**Washer Method Practice FRQ Problem (Non-Calculator)**

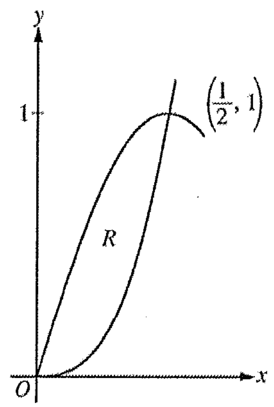
Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, p as shown in the figure above.

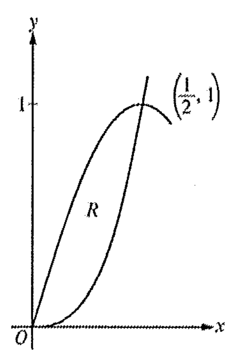
(a) Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.

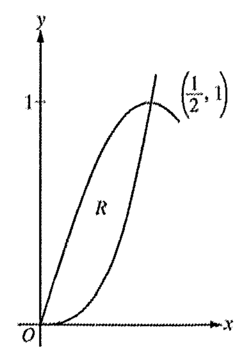
(b) Find the area of R.

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line y = 1.

(d) Write, but do not evaluate, an integral expression for the volume of solid generated when R is rotated about the vertical line x = -3



**Washer Method Practice Problem** (Non-Calculator)

Key

Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$ , p as shown in the figure above.

) Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.

(b) Find the area of R.

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line y = 1.

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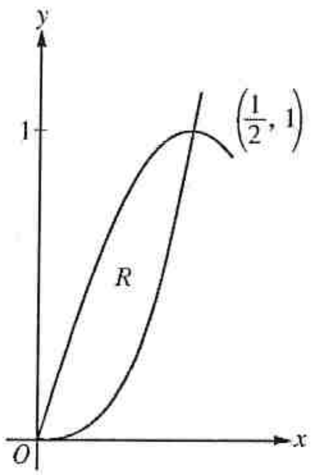



a) Equation of tangent line: $y - y_1 = m(x - x_1)$

point: $f(\tfrac{1}{2}) = 8\left(\tfrac{1}{2}\right)^3 = 8\left(\tfrac{1}{8}\right) = 1$

slope: $f'(x) = 24x^2$

$f'(\tfrac{1}{2}) = 24\left(\tfrac{1}{2}\right)^2 = \frac{24}{4} = 6$

point: $(\tfrac{1}{2}, 1)$

slope: $m = 6$

$$\boxed{y - 1 = 6(x - \tfrac{1}{2})}$$

b) Area: $\int_{x_1}^{x_2} \text{Top} - \text{Bottom}\, dx \rightarrow \int_0^{1/2} \sin(\pi x) - 8x^3\, dx$

u-sub

$\int \sin(\pi x)\, dx$

$u = \pi x$

$\frac{du}{dx} = \pi$

$dx = \frac{du}{\pi}$

$\int \sin(u) \frac{du}{\pi}$

$\frac{1}{\pi}\int \sin u\, du$

$\frac{1}{\pi}(-\cos u)$

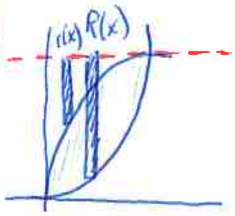
$-\frac{1}{\pi}\cos(\pi x)$

$-\frac{1}{\pi}\cos(\pi x) - 8 \cdot \frac{x^4}{4} \Big]_0^{1/2}$

$-\frac{1}{\pi}\cos(\tfrac{\pi}{2}) - 2\left(\tfrac{1}{2}\right)^4 - \left(-\frac{1}{\pi}\cos(0) - 2(0)^4\right)$

$-\frac{1}{\pi}(0) - \frac{1}{8} + \frac{1}{\pi}(1) = \boxed{-\frac{1}{8} + \frac{1}{\pi}}$

c)




AOR $y = 1$

* washer method
* Top−Bottom

$R(x) = 1 - 8x^3$

$r(x) = 1 - \sin(\pi x)$

$$\boxed{V = \pi \int_0^{1/2} \left[1 - 8x^3\right]^2 - \left[1 - \sin(\pi x)\right]^2 dx}$$

d)
* washer
* Right−Left

AOR $x = -3$

$R(y) = \left(\frac{y}{8}\right)^{1/3} - (-3)$

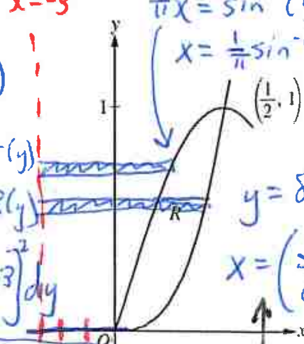
$r(y) = \frac{1}{\pi}\sin^{-1}(y) - (-3)$

$V = \pi \int_0^1 \left[\left(\frac{y}{8}\right)^{1/3} + 3\right]^2 - \left[\frac{1}{\pi}\sin^{-1}(y) + 3\right]^2 dy$

$y = \sin(\pi x)$

$\pi x = \sin^{-1}(y)$

$x = \frac{1}{\pi}\sin^{-1}(y)$




$y = 8x^3$

$x = \left(\frac{y}{8}\right)^{1/3}$

$x = \left(\frac{y}{8}\right)^{1/3}$

$$\boxed{V = \pi \int_0^1 \left[\left(\frac{y}{8}\right)^{1/3} + 3\right]^2 - \left[\frac{1}{\pi}\sin^{-1}(y) + 3\right]^2 dy}$$