y)]^2 \, dy$ (iv) $y = b$

Dividing a Solid **In Exercises 55 and 56, consider the solid formed by revolving the region bounded by $y = \sqrt{x}$, $y = 0$, and $x = 4$ about the x-axis.**

55. Find the value of x in the interval $[0, 4]$ that divides the solid into two parts of equal volume.

56. Find the values of x in the interval $[0, 4]$ that divide the solid into three parts of equal volume.

57. **Manufacturing** A manufacturer drills a hole through the center of a metal sphere of radius R. The hole has a radius r. Find the volume of the resulting ring.

58. **Manufacturing** For the metal sphere in Exercise 57, let $R = 6$. What value of r will produce a ring whose volume is exactly half the volume of the sphere?

59. **Volume of a Cone** Use the disk method to verify that the volume of a right circular cone is $\frac{1}{3}\pi r^2 h$, where r is the radius of the base and h is the height.

60. **Volume of a Sphere** Use the disk method to verify that the volume of a sphere is $\frac{4}{3}\pi r^3$, where r is the radius.

61. **Using a Cone** A cone of height H with a base of radius r is cut by a plane parallel to and h units above the base, where $h < H$. Find the volume of the solid (frustum of a cone) below the plane.

62. **Using a Sphere** A sphere of radius r is cut by a plane h units above the equator, where $h < r$. Find the volume of the solid (spherical segment) above the plane.

63. **Volume of a Fuel Tank** A tank on the wing of a jet aircraft is formed by revolving the region bounded by the graph of $y = \frac{1}{8}x^2\sqrt{2 - x}$ and the x-axis $(0 \le x \le 2)$ about the x-axis, where x and y are measured in meters. Use a graphing utility to graph the function and find the volume of the tank.

64. **Volume of a Lab Glass** A glass container can be modeled by revolving the graph of

$$y = \begin{cases} \sqrt{0.1x^3 - 2.2x^2 + 10.9x + 22.2}, & 0 \le x \le 11.5 \\ 2.95, & 11.5 < x \le 15 \end{cases}$$

about the x-axis, where x and y are measured in centimeters. Use a graphing utility to graph the function and find the volume of the container.

65. **Finding Volumes of a Solid** Find the volumes of the solids (see figures) generated if the upper half of the ellipse $9x^2 + 25y^2 = 225$ is revolved about (a) the x-axis to form a prolate spheroid (shaped like a football), and (b) the y-axis to form an oblate spheroid (shaped like half of a candy).

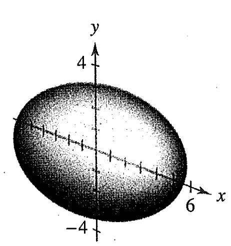


Figure for 65(a)

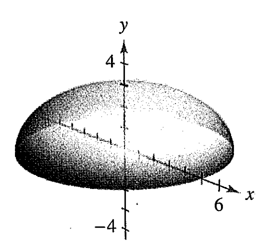
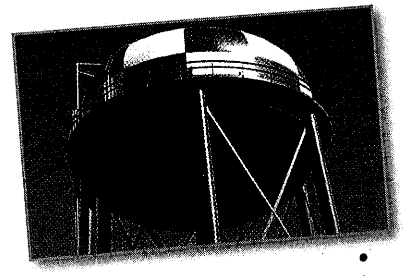


Figure for 65(b)

66. **Water Tower**

A tank on a water tower is a sphere of radius 50 feet. Determine the depths of the water when the tank is filled to one-fourth and three-fourths of its total capacity. (*Note:* Use the *zero* or *root* feature of a graphing utility after evaluating the definite integral.)



67. **Minimum Volume** The arc of $y = 4 - (x^2/4)$ on the interval $[0, 4]$ is revolved about the line $y = b$ (see figure).

(a) Find the volume of the resulting solid as a function of b.

(b) Use a graphing utility to graph the function in part (a), and use the graph to approximate the value of b that minimizes the volume of the solid.

(c) Use calculus to find the value of b that minimizes the volume of the solid, and compare the result with the answer to part (b).

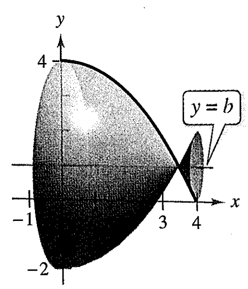


Figure for 67

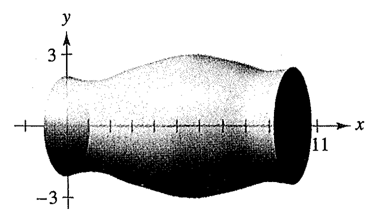


Figure for 68

68. **Modeling Data** A draftsman is asked to determine the amount of material required to produce a machine part (see figure). The diameters d of the part at equally spaced points x are listed in the table. The measurements are listed in centimeters.

x	0	1	2	3	4	5
d	4.2	3.8	4.2	4.7	5.2	5.7

x	6	7	8	9	10
d	5.8	5.4	4.9	4.4	4.6

(a) Use these data with Simpson's Rule to approximate the volume of the part.

(b) Use the regression capabilities of a graphing utility to find a fourth-degree polynomial through the points representing the radius of the solid. Plot the data and graph the model.

(c) Use a graphing utility to approximate the definite integral yielding the volume of the part. Compare the result with the answer to part (a).

69. Think About It Match each integral with the solid whose volume it represents, and give the dimensions of each solid.

(a) Right circular cylinder (b) Ellipsoid

(c) Sphere (d) Right circular cone (e) Torus

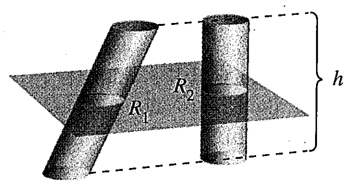
(i) $\pi \int_0^h \left(\frac{rx}{h}\right)^2 dx$ (ii) $\pi \int_0^h r^2\, dx$

(iii) $\pi \int_{-r}^{r} \left(\sqrt{r^2 - x^2}\right)^2 dx$

(iv) $\pi \int_{-b}^{b} \left(a\sqrt{1 - \frac{x^2}{b^2}}\right)^2 dx$

(v) $\pi \int_{-r}^{r} \left[\left(R + \sqrt{r^2 - x^2}\right)^2 - \left(R - \sqrt{r^2 - x^2}\right)^2\right] dx$

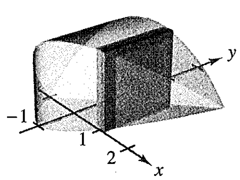
70. Cavalieri's Theorem Prove that if two solids have equal altitudes and all plane sections parallel to their bases and at equal distances from their bases have equal areas, then the solids have the same volume (see figure).



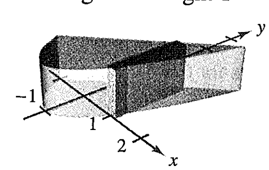
Area of R_1 = area of R_2

71. Using Cross Sections Find the volumes of the solids whose bases are bounded by the graphs of $y = x + 1$ and $y = x^2 - 1$, with the indicated cross sections taken perpendicular to the x-axis.

(a) Squares

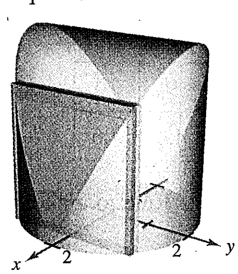


(b) Rectangles of height 1

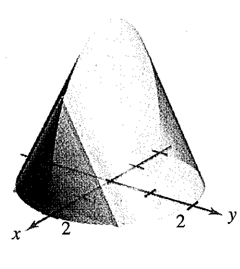


72. Using Cross Sections Find the volumes of the solids whose bases are bounded by the circle $x^2 + y^2 = 4$, with the indicated cross sections taken perpendicular to the x-axis.

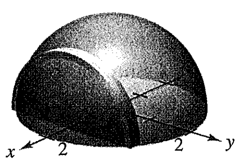
(a) Squares



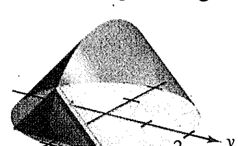
(b) Equilateral triangles



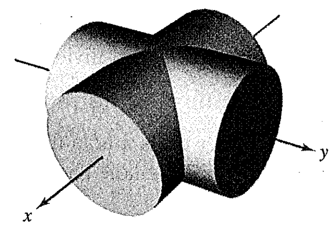
(c) Semicircles



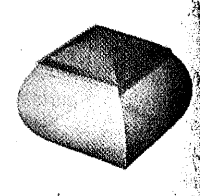
(d) Isosceles right triangles



73. Using Cross Sections Find the volume of the solid of intersection (the solid common to both) of the two right circular cylinders of radius r whose axes meet at right angles (see figure).



Two intersecting cylinders



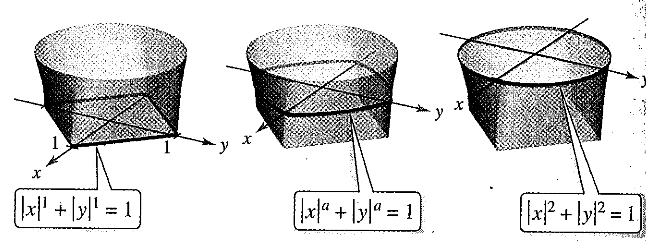
Solid of intersection

■ **FOR FURTHER INFORMATION** For more information on this problem, see the article "Estimating the Volumes of Solid Figures with Curved Surfaces" by Donald Cohen in *Mathematics Teacher*. To view this article, go to *MathArticles.com*.

74. Using Cross Sections The solid shown in the figure has cross sections bounded by the graph of $|x|^a + |y|^a = 1$, where $1 \le a \le 2$.

(a) Describe the cross section when $a = 1$ and $a = 2$.

(b) Describe a procedure for approximating the volume of the solid.



75. Volume of a Wedge Two planes cut a right circular cylinder to form a wedge. One plane is perpendicular to the axis of the cylinder and the second makes an angle of θ degrees with the first (see figure).

(a) Find the volume of the wedge if $\theta = 45°$.

(b) Find the volume of the wedge for an arbitrary angle θ. Assuming that the cylinder has sufficient length, how does the volume of the wedge change as θ increases from 0° to 90°?

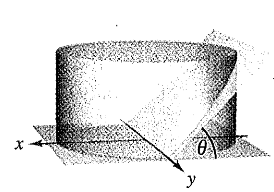


Figure for 75

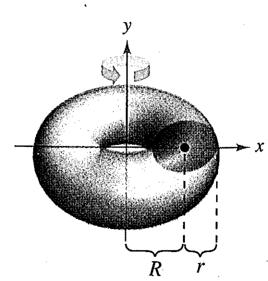


Figure for 76

76. Volume of a Torus

(a) Show that the volume of the torus shown in the figure is given by the integral $8\pi R \int_0^r \sqrt{r^2 - y^2}\, dy$, where $R > r > 0$.

(b) Find the volume of the torus.